Chapter 15
Quantum Gravity: A Heretical Vision

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Abstract The goal of this work is to contribute to the development of a background-independent, non-perturbative approach to quantization of the gravitational field based on the conformal and projective structures of space-time. But first I attempt to dissipate some mystifications about the meaning of quantization, and foster an ecumenical, non-competitive approach to the problem of quantum gravity (QG), stressing the search for relations between different approaches in any overlapping regions of validity. Then I discuss some topics for further research based on the approach we call unimodular conformal and projective relativity (UCPR).

15.1 Only Theories

Perhaps it will be helpful if I recall a tripartite classification of theories that I proposed many years ago. The three categories are:

(1) *Perfectly perfect theories*: The range of these theories includes the entire universe: There is nothing In the world that these theories do not purport to explain, and they correctly explain all these phenomena. Today we call such theories TOEs— Theories of Everything.
(2) *Perfect theories*: These are more modest. They correctly explain all phenomena within their range of application, but there are phenomena that they do not purport to explain.
(3) Then there are just plain *Theories*: There are phenomena that they do not purport to explain, and there are phenomena that they do purport to explain, but do not explain correctly.

Both the history of science and my own experience have taught me that all we have now, ever have had in the past, or can hope to have in the future are just plain theories. This tale had two morals:

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(1) Every theory has its range of validity and its limits; to understand a theory better we must find its limits. In this sense, we understand Newtonian gravity better than general relativity (GR).
(2) There will be theories with over-lapping ranges of validity; to understand each of these theories better we must explore the relations between them in the overlap regions. Some examples will be given in the next section.

15.2 What Is Quantum Theory? What Quantization Is and Is Not

A certain mystique surrounds the words "quantum theory." The very words conjure up visions of probing the depths of reality, exploring the paradoxical properties of the exotic building blocks of the universe: fundamental particles, dark matter, dark energy—dark thoughts.

> But the scope of the quantum mechanical formalism is by no means limited to such (presumed) fundamental particles. There is no restriction of principle on its application to any physical system. One could apply the formalism to sewing machines if there were any reason to do so! [17].

Then what is quantization? Quantization is just a way accounting for the effects of the existence of *h*, the quantum of action, on any process undergone by some system–or rather on some theoretical model of such a system. This is the case whether the system to be quantized is assumed to be "fundamental" or "composite." That is, whether the model describes some (presumed) fundamental entities, or whether it describes the collective behavior of an ensemble of such entities.

> [T]he universal quantum of action … was discovered by Max Planck in the first year of this [20th] century and came to inaugurate a whole new epoch in physics and natural philosophy. We came to understand that the ordinary laws of physics, i.e., classical mechanics and electrodynamics, are idealizations that can only be applied in the analysis of phenomena in which the action involved at every stage is so large compared to the quantum that the latter can be completely disregarded [4].

We all know examples of the quantization of fundamental systems, such as electrons, quarks, neutrinos, etc.; so I shall just remind you of some examples of *non-fundamental quanta*, such as *quasi-particles*: particle-like entities arising in certain systems of interacting particles, e.g., phonons and rotons in hydrodynamics (see, e.g., [13]); and *phenomenological field quanta*, e.g., quantized electromagnetic waves in a homogeneous, isotropic medium (see, e.g., [11]).

So, successful quantization of some classical formalism does *not necessarily* mean that one has achieved a deeper understanding of reality—or better, an understanding of a deeper level of reality. What is does mean is that one has successfully understood the effects of the quantum of action on the phenomena (Bohr's favorite word), or processes (Feynman's favorite) described by the formalism being quantized.

Having passed beyond the quantum mystique, one is free to explore how to apply quantization techniques to various formulations of a theory without the need to single one out as the unique "right" one. One might say, with Jesus: "In my Father's house are many mansions" (John 14:2); or with Chairman Mao (in his more tolerant moments): "Let a hundred flowers blossom, let a hundred schools contend."

Three Morals of This Tale:

1. *Look for relations between quantizations*: If two such quantizations at *different*levels are carried out, one may then investigate the relation between them. *Example*: ([7]) have investigated the relation between microscopic and macroscopic quantizations of the electromagnetic field in a dielectric.
   If two such quantizations at the *same* level exist, one may investigate the relation between them. *Example:* [2] studied the relation between loop quantization and the usual field quantization of the electromagnetic field: If you "thicken" the loops, the two are equivalent.
2. *Don't Go "Fundamental"*: The search for a method of quantizing space-time structures associated with the Einstein equations is distinct from the search for an underlying theory of all "fundamental" interactions.
   I see no reason why a quantum theory of gravity should not be sought within a standard interpretation of quantum mechanics (whatever one prefers). … We can consistently use the Copenhagen interpretation to describe the interaction between a macroscopic classical apparatus and a quantum-gravitational phenomenon happening, say, in a small region of (macroscopic) spacetime. The fact that the notion of spacetime breaks down at short scale within this region does not prevent us from having the region interacting with an external Copenhagen observer ([14], p. 370).
3. *Don't go "Exclusive"*: Any attempt, such as ours (see [5, 19]), to quantize the conformal and projective structures does not negate, and need not replace, attempts to quantize other space-time structures. Everything depends on the utility of the results of formal quantization in explaining some physical processes depending on the quantum of action.
   One should not look at different approaches to QG as "*either-or*" alternatives, but "*both-and*" supplements. The question to ask is not: "Which is right and which is wrong?" but: "In their regions of overlapping validity, what is the relation between these different models of quantized gravitational phenomena?".

15.3 Measurability Analysis

A physical theory consists of more than a class of mathematical models. Certain mathematical structures within these models must be singled out as corresponding to physically significant concepts. And these concepts must be in principle measurable. This is *not* operationalism: What is measurable is real. Rather, it is the opposite: What is real must be measurable by some idealized physical procedure that is consistent with the theory. This test of the physical validity of a theory is called *measurability analysis*

> Measurability analysis identifies those dynamic field variables that are susceptible to observation and measurement ("observables"), and investigates to what extent limitations inherent in their experimental determination are consistent with the uncertainties predicted by the formal theory [3].

15.4 Process is Primary, States are Secondary

I cannot put this point better than Lee Smolin has done:

> [R]elativity theory and quantum theory each …tell us– no, better, they scream at us- that our world is a history of processes. Motion and change are primary. Nothing is, except in a very approximate and temporary sense. How something is, or what its state is, is an illusion. … So to speak the language of the new physics we must learn a vocabulary in which process is more important than, and prior to, stasis [15].

Carlo Rovelli has helped us to develop that vocabulary for QG:

> The data from a local experiment (measurements, preparation, or just assumptions) must in fact refer to the state of the system on the entire boundary of a finite spacetime region. The field theoretical space …is therefore the space of surfaces Σ [a three-dimensional hypersurface bounding a finite four-dimensional spacetime region] and field configurations φ on Σ. Quantum dynamics can be expressed in terms of an [probability] amplitude $W[\Sigma, \varphi]$ [for some process].

Background dependence versus background independence:

> Notice that the dependence of $W[\Sigma, \varphi]$ on the geometry of Σ codes the spacetime position of the measuring apparatus. In fact, the relative position of the components of the apparatus is determined by their physical distance and the physical time elapsed between measurements, and these data are contained in the metric of Σ. Consider now a background independent theory. Diffeomorphism invariance implies immediately that $W[\Sigma, \varphi]$ is independent of Σ … Therefore in gravity W depends only on the boundary value of the fields. However, the fields include the gravitational field, and the gravitational field determines the spacetime geometry. Therefore the dependence of W on the fields is still sufficient to code the relative distance and time separation of the components of the measuring apparatus! ([14], p. 23).

15.5 Poisson Brackets Versus Peierls Brackets

One central method of taking into account the quantum of action is by means of introducing commutation relations between various particle or field quantities entering into the classical formalism. These commutation relations have more than a purely formal significance

> We share the point of view emphasized by Heisenberg and Bohr and Rosenfeld, that the limits of definability of a quantity within any formalism should coincide with the limits of measurability of that quantity for all conceivable (ideal) measurement procedures. For well-established theories, this criterion can be tested. For example, in spite of a serious challenge, source-free quantum electro-dynamics was shown to pass this test. In the case of quantum

> gravity, our situation is rather the opposite. In the absence of a fully accepted, rigorous theory, exploration of the limits of measurability of various quantities can serve as a tool to provide clues in the search for such a theory: If we are fairly certain of the results of our measurability analysis, the proposed theory must be fully consistent with these results ([1]).

It follows that one should replace canonical methods, based on the primacy of states, by some covariant method, based on the primacy of processes. As Bryce DeWitt emphasizes, Peierls found the way to do this:

> When expounding the fundamentals of quantum field theory physicists almost universally fail to apply the lessons that relativity theory taught them early in the twentieth century. Although they usually carry out their calculations in a covariant way, in deriving their calculational rules they seem unable to wean themselves from canonical methods and Hamiltonians, which are holdovers from the nineteenth century, and are tied to the cumbersome (3 + 1)-dimensional baggage of conjugate momenta, bigger-than-physical Hilbert spaces and constraints. One of the unfor-tunate results is that physicists, over the years, have almost totally neglected the beautiful covariant replacement for the canonical Poisson bracket that Peierls invented in 1952 ([8], Preface, p. *v*; see also Sect. 15.5, "The Peierls Bracket").

15.6 What Is Classical General Relativity?

GR is often presented as if there were only *one* primary space-time structure: the pseudo-Riemannian metric tensor g. Once one realizes that GR is based on *two* distinct space-time structures, the *chrono-geometry* (metric g) and the *inertio-gravitational field* (affine connection Γ), and the *compatibility conditions* between the two ($Dg = 0$), the question arises: What structure(s) shall we quantize and how?

Usually, it is taken for granted that all the space-time structures must be simultaneously quantized. Traditionally, one attempts to quantize the chrono-geometry, or some canonical (3 + 1) version of it, such as the first fundamental form of a Cauchy hypersurface; and introduces the inertia-gravitational field, again in canonical version as the second fundamental form of the hypersurface, disguised as the momenta conjugate to the first fundamental form (see, e.g., [20], pp. 160–170). More recently, the inverse approach has had great success in loop QG: One starts from a (3 + 1) breakup of the affine connection that makes it analogous to a Yang-Mills field, and introduces some (3 + 1) version the metric as the momenta conjugate to this connection (see, e.g., [14]).

Both approaches have one feature in common: the (3 + 1) canonical approach adopted naturally favors states over processes, leading to a number of problems. In particular, the state variables (the "positions") are primary; their time derivatives (the "momenta") are secondary.

However, there is no need to adopt a canonical approach to GR, nor to initially conflate the two structures g and Γ. From the point of view of a first-order Palatini-type variational principle, *the compatibility conditions* between the two are just one of the two sets of dynamical field equations derived from the Lagrangian, linking g and Γ, which are initially taken to be independent of each other. The other set of field equations, of course, links the trace of the affine curvature tensor, the affine Ricci tensor, to the non-gravitational sources of the inertio-gravitational field. There is a sort of electromagnetic analogy: In the first order formalism, $\boldsymbol{G}^{\mu\nu}$ and $F_{\mu\nu}$ (or A_μ) are initially independent fields, which are then made compatible by the constitutive relations [16].

Both the canonical approach and the first-order Palatini-type approach take it for granted that the compatibility conditions must be preserved exactly, whether from the start or as a result of the field equations. As we shall see, in UCPR this is no longer the case.

15.7 The Newtonian Limit, Multipole Expansion of Gravitational Radiation

The remarkable accuracy of the Newtonian approximation for the description of so many physical systems suggest that the Newtonian limit of GR might provide a convenient starting point for a discussion of quantization of the gravitational field. In the version of Newtonian theory that takes into account the equivalence principle (see [18]), the chronometry (universal time) and the geometry (Euclidean in each of the preferred frame of reference picked out by the symmetry group, i.e., all frames of reference that are rotation-free, but linearly accelerated with respect to each other) are absolute, i.e., fixed background structures; while the inertia-gravitational field is dynamical and related by field equations relating the affine Ricci tensor to the sources of the field. The compatibility conditions between connection and chronometry and geometry allow just sufficient freedom to introduce a dynamical gravitational field. Thus, the quantum theory *must* proceed by quantization of the connection while leaving the chronometry and geometry *fixed* (see [6]).

This suggests the possibility of connecting the Newtonian near field and the far radiation field by the method of matched asymptotic expansions. Kip Thorne explained this approach:

> Previous work on gravitational-wave theory has not distinguished the local wave zone from the distant wave zone. I think it is useful to make this distinction, and to split the theory of gravitational waves Into two corresponding parts: Part one deals with the source's generation of the waves, and with their propagation into the local wave zone; thus it deals with … all of spacetime except the distant wave zone. Part two deals with the propagation of the waves from he local wave zone out through the distant wave zone to the observer … The two parts, wave generation and wave propagation, overlap in the local wave zone; and the two theories can be matched together there. … [F]or almost all realistic situations, wave propagation theory can do its job admrably well using the elementary formalism of geometric optics ([21], p. 316).

If one looks at this carefully, there are really three zones:

(1) Near zone, where field is generated by the source.
(2) Intermediate zone, where the transition takes place between zones (1) and (3).

(3) Far zone, where pure radiation field has broken free from the source.

But before proceeding any further with the discussion of quantization in this Newtonian limit, it will be helpful first to discuss UCPR.

15.8 Unimodular Conformal and Projective Relativity

Einstein was by no means wedded to general covariance when he started his search for a generalized theory of relativity that would include gravitation. The equivalence principle:

> made it not only probable that the laws of nature must be invariant with respect to a more general group of transformations than the Lorentz group (extension of the principle of relativity), but also that this extension would lead to a more profound theory of the gravitational field. That this idea was correct in principle I never doubted in the least. But the difficulties in carrying it out seemed almost insuperable. First of all, elementary arguments showed that the transition to a wider group of transformations is incompatible with a direct physical interpretation of the space-time coordinates, which had paved the way for the special theory of relativity. Further, at the outset it was not clear how the enlarged group was to be chosen [10].

He actually considered restricting the group of transformations to those that preserved the condition that the determinant of the metric be equal to -1, both when formulating GR and when investigating whether the theory could shed light on the structure of matter (see Einstein [9], the translation of his 1919 paper). So the choice of SL(4, R) as the preferred invariance group is actually in the spirit of Einstein's original work.

I suspect that the restriction to such *unimodular diffeomorphisms*, which guarantees the existence of a volume structure, may be the remnant, at the continuum level, of a discrete quantization of four-volumes, which would form the fundamental space-time units, as in causal set theory. Quantization of three-volumes, etc., would be "perspectival" effects, dependent on the (3 + 1) breakup chosen for space-time. The fact that one can impose the unimodularity condition prior to, and independently of, any consideration of the conformal or projective structures lends some credence to this speculation.

If we confine ourselves to *unimodular diffeomorphisms*, we can easily go from compatible metric and connection to compatible conformal and projective structures. Many of the questions discussed above must then be reconsidered in this somewhat different light. One will now have to take into account both the conformal and projective connections and their compatibility conditions; and the conformal and projective curvature tensors.

Now we are ready to return to the Newtonian limit, and propose a *conjecture*:

In zone (1), the projective structure dominates; the field equations connect it with the sources of the field. In zone (3), the conformal structure dominates; the radiation field obeys Huygens' principle (see the next section). In zone (2), the compatibility conditions between the conformal and projective structures dominate, assuring that the fields of zones (1) and (3) describe the same field.

In order to verify these conjectures, we shall have to find the answers to the following *questions*: How do the field equations look in the near zone? Which projective curvature tensor is related to the sources in the near zone? In the far zone, which conformal curvature tensor obeys Huygens' principle? In the intermediate zone, which conformal and projective connections/curvatures should be made compatible?

15.9 Zero Rest Mass Radiation Fields, Huygens' Principle, and Conformal Structure

The name "Huygens' Principle" is given to several versions (see, e.g., [12]), but I shall consider only one. Let $u(x)$ be a function obeying some hyperbolic field equation on an n-dimensional differentiable manifold V_n, with a pseudo-Riemannian metric. As Hadamard showed, if the Cauchy problem is well-posed on some initial space-like hypersurface S, the solution at any future point $x_0 \in \mathrm{V}_n$, depends on some set of initial data given on the boundary and in the interior of the intersection of the retrograde characteristic conoid of $u(x_0)$ with the initial surface S. If, for every Cauchy problem on any S and every x_0, the solution depends *only* on *the initial data on the boundary*, the equation is said to satisfy *Huygens' principle*.

It's importance for our purposes lies in the fact that, only if Huygens' principle holds for a solution to the field equations of massless fields, such as the electromagnetic and the gravitational, does geometrical optics, i.e., the null-ray representation of the field, make sense. In that case one may carry out the analysis of the radiation field in terms of the shear tensor of a congruence of null rays, the components of the conformal curvature tensor projected onto these rays, etc. Similarly, ideal measurement of these quantities become possible; for example of the shear by means of two screens: one with a circular hole and one behind it to register the distortion of the shadow cast by the first screen.

In an arbitrary space-time, whether it is a fixed background chrono-geometry or one that is interacting with the Maxwell field, solutions to either the empty space Maxwell or Einstein-Maxwell equations, respectively, do *not* obey Huygens' principle. However, in a conformally flat space-time they do; and the interacting Einstein-Maxwell plane wave metric, which is type N in the Pirani-Petrov classification (see, e.g.[20]), also does. And in all such cases, the conformal structure is all that is needed to carry out the conceptual analysis and the corresponding ideal measurements

I assume that asymptotically "free," locally plane-wave solutions to the Einstein-Maxwell equations that are regular at past or full null infinity (Penrose's scri-minus and scri-plus) do obey the Huygens condition. In addition to the above considerations, this condition is also necessary for an analysis of scattering in terms of the probability amplitude :<*incoming free wave l outgoing scattered free wave*> to be valid.

If these assumptions are correct, then the free radiation field can be analyzed and presumably quantized entirely in terms of the conformal structure. However, all of these assumptions must of course be carefully checked.

15.10 Zero Rest-Mass Near Fields and Projective Structure

Local massless fields, still tied to the sources, do not obey Huygens' principle, and hence cannot be so analyzed. However, the gravitational analogue of the Bohr-Rosenfeld method of measuring electromagnetic field averages over four-dimensional volumes should still hold in this case. In UCPR, four-volumes are invariantly defined independently of any other space-time structures. If we want the four volume elements to be parallel (i.e., independent of path), we introduce a one form related to the gradient of the four-volume field and require this to be the trace of the still unspecified affine connection. So we are still left with full freedom to choose the conformal and projective structures [5].

The so-called equation of "geodesic deviation" (it should really be called "autoparallel deviation" since it involves the affine connection) will ultimately govern this type of analysis. And if we abstract from the parameterization of the curves, the projective structure should govern the resulting equations for the autoparallel paths. And in terms of amplitudes connecting asymptotic in- and out-states, one would expect that projective infinity will take the place of conformal infinity. Again, these expectations, and their implications for quantization of the near fields and their sources must be carefully investigated.

For further details on many points, see the paper by Kaća Bradonjić, "Unimodular Conformal and Projective Relativity: an Illustrated Introduction," in this volume.

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