Educational Signaling, Credit Constraints and Inequality Dynamics

Marcello D'Amato and Dilip Mookherjee[1]

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Abstract

We present a dynamic OLG model of educational signaling, inequality and mobility with missing credit markets. Agents are characterized by two sources of unobserved heterogeneity: ability and parental income, consistent with empirical evidence on returns to schooling. Both quantity and quality of human capital evolve endogenously. The model generates a Kuznets inverted-U pattern in skill premia similar to historical US and UK experience. In the first (resp. later) phase the skill premium rises (falls), social returns to education exceed (falls below) private returns: under-investment owing to financial imperfections dominate (are dominated by) over-investment owing to signaling distortions. There always exist Pareto-improving policy interventions reallocating education between poor and rich children.

[1]University of Salerno and Boston University, respectively. D'Amato's research was supported by Farb-Unisa Research Grant, 2010 and 2011, while Mookherjee's research was supported by NSF Grant no. SES-0617874. We are grateful to Stefan Napel for sharing with us his notes on a related model, and to Bob Margo for his detailed comments and references to the historical literature on skill premia in the US and in the UK. We have benefited from comments and questions by participants at the 2008 CSEF Conference in Capri and the 2011 ISI Conference on Growth and Development, and at seminars at Universities of Bologna, Boston, Bruxelles-Ecares, Glasgow-Strathclyde, Padua and Paris-Douphine.

1 Introduction

Discussions of education policy in both developed and developing countries generally presume there is a role for government interventions to encourage schooling, especially among poor households, on efficiency grounds as well as the need to promote occupational mobility and equality of opportunity. There is a large empirical literature on rates of return to education in both developed and developing countries, intended to inform policy makers concerning the importance of educational interventions. Yet there are few theoretical models that clarify the welfare arguments for educational policy interventions, or the source of underlying divergences between social and market rates of return.

One possible source of such divergence is 'ability bias', where measured education differences proxy for unobserved ability attributes, owing to Spencian signaling for instance. There is compelling evidence of education as a signaling device in labor markets (Bedard (2001), Lang and Kropp (1986), Riley (1979) and Wolpin (1977)). However, the Spencian theory implies that ability bias is positive, i.e., the social rate of return to education is lower than the market rate of return. This is inconsistent with the notion that government interventions to promote schooling would enhance efficiency or per capita income. Moreover, the empirical literature finds little concrete evidence for positive ability bias, with some evidence of a negative bias instead (e.g., see Card (2001) or Angrist and Pischke (2009)). How to reconcile this with the evidence concerning the role of signaling in labor markets constitutes an intriguing puzzle.

One way of resolving this puzzle is to argue that signaling distortions co-exist with another significant distortion — capital market imperfections – which create an opposite effect of under-investment in education.[2] Our model features two sources of heterogeneity that explain differences in schooling in the population: differences in ability, and in family

[2]An alternative might be to incorporate externalities associated with education, which has been well studied in the endogenous growth literature (e.g., Lucas (1988), Tamura (1991). We take the route of capital market imperfections because of the significant empirical evidence of such imperfections, as well as our interest in issues of inequality and social mobility which most endogenous growth models abstract from (see however, Glomm and Ravikumar (1992) for a notable exception).

income. This is consistent with empirical evidence concerning schooling. For instance, in reviewing empirical studies on returns to schooling in a diverse set of countries, Card (2001) argues that there is heterogeneity in the returns to education, with bias-corrected (IV) estimates based on supply-side interventions that measure returns for subsets of the population with relatively high returns (who could not access schooling owing to low family income or distance from educational institutions, rather than lack of ability).[3] In this view of the labor market, credit market imperfections co-exist with variations in ability. Differences in the cost of schooling indicate the importance of the former, whereby children from poorer families obtain less schooling. Moreover, reductions in the cost of schooling through institutional interventions induce higher schooling especially among such children, the return to whose education is higher than the average marginal return to education, indicating differences in ability between entrants and incumbents.

Our model features overlapping generations, fusing a signaling model in the Spence (1974) tradition with an occupational choice model based on credit constraints (e.g., Loury (1981), Ray (1990), Galor-Zeira (1993), Ljungqvist (1993), Freeman (1996), Mookherjee and Ray (2003)). Education and ability are assumed to matter only in the modern sector, not in the traditional sector where wages are exogenously fixed at a low level. Agents need an education to enter the modern sector, where their productivity depends on their ability which they privately observe. Parents are altruistically motivated and pay for their chil-

[3]Quoting Card (2001, pp 1156-57): "...there is underlying heterogeneity in the returns to education, and that many of the IV estimates based on supply-side innovations tend to recover returns to education for a subset of individuals with relatively high returns to education. Institutional features like compulsory schooling or the accessibility of schools are most likely to affect the schooling choices of individuals who would otherwise have relatively low schooling. If the main reason that these individuals have low schooling is because of higher-than-average *costs* of schooling, rather than because of lower-than-average *returns* to schooling, then "local average treatment effect" reasoning suggests that IV estimators based on compulsory schooling or school proximity will yield estimated returns to schooling above the average marginal return to schooling in the population, and potentially above the corresponding OLS estimates. Under this scenario, both the OLS and IV estimates are likely to be upward-biased estimates of the average marginal return to education. For policy evaluation purposes, however, the average marginal return to schooling in the population may be less relevant than the average return for the group who will be impacted by a proposed reform."

dren's education, owing to lack of borrowing opportunities. Pecuniary costs of education are decreasing in child ability; the utility costs of educating children are decreasing in parental income. Hence education is correlated positively with both ability and parental income. Competitive equilibrium dynamics endogenously determine the evolution of education and skill premia in wages across successive generations. With abilities of any given generation drawn randomly from a given distribution, we obtain a model of occupational and income mobility.

We show that the interaction between signaling and missing credit markets produces a theory of development based on quality as well as quantity of human capital accumulation. The model has a unique steady state, so the process of development is associated with the non-steady-state dynamics, starting from an initial level of education in the economy below the steady state level. The novel feature here is the endogenous evolution of quality of human capital, which helps explain historical patterns such as non-monotonic evolution of wage inequality (the Kuznets inverted-U) for 19th century US and UK. The first phase of development is marked by rising levels of education as well as quality of workers in the modern sector, owing to faster entry of children from poor families in the traditional sector with abilities higher than incumbents. During this phase the social rate of return to education can be shown to exceed the private return, owing to the pecuniary externality associated with entry of relatively high ability people into the modern sector. As development proceeds, the quality of marginal entrants declines; eventually it falls below the ability of incumbents. From that point onwards the average quality of workers in the modern sector (and hence the education premium in wages) falls. The Spencian signaling effect now dominates the effect of the credit constraints, and the social rate of return to education falls below the private return. Hence whether ability bias is positive or negative depends on the stage of economic development.

In this context we examine the basis of the argument for public intervention in education. This argument can be traced back to Friedman (1955):

> "Existing imperfections in the capital market tend to restrict the more expensive vocational and professional training to individuals whose parents or bene-

> factors can finance the training required. They make such individuals a "non-competing" group sheltered from competition by the unavailability of the necessary capital to many individuals, among whom must be large numbers with equal ability. The result is to perpetuate inequalities in wealth and status. The development of arrangements such as those outlined above would make capital more widely available and would thereby do much to make equality of opportunity a reality, to diminish inequalities of income and wealth, and to promote the full use of our human resources. (Friedman (1955))

To this end, Friedman suggested a program of educational loans provided by the government, which would be better placed to enforce repayments compared with private lenders owing to its ability to collect taxes:

> For vocational education, the government, this time however the central government, might likewise deal directly with the individual seeking such education. If it did so, it would make funds available to him to finance his education, not as a subsidy but as "equity" capital. In return, he would obligate himself to pay the state a specified fraction of his earnings above some minimum, the fraction and minimum being determined to make the program self-financing. Such a program would eliminate existing imperfections in the capital market and so widen the opportunity of individuals to make productive investments in themselves while at the same time assuring that the costs are borne by those who benefit most directly rather than by the population at large." (Friedman (1955))

Similar arguments for governmental education programs have been explored in formal models in recent literature (Galor and Zeira (1993), Ljungqvist (1993), Glomm and Ravikumar (1992), Benabou (2002), Mookherjee and Ray (2003)). It is commonly argued that the government has a comparative advantage *vis-a-vis* private lenders with respect to its ability to enforce repayment on educational loans, owing to the power of the government in collecting taxes. This constitutes the rationale of government interventions from an efficiency perspective. Even if this were granted, the interventionist argument ignores the possibility of adverse selection problems, arising from the difficulty of verifying ability. As explained above, heterogeneity and unobservability of ability are important ingredients of a policy-relevant model of education.

We use our model to explore the welfare argument for public interventions in schooling. We assume that the government can collect of loan repayments or special taxes imposed on recipients of public loans or public education, unlike private lenders, but it is equally subject to adverse selection problems. The presence of unobserved abilities and the possibility of over-investment in education by some households that may co-exist with under-investment by others complicates the design of such programs. If the government cannot observe abilities of children, such programs could aggravate signaling distortions. Is the argument for intervention valid only at early stages of development when the capital market imperfection dominates the signaling distortion?

To the contrary, irrespective of the stage of development, we show there is always scope for an educational intervention which generates an *ex post* Pareto-improvement. This owes to heterogeneity in marginal (social) rate of return to education across poor and rich families. The intervention is designed to improve quality rather than quantity of human capital. The mechanism that we construct involves provision of schooling by the government to the most able children from the traditional sector who would not receive an education under *laissez faire*. It is funded by bonds contributed by parents of rich parents of the least able children to receive an education under *laissez faire*. These parents therefore substitute educational provision by financial bequests. This results in a change in the composition of the educated labor force, with children from poorer backgrounds with superior abilities displacing those from richer backgrounds of lower ability. Such policies substitute for the missing credit market and tend to equalize educational opportunity across children of poor and rich families. Nevertheless, the intervention has to be carefully designed to overcome problems of private information of ability and resulting incentive problems, as well as dynamic general equilibrium effects. The only information available to the government is education status of parents and children, i.e., the program can be means-tested and thus discriminate according to family background, and can be conditioned on whether or not the child receives an education. The government does not need to monitor underlying abilities or educational expenditures incurred by parents.

The paper is organized as follows. Section 2 presents the model. Section 3 analyses steady states, while Section 4 deals with non-steady-state dynamics, including illustrative numerical calculations that demonstrate the Kuznets pattern under varying specifications of parameter values and technologies. Section 5 discusses normative implications. Section 6 describes related theoretical literature, as well as supporting evidence concerning skill premium dynamics in the context of the US and UK. Section 7 concludes with a summary and issues that remain to be explored further. Proofs are collected separately following the list of references. Appendices A-C respectively discuss implications of altering key assumptions of the model pertaining to returns to scale, ability of private employers to condition wages on family background, and absence of capital market imperfections (i.e., linearity of utility functions).

2 Model

The traditional sector has a fixed wage v. The endogenous wage in the modern sector is denoted w. Agents' innate abilities are denoted by n; in any generation t these are drawn randomly from a given distribution with c.d.f. F, which has full support on $[0, \bar{n}]$, and has a continuous density function f. Education is a $0-1$ decision. Productivity in the modern sector equals $e.n$, where e denotes education and n the ability of a worker. Productivity in the traditional sector equals v for all agents. Hence working in the modern sector requires education, unlike the traditional sector. Production operates according to constant returns to scale, and both sectors produce a common consumption good. We assume that the average ability in the population En exceeds v — this will ensure that the modern sector wage will always exceed the traditional sector wage.

There is a continuum of families indexed $i \in [0, 1]$. Each family has a single agent in a given generation, whose payoff is $U(c_{it}) + V(y_{i,t+1})$, where c_{it} denotes consumption of this parent, $y_{i,t+1}$ denotes the income of its child, and both U, V are strictly increasing, strictly concave and twice differentiable functions.

The parent in household i at t observes the ability draw of its child $n_{i,t+1}$ and then decides whether to invest in the latter's education. Education costs $x(n)$ for a child of ability n, where x is strictly decreasing, differentiable, with $x(\bar{n}) = 0$. If w_{t+1} is the skilled wage expected to prevail at $t+1$, a parent with income $y_{it} \in \{v, w_{it}\}$ and a child with ability n will select an education decision $e = e_{i,t+1} \in \{0, 1\}$ to maximize

$$U(y_{it} - e.x(n)) + V(ew_{i,t+1} + (1-e)v). \tag{1}$$

Here U represents the utility of the parent from its own consumption, and V the altruistic benefit it derives from the future earnings of its child. Implicit in this formulation is the assumption that education loan markets are missing.

Clearly if $w_{i,t+1} < v$ then no parent in generation at t will invest, whereas if $w_{i,t+1} > v$ some parents (with gifted children) will invest. In case of indifference we shall assume that investment will take place. For any given skilled wage $w^e \geq v$ expected in the next generation, the investment decision of a parent with income y is described by an ability threshold $n^*(w^e, y)$ at which the parent is indifferent:

$$U(y) - U(y - x(n^*(w^e, y))) = V(w^e) - V(v). \tag{2}$$

Then children with ability at or above this threshold receive education, and others do not.

Let λ_t denote the fraction of population that is skilled at t, and w^e_t the skilled wage at $t+1$ anticipated by parents of generation t. Then the evolution of the skill proportion is given as follows:

$$\lambda_{t+1} = \tilde{\lambda}(w^e_t; w_t, \lambda_t) \equiv \lambda_t[1 - F(n^*(w^e_t, w_t))] + (1-\lambda_t)[1 - F(n^*(w^e_t, v))] \tag{3}$$

Bertrand competition among employers in the modern sector implies the skilled wage in the next generation is : $w_{t+1} = \tilde{q}(w^e_t; w_t, \lambda_t)$ where:

$$\tilde{q}(.;.) \equiv \frac{[m(n^*(w^e_t, w_t))\lambda_t[1 - F(n^*(w^e_t, w_t))] + m(n^*(w^e_t, v))(1-\lambda_t)[1 - F(n^*(w^e_t, v))]}{\lambda_{t+1}}, \tag{4}$$

(with $m(n^*)$ denoting $E[n|n \geq n^*]$), provided $\lambda_{t+1} > 0$. In case $\lambda_{t+1} = 0$, we shall set $w_{t+1} = \bar{n}$.[4]

[4] In other words, if there are no agents that are educated at $t+1$, the skilled wage is set equal to the

It remains to specify wage expectations. We shall consider two expectational processes: *static expectations (SE)* where $w_t^e = w_t$ and *rational expectations (RE)* where $w_t^e = w_{t+1}$. This generates the following definitions of competitive equilibrium dynamics.

Definition 1 *A* **dynamic competitive equilibrium sequence with static expectations (ESE)** *given initial conditions* (w_0, λ_0) *is a sequence* $(w_t, \lambda_t), t = 1, 2, \ldots$ *such that* $\lambda_{t+1} = \tilde{\lambda}(w_t; w_t, \lambda_t), w_{t+1} = \tilde{q}(w_t; w_t, \lambda_t)$ *for all* $t = 0, 1, 2, \ldots$ *A* **dynamic competitive equilibrium sequence with rational expectations (ERE)** *given initial conditions* (w_0, λ_0) *is a sequence* $(w_t, \lambda_t), t = 1, 2, \ldots$ *such that* $\lambda_{t+1} = \tilde{\lambda}(w_{t+1}; w_t, \lambda_t), w_{t+1} = \tilde{q}(w_{t+1}; w_t, \lambda_t)$ *for all* $t = 0, 1, 2, \ldots$

Note that ESE is recursively determined: the wage and skill proportion at any date uniquely determine the wage and skill proportion at the next date. Not so for ERE, where the market-clearing wage in the modern sector at $t + 1$ is a fixed point of the function $\tilde{q}(.; w_t, \lambda_t)$.

3 Steady State

It is obvious from the definitions above that both static and rational expectations processes are associated with the same steady states.

Definition 2 *A steady state (SS) is* w^*, λ^* *such that* $\lambda^* = \tilde{\lambda}(w^*; w^*, \lambda^*), w^* = \tilde{q}(w^*; w^*, \lambda^*)$*.*

Hence in looking for steady states we may as well confine attention to stationary points of the static expectations dynamic.

Proposition 1 *There exists a unique SS.*

highest ability in the population. This assumption prevents the possibility of the economy getting trapped in trivial steady states where $w < v$ and $\lambda = 0$. We do this to ensure that perceived average quality is continuous with respect to the expected wage at $w^e = v$.

An important reason for steady state uniqueness is the fixed nature of the wage v in the traditional sector. These owes to the constant returns assumption, as well as the irrelevance of ability in that sector. With diminishing returns to labor, increasing out-migration would drive up the traditional wage. Then (as in Mookherjee-Napel (2007)) there could be multiple steady states, as higher wages in the traditional sector relax liquidity constraints and allow more unskilled households to educate their children.[5]

The argument for uniqueness depends on whether q the average quality of the workforce in the modern sector is decreasing in the wage w. One reason for this is the greater 'pull' of the modern sector when wages in that sector rise, inducing a decline in the ability of the marginal type from within the traditional or modern sector that receive education. There is however a complicating compositional effect: those migrating into the sector from the traditional sector come from poorer families, compared with children of families already in the modern sector. Hence the former are more talented than those coming from within the modern sector. If the proportion of the former rises appreciably, average quality in the modern sector could rise following a rise in w. If the proportions are such as to maintain steady state (i.e., $\lambda = \lambda(w)$) the proof shows that this compositional effect is not powerful enough to allow multiple steady states. Out of steady state, however, it can cause quality and wage in the modern sector move in the same direction, as we shall see in the next section.

4 Non-Steady State Dynamics

4.1 Static Expectations

We start with the case of static expectations. This may be considered plausible from a behavioral standpoint. In any case it is simpler to work with, being recursively determined. We shall show later that the main results continue to apply with rational expectations.

[5]In Appendix A we provide an example of multiple steady states with diminishing returns in the traditional sector.

Proposition 2 *Consider any competitive equilibrium sequence with static expectations. There exist functions $\lambda(w), \lambda_1(w)$ mapping $[v, \bar{n}]$ into $[0,1]$ both of which pass through the steady state (λ^*, w^*), with $\lambda(w)$ given by (11), and $\lambda_1(w) < (>) \lambda(w)$ according as $w > (<) w^*$, such that (as depicted in Figure 1):*

(a) $\lambda_{t+1} > (=, <) \lambda_t$ according as $\lambda_t < (=, >) \lambda(w_t)$, and

(b) $w_{t+1} > (=, <) w_t$ according as $\lambda_t < (=, >) \lambda_1(w_t)$.

While it is straightforward to see that $\lambda(w)$ is upward-sloping, it is more difficult to sign the slope of $\lambda_1(w)$. To understand this better, we introduce the following notation.

Define $n^R(w) \equiv n^*(w, w)$ and $n^P(w) \equiv n^*(w, v)$, thresholds pertaining to abilities of children from the modern and traditional sectors in steady state when the modern sector wage is w. These are both continuous functions mapping $[0, \bar{n}]$ to itself. Next, define the corresponding steady state quantity and quality of the workforce in the modern sector when the current proportion in the modern sector is λ:

$$L(w, \lambda) = \lambda[1 - F(n^R(w))] + (1 - \lambda)[1 - F(n^P(w))] \tag{5}$$

$$Q(w, \lambda) = \frac{m(n^R(w))\lambda[1 - F(n^R(w))] + m(n^P(w))(1 - \lambda)[1 - F(n^P(w))]}{\lambda}. \tag{6}$$

These functions map $[En, \bar{n}] \times [0, 1]$ to itself.[6]

The properties of the Q function play a key role in the analysis. Applying the Implicit Function Theorem to this function, we obtain $\lambda_1' = \frac{1 - Q_w}{Q_\lambda}$. Hence $1 > Q_w$ ensures that $\lambda_1(w)$ is downward sloping (since $Q_\lambda < 0$). Note that $Q(w, \lambda) = \alpha m(n^R(w)) + (1 - \alpha) m(n^P(w))$ where α denotes $\frac{\lambda[1 - F(n^R)]}{\lambda[1 - F(n^R)] + (1 - \lambda)[1 - F(n^P)]}$. Hence

$$Q_w = (1 - \alpha) m'(n^P(w)) n_w^P + \alpha m'(n^R(w)) n_w^R + \alpha_w [m(n^R) - m(n^P)]. \tag{7}$$

The first two terms on the right-hand-side of (7) are negative, reflecting the lowering of quality of the marginal person receiving education from within the pool of unskilled and skilled families as the wage in the modern sector grows. The third term involves changing

[6] Note that $En > v$ ensures that $L(w, \lambda)$ is strictly positive for every $w \geq En$, so Q is well-defined.

composition of the pool of the educated between these two groups. This compositional effect cannot be signed unambiguously, since $\alpha_w > 0$ if and only if

$$\frac{f(n^P)}{1-F(n^P)}[-n^P_w] < \frac{f(n^R)}{1-F(n^R)}[-n^R_w]. \tag{8}$$

In other words, it depends on the relative hazard rates of the ability distribution at the respective thresholds of the two groups, weighted by the slope of the threshold with respect to the wage. If (8) holds, then $Q_w < 0$, and $\lambda_1(w)$ is downward-sloping. But it is possible that (8) does not hold at some w, i.e., an increase in the modern sector wage elicits a much larger response from children in families located in the traditional sector, than those in the modern sector. In that case the compositional effect contributes to an improvement in the quality of the workforce in the modern sector. If it is strong enough to overwhelm the direct effect of quality of each group separately, it is possible that increasing wages improve the quality of the skilled workforce.

However, "on average" the $\lambda_1(w)$ function must be downward sloping, in the following sense. Define $\tilde{w}$ by the solution to $m(n^P(w)) = w$, and $\hat{w}$ by the solution to $m(n^R(w)) = w$, if these solutions exist. Clearly $\tilde{w} > w^*$ since at w^* we have $m(n^P(w^*)) > q(w^*) = w^*$. If $\bar{n}$ is large enough in the sense that $m(n^P(\bar{n})) < \bar{n}$, then $\tilde{w}$ is well defined and lies in the interval $(w^*, \bar{n})$. Then $\lambda_1(w) = 0$ for all $w > \tilde{w}$, since $Q(w, \lambda) < Q(w, 0) = m(n^P(w)) < w$ for all $w > \tilde{w}$ and all $\lambda > 0$. Conversely, note that $\hat{w}$ is well-defined and lies in the interval (v, w^*) since $m(n^R(v)) \geq En > v$, and $m(n^R(w^*)) < q(w^*) = w^*$. Then for all w in the interval $(v, \hat{w})$ we must have $\lambda_1(w) = 1$. So the $\lambda_1(.)$ function slopes down on average in the sense that it equals 0 above $\tilde{w}$, w^* at w^*, and 1 below $\hat{w}$. It will slope downwards at any point where the compositional effect is not strong enough in the sense that $Q_w < 1$.

One set of sufficient conditions for $\lambda_1(w)$ to be downward-sloping throughout the interior of the state space is provided below.

Remark 1 *Suppose the hazard rate of the ability distribution $\frac{f(n)}{1-F(n)}$ is non-increasing in n[7], and education cost $x(n)$ is linear or concave in n. Then $\lambda_1(w)$ is everywhere decreasing in the interior of the state space.*

[7] An example is an exponential distribution, where $f(n) = ke^{-\mu n}$, whence the hazard rate is constant.

In numerical computation of the equilibrium dynamics for log utility and uniform ability distributions (described in a subsequent section), the $\lambda_1(w)$ function turns out to be downward-sloping throughout. So for the purpose of the remaining discussion of this section we shall proceed on this assumption, whence the inverse of λ_1 function is well-defined.

Proposition 2 shows that the non-steady-state dynamics can be characterized by a partition of the state space (λ, w) into four regions, as depicted in Figure 1.a:

I. $\lambda_t < \lambda(w_t), \lambda_t < \lambda_1(w_t)$: here $w_{t+1} > w_t, \lambda_{t+1} > \lambda_t$. Both quality and quantity of the modern work force grows.

II. $\lambda_t < \lambda(w_t), \lambda_t > \lambda_1(w_t)$: here $w_{t+1} < w_t, \lambda_{t+1} > \lambda_t$. The quantity of the modern work force grows, but its quality declines.

III. $\lambda_t > \lambda(w_t), \lambda_t > \lambda_1(w_t)$: here $w_{t+1} < w_t, \lambda_{t+1} < \lambda_t$. Both quality and quantity of the modern work force shrink.

IV. $\lambda_t > \lambda(w_t), \lambda_t < \lambda_1(w_t)$: here $w_{t+1} > w_t, \lambda_{t+1} < \lambda_t$. Quality improves, but quantity declines.

Consider a country with low per capita income owing to a low proportion and quality of workforce in the modern sector. The quantity of skilled workforce is low in the sense that $\lambda_t < \lambda(w_t)$. Then we are in either region I or II. If the quality is also low in the sense that $w_t < \lambda_1^{-1}(\lambda_t)$, we are in region I. Both quality and quantity of the modern work-force will grow from t to $t+1$. Both will contribute to a rise in per capita income:

$$y_{t+1} - y_t = [\lambda_{t+1} - \lambda_t][w_t - v] + \lambda_{t+1}[w_{t+1} - w_t] \tag{9}$$

and the social rate of return to education exceeds the market rate of return:

$$\frac{y_{t+1} - y_t}{\lambda_{t+1} - \lambda_t} = (w_t - v) + \lambda_{t+1}\frac{w_{t+1} - w_t}{\lambda_{t+1} - \lambda_t} \tag{10}$$

During this early phase of development, there are a sufficiently large proportion of new entrants into the modern sector from the traditional sector. These new entrants come from poorer backgrounds and are more able than those in the modern sector in the previous generation. Upward mobility goes hand-in-hand with a positive externality: the marginal entrants from the traditional sector are smarter on average than those already in the modern sector, causing the wage to rise, which benefits all others in the modern sector.[8] Over this range, the under-investment effect owing to the capital market imperfection dominates the over-investment effect owing to signaling.

This dynamic will propel the economy into region II, as enough people migrate into the modern sector.[9] Subsequently the proportion of the educated will continue to grow, while quality will fall. During this subsequent stage, the rise in the modern sector wage attracts new types with lower ability than those in the modern sector in the previous generation, which causes the wage to fall. In this case, the 'ability bias' is positive: the social return to education falls below the private return. Per capita income growth is likely to slow down both because the increase in the skill proportion is likely to slow down, and the quality of the skilled workforce starts to fall. Over this range the signaling externality overwhelms the capital market imperfection.

It is possible that the economy converges thereafter to a steady state, though we have not been able to prove any results concerning convergence. We explore this issue in the context of the numerical solutions below.

In general, however, the dynamics are quite complicated. Regions III and IV are those where there is 'too much" education in the economy relative to the wage, causing the proportion of educated to fall. Quality also declines in region III, so per capita income definitely falls. In region IV, quantity declines and quality increases, so the effect on per capita income is ambiguous. The economy could converge to the steady state if the initial position is to the south-east of the steady state. If it is to the south-west, it could transit

[8] If increasing scarcity of labor in the traditional sector causes wages there to rise, then the migration benefits those remaining in the traditional sector as well.

[9] If $Q_w < 0$, as in the case described in Remark (1), and $w_t < w^*$, then the economy must move to Region II in the next generation. This is because the skill ratio will move towards λ^* but cannot overshoot it (since $\lambda_{t+1} < \lambda(w_t) < \lambda(w^*) = \lambda^*$). On the other hand, the fact that Q is decreasing, $\lambda_1^{-1}(\lambda_t)$ is the unique fixed point of $Q(.;\lambda_t)$ and $w_t < \lambda_1^{-1}(\lambda_t)$ implies that $w_{t+1} > \lambda_1^{-1}(\lambda_t) > \lambda_1^{-1}(\lambda_{t+1})$, the last inequality following from the fact that $\lambda_{t+1} > \lambda_t$ and the assumption that λ_1 is a decreasing function.

into Region I.

Note also that reverse transitions from region II to region I cannot be ruled out. It appears possible then that even with $\lambda_1(.)$ downward sloping, the economy could flip-flop between these two regions. Hence the dynamics could be more complicated than the simple Kuznets pattern: periods of falling skill premia can be interspersed with periods of rising skill premia.

Nevertheless the dynamics of the skill premium provides a useful guide to the divergence between social and private returns to education. A rising premium indicates the social return lies above the private return, while a falling one indicates a positive 'ability bias'.

4.2 Rational Expectations

We now consider the case of rational expectations. With forward-looking agents, the equilibrium sequence cannot be recursively computed. A related problem is that short-run competitive equilibrium of the modern sector labor market may not be unique. Recall the definition of the perfect foresight equilibrium skilled wage w_{t+1}, i.e., given the state (w_t, λ_t), it is a fixed point of $\tilde{q}(.; w_t, \lambda_t)$. Owing to the compositional effect (explained above) this function can be non-monotone: a rise in w^e_{t+1} could raise the average quality of the workforce in the modern sector over some ranges. So there may be multiple wage equilibria.

If we focus on a locally stable equilibrium (where $\tilde{q}$ is downward-sloping), the wage will be locally decreasing in λ_t. An increase in λ_t (for given w^e_{t+1}) raises the proportion of children coming from wealthier backgrounds, which lowers the average quality of the workforce in the next generation. It is therefore natural to select equilibria so that this property is globally satisfied.

Similarly, an increase in w_t for given w^e_{t+1} raises the proportion of children with educated parents that choose to be educated, lowering average quality of the educated workforce at $t + 1$.

If the highest fixed point or the lowest fixed point (corresponding to the most optimistic or most pessimistic expectations) is always selected, the perfect foresight equilibrium wage function $w_{t+1} \equiv Q^R(w_t, \lambda_t)$ will be decreasing and (almost everywhere) continuously differentiable in both w_t and λ_t.[10]

Proposition 3 *Suppose that with rational expectations, the equilibrium wage w_{t+1} is given by a function $Q^R(w_t, \lambda_t)$ which is decreasing and (almost everywhere) continuously differentiable in w_t and in λ_t. Then there exists a non-increasing function $w^R(\lambda)$ mapping $[0,1]$ into $[v, \bar{n}]$, and an (a.e.) continuous function $\lambda^R(w)$ mapping $[v, \bar{n}]$ into $[0,1]$ such that (as depicted in Figure 1.b):*

(a) $\lambda_{t+1} > (=,<)\lambda_t$ according as $\lambda_t < (=,>)\lambda^R(w_t)$;

(b) $w_{t+1} > (=,<)w_t$ according as $w_t < (=,>)w^R(\lambda_t)$;

(c) both functions pass through the steady state λ^*, w^*;

(d) the function $\lambda^R(w)$ is nondecreasing at w if $L^R(w, \lambda)$ is nondecreasing in w.

We thus obtain qualitatively similar dynamics with rational expectations, as shown in Figure 1.b. The difference from static expectations is that the threshold function $w^R(\lambda)$ dividing the space between states where wages are rising and where they are falling, is now a nonincreasing function in general. On the other hand, the threshold $\lambda^R(w)$ defining the condition for λ to increase, cannot be guaranteed in general to be upward sloping. The reason is that an increase in w_t raises the supply of skilled people from skilled households, lowering w_{t+1}. *This causes the supply of skilled people from unskilled households to decrease: n^P rises.* This is in contrast to the case of static expectations, where the supply from both types of households increase with higher w, since everyone expects the current wage next

[10] Existence is ensured by the fact that for any w, λ, the function $\tilde{q}(.; w, \lambda)$ maps $[v, \bar{n}]$ into itself continuously. Since utility functions and the distribution functions are C^1 functions, $\tilde{q}$ is C^1. Standard arguments imply that for a generic set of values of w, λ, the function $\tilde{q}(.; w, \lambda)$ will have a finite number of equilibria that are locally stable and locally C^1. The Implicit Function Theorem ensures each locally stable equilibrium is locally decreasing. Next, note that $\tilde{q}$ approaches $\bar{n}$ as w^e_{t+1} approaches v, the lowest fixed point must be locally stable. The highest fixed point must also be locally stable, since $\tilde{q}$ is bounded away from $\bar{n}$ as w^e_{t+1} approaches $\bar{n}$. Since an increase in w or λ causes the function $\tilde{q}$ to shift downwards, the highest or lowest fixed point must be everywhere decreasing.

period. With rational expectations it is therefore possible that increasing the skilled wage at t lowers the aggregate supply of skilled people at $t+1$. Then the $\lambda^R(w)$ locus could be downward sloping.

4.3 Numerical Analysis

Numerical solutions for equilibrium dynamics can be computed with static expectations for specific utility functions and ability distributions. These permit us to check convergence to steady state, and verify theoretical results concerning skill premia dynamics.

Figure 2(a,b) presents the equilibrium dynamic for the skilled wage corresponding to logarithmic utility (for both U and V), uniform ability distribution on $[0,1]$, education cost $x(n) = 1 - n$, initial values $w(0) = 0.9, \lambda(0) = 0.01$, and two values for $v = 0.1$ and 0.2. A Kuznets pattern is evident: both the wage and skill ratio rise initially. Then the skilled wage falls while the skill ratio rises, converging eventually to a steady state (in the sense that all the trajectories plotted include up to around 15-20 observations where the differences between subsequent observations is zero up to five decimal points). However, the process of convergence does not involve a falling modern sector wage throughout the second stage: it alternately declines and increases across successive generations. Hence variations from the Kuznets pattern are also possible over some ranges, with increases in the modern wage in one generation inducing entry into the modern sector by less able agents in the next generation, causing the wage to drop. This in turn discourages entry in the succeeding generation, inducing greater selectivity and a subsequent rise in the wage, and so on.

Figure 2(c,d) shows the effects of lowering the initial level of the skilled wage $w(0)$ to 0.6, while keeping other parameters the same. This lowers the motivation of parents to educate their children, raising the ability thresholds in both sectors, and causing a steeper initial rise in the skilled wage. The skilled wage in generation 1 is now higher. This causes a steeper fall in the skilled wage from generation 1 to 2, as parents are now more motivated to educate their children, and those in the modern sector are less credit-constrained. Hence the Kuznets pattern is more pronounced if the skilled wage is lower at the outset. The process converges eventually to the same steady state.

In all these cases, the first phase of the Kuznets pattern where the skill premium and skill ratio rise at the same time lasts only for one period, while the second phase operates for all successive periods. Even if the economy starts in Region I (where the social return to education exceeds the private return) it seems to spend a negligible proportion of time in the long run in that region. This may owe to the lack of a realistic age structure in the model. We now explore the implications of more realistic demographic patterns.

Consider the following extension of the model. Any given cohort works for K periods. A date t cohort is educated at $t-1$, starts working at t, and works until $t+K$. The parent of cohort t belongs to cohort $t-T$, so T is the age gap between parents and children. The proportion of cohort t that becomes educated depends on the wage of their parent and on the wage at $t-1$ (the latter representing the wage they expect in their lifetime):

$$\lambda_t^c = \lambda_{t-T}^c[1 - F(n(w_{t-1}, w_{t-T}))] + (1 - \lambda_{t-T}^c)[1 - F(n(w_{t-1}, v))]$$

All cohorts are equal in size, so the workforce size is constant. The proportion of the entire economy's workforce that is skilled at t is then given by

$$\lambda_t = \sum_{k=0}^{K} \lambda_{t-k}^c$$

Assuming that employers cannot discriminate by age, the wage at t equals

$$\begin{aligned} w_t &= \frac{1}{\sum_{k=0}^{K} \lambda_{t-k}^c} \sum_{k=0}^{K} \lambda_{t-k}^c [1 - F(n(w_{t-k-1}, w_{t-T-k}))] m(n(w_{t-k-1}, w_{t-T-k})) \\ &\quad + (1 - \lambda_{t-k}^c)[1 - F(n(w_{t-k-1}, v))] m(n(w_{t-k-1}, v)) \end{aligned}$$

The equilibrium sequence can now be recursively computed.

Figure 3(a,b) presents computations of the equilibrium dynamics where we set $K = 5$, and initial values of the skilled wage and skill ratio for periods 0–4 are 0.65 and 0.3 respectively, while λ_c is set at 0.041. The ability distribution is uniform on $[0, 1]$, and values of v are varied from 0.1 to 0.2. The first phase of the Kuznets pattern now lasts the first five generations, with the second phase operating thereafter. However there is a tendency for the dynamics to overshoot the steady state and loop back thereafter before converging to the steady state.

Finally Figure 3(c,d) considers the effect of imperfect substitutability between skilled and unskilled labor, as well as variable quality of unskilled labor.[11] The production function now has constant elasticity of substitution between efficiency units of skilled and unskilled labor. Efficiency units of either kind of labor are obtained by weighting proportions of the labor force in each category by their average ability. The ability distribution is uniform as before on $[0, 1]$. Initial values of the skilled wage, unskilled wage and skill ratio are set at $0.3, 0.1$ and 0.01 respectively. The skilled (resp. unskilled) wage is calculated by multiplying the average ability of skilled (resp. unskilled) workers by the marginal product of skilled (resp. unskilled) work. Dynamics for two values of elasticity of substitution are shown in parts (c–d). Raising the elasticity of substitution prolongs the duration of the first phase of the Kuznets pattern. It also causes the steady state skill ratio and skilled wage to fall, a natural consequence of the increasing ability of firms to substitute skilled with unskilled labor.

5 Normative Implications

In this section we consider normative properties of *laissez faire* competitive equilibria and corresponding implications for educational policy interventions.

There are a variety of normative criteria employed in discussions of educational policy. One criterion is the social rate of return to education and its relation to the market rate of return. Another is whether or not there is under-investment or over-investment in education. A third criterion is welfare-based: do there exist feasible policy interventions that are Pareto improving, or those that raise a suitable notion of welfare (utilitarian, or Rawlsian). All of these are related to one another, though there exist no general presumptions here owing to the fact that we are dealing with an overlapping generations economy with missing markets and asymmetric information. Governments may also be constrained with regard to their access to credit from international agencies or markets, and have less information than available to private agents concerning abilities and educational costs. Criteria based on

[11] Appendix A shows that steady states may no longer be unique under this conditions. However we do not encounter any such steady state multiplicity in these simulations.

measures of rates of return, or of under- or over-investment therefore do not have *a priori* obvious implications for the welfare effects of interventions or policy design.

5.1 Macro Rates of Return to Education

The most common normative criterion used in discussions of educational policy concerns a macro measure of the social rate of return to education, and how it deviates from market rates of return. A key aspect of our model is the heterogeneity of households and agents with regard to abilities and incomes, which makes it difficult to give any meaning to a notion of *the* social rate of return to education. Much depends on the ability and parental backgrounds of those who are being educated at the margin, and the associated pecuniary externalities.

One way to measure the social rate of return to education at the margin is to evaluate the change in national income per additional person educated along a non-steady-state path involving rising educational attainment in the population. Here as we have already discussed in Section 4 (see in particular (10)), the social rate of return lies above or below the market-based measure of the rate of return depending on whether the latter are rising or falling over time. It suggests that policy ought to subsidize education in the first phase of development when skill premia rise, and tax it in the second stage when premia are falling.

5.2 Micro-based Criteria: Under- and Over-Investment

Our model highlights heterogeneity of abilities and parental backgrounds of agents, indicating that the notion of social rate of return differs substantially across the population. In particular credit market imperfections create a divergence in educational decisions across households located in the traditional and modern sectors. Hence the economy-wide implications of education of children are likely to be different across households located in the traditional and modern sectors.

For a child located at or near the threshold n^P used by 'poor' parents in the traditional sector, education of this child is associated with a switch from working in the traditional to the modern sector, whose effect on output in the economy is $n^P - v$ but involves a resource cost of $x(n^P)$. The output implications appear one period after the educational investments are made. Owing to missing credit markets, there is no market rate of interest, so one needs some notion of a rate of time preference.

To simplify matters, therefore, suppose that all households have a common rate of time preference (which corresponds to the degree of parental altruism): $V \equiv \delta U$, for some positive scalar δ. Using this as the social rate of time preference, then, we obtain the following notions of under- or over-investment.

Definition 3 *Consider a competitive equilibrium sequence* $\{w_t, \lambda_t\}$ *with rational expectations and associated ability thresholds* $n_t^P = n^*(w_{t+1}, v), n_t^R = n^*(w_{t+1}, w_t)$ *used in educational decisions by poor and rich households respectively at date* t*. Suppose that* $V \equiv \delta U$*. Then there is* **under-investment among the poor (resp. rich) at** t *if* $\delta[n_t^P - v] > x(n_t^P)$ *(resp. if* $\delta[n_t^R - v] > x(n_t^R)$*). There is* **over-investment among the poor (resp. rich) at** t *if these inequalities are reversed.*

This is essentially a measure of production inefficiency. Whether it corresponds to some notion of Pareto or welfare inefficiency will be discussed in the next subsection. For the time being, we present some results concerning when competitive equilibria involve over or under-investment for rich and poor households respectively.

Define $\tilde{n}$ by the property that $\delta[\tilde{n} - v] = x(\tilde{n})$. First-best productive efficiency involves ability threshold $\tilde{n}$ for all households. Hence whether or not there is over or under-investment in any sector of the economy depends on how the corresponding threshold used in that sector compares with $\tilde{n}$.

Proposition 4 *Consider a competitive equilibrium sequence* $\{w_t, \lambda_t\}$ *with rational expectations and associated ability thresholds* $n_t^P = n^*(w_{t+1}, v), n_t^R = n^*(w_{t+1}, w_t)$ *used in educational decisions by poor and rich households respectively at date* t*. Suppose that* $V \equiv \delta U$ *for some positive discount factor* δ*.*

(a) There is under-investment among the poor at $t - 1$ *if either of the following is satisfied:*

(i) $w_t < \tilde{n}$ *or* $w_t > m(\tilde{n})$

(ii) $\lambda_t < 1 - F(\tilde{n})$

(iii)[12] *The economy is operating in the 'first phase of development' with rising skill premia and ratios, i.e.,* $\lambda_t > \lambda_{t-1}, w_{t+1} > w_t > w_{t-1}$.

(b) There is over-investment among the rich if $w_t < m(\tilde{n})$.

This result provides some conditions for under-investment among the poor (what we might expect from the presence of capital market imperfections), and for over-investment among the rich (expected owing to signaling distortions). The former results when the modern sector wage or its relative size are small (parts (i) and (ii) respectively of (a)) — i.e, at 'early' stages of development. If the modern sector wage is small (smaller than $\tilde{n}$) we also have over-investment among the rich — even though they are then not 'that rich'. Intuitively, a low modern sector wage exerts a low 'pull' among poor households to educate their children, generating under-investment among them. At the same time it reflects a low quality of those coming from the modern sector, i.e., over-investment among them.

Part (iii) of (a) relates under-investment among the poor to the nature of the equilibrium dynamic: if the size and the quality of the modern sector are both rising then there must be under-investment among the poor. The new entrants to the modern sector coming from the traditional sector must be better than the average quality of previously in the modern sector, i.e., the modern sector wage. And the existence of the capital market imperfection implies that market-based rates of return exceed the education costs. Hence valuing the contribution of the new entrants at their true productivity in the modern sector must generate a higher return than the costs of educating them.

The gap in the sufficient condition (i) in part (a) of the preceding Proposition gives rise to the question whether there may be cases when under-investment among the poor does not obtain. In the case of linear utility (described in Appendix C) competitive equilibrium allocations are unchanging over time, with poor and rich households using the same threshold, which is characterized by over-investment. Hence for 'very slightly' concave utility

[12] If expectations are static, the same result follows under the weaker condition $\lambda_t > \lambda_{t-1}, w_t > w_{t-1}$.

functions one would expect over-investment among both rich and poor as well at all dates. In that case the capital market imperfection has little bite and the signaling distortions dominate; the modern sector wage must be wedged in between $\tilde{n}$ and $m(\tilde{n})$ at all dates.

5.3 Pareto Improving Policies

What are the policy implications of the preceding results? Do the notions of under or over-investment among specific groups in the population correspond to suitably corrective policy interventions?

Much depends on the constraints that bind governments, and how these relate to those that bind private agents. In the following we shall assume that the government cannot borrow or lend on par with private agents: the economy is closed, or the government lacks access to international capital markets. Hence all interventions must balance the government budget period-by-period. In Mookherjee and Ray (2003), this constraint alone prevented a class of steady states with over-investment from admitting any Pareto-improving interventions.[13]

On par with private employers, it is also reasonable to suppose that the planner cannot observe abilities of children. We do, however, assume that the planner can observe educational status, though the actual educational expenditures incurred by parents cannot be observed. The planner can ask parents to report the abilities of their children, and impose taxes or transfers as functions of these reports and educational status of parents and children. This confers upon planners superior monitoring and enforcement powers compared with private employers or lenders, but is a natural description of the powers of the government (cf. Friedman (1955) or Galor and Zeira (1993)).

In the following proposition, we endow the government with the additional option of running a *public school*, in which children of specified types (defined by parental educational status and their abilities as reported by their parents) can be enrolled. The government can decide how much to spend per child enrolled in the public school. If it spends an amount

[13]The current context differs from Mookherjee and Ray (2003) owing to its incorporation of ability heterogeneity, besides the nature of altruism which is paternalistic rather than dynastic (parents care intrinsically about their children's future wealth rather than utility).

x^* per child, those with abilities satisfying $x(n) \leq x^*$ will complete their education and become skilled, while those with $x(n) > x^*$ will remain unskilled.

With these policy instruments, we now show that the market equilibrium is *always* constrained Pareto-inefficient: there exists an intervention by the government involving provision of public schooling for children of specified abilities from households in the traditional sector, who do not receive education in the market equilibrium. The schools are funded by a government bond to which parents in the modern sector contribute. These are parents whose children have the least ability among those who get educated in the market equilibrium. Their parents switch from educating these children, to purchasing the government bond. In the next generation the children (who will become unskilled) will receive the proceeds from the bond that their parent purchased. These bond payoffs are financed by those who went to public school in the previous generation, in the form of repayment to the government of the benefits of the free schooling they had received.

The intervention achieves a Pareto improvement owing to the mis-allocation of education investments between rich and poor households. Owing to the missing credit markets, rich households with children just above the ability threshold are 'earning' a lower rate of return on their educational investment than corresponding poor households. In the presence of an efficient credit market, the former would lend to poor households whose children have abilities just below the threshold in that sector. Frictions in financial markets arising from difficulties in enforcing loan repayments prevent such Pareto-improving re-allocations. A planner can simulate such reallocations by designing a suitable mechanism which encourages 'marginal' rich households to purchase a government bond, which finances educational loans to poor households just below the threshold.

However the construction of the scheme is complicated owing to the informational constraints faced by the government. Not knowing the abilities of children, the government has to rely on reports made by parents and has to design the scheme to encourage truthful revelation. In effect the government is providing a combination of public schooling and educational loans for children from poor households, which are subject to problems of adverse selection. On the other side, it is designing a system of financing public schools with educational bonds which are also subject to adverse selection, owing to heterogeneous preferences among well-to-do parents concerning investment in their children's education *vis-a-vis* and financial bond as alternative ways of providing for their children's future well-being.

An additional difficulty is that the interventions are designed in a dynamic framework, where income changes that accrue to agents are going to have repercussions for future generations via pecuniary externalities. An increase in incomes of some agents in any generation which leaves the incomes of all others in that generation unaffected, may end up hurting some people in the next generation owing to induced effects on education and wages. For instance, if incomes in the modern sector rise, they will tend to invest more in their children's education, which will reduce modern sector wages owing to quality dilution effects. So the government intervention needs to be constructed in a way as to generate increases in consumption rather than incomes for parents, which will not generate any such future repercussions.

Proposition 5 *Assume that $u(0)$ is finite and $u'(c)$ tends to ∞ as c tends to 0. Then given any competitive equilibrium, the planner can design an intervention involving any pair $t-1, t$ of successive generations with the following properties:*

(a) in generation $t-1$ it provides public schooling for children of parents in the traditional sector with specified abilities, funded by a bond which is purchased by parents in the modern sector whose children have specified abilities;

(b) in generation t the recipients of public schooling pay a tax to the government, which the government uses to pay children of bondholders;

(c) the government imposes a tax on modern sector wages at t; and

(d) the scheme generates an ex post Pareto improvement, is incentive compatible and runs a government budget surplus at every date.

Whether Pareto-improving interventions using educational loans instead of public schooling can be constructed remains an open question. Educational loan programs entail additional incentive problems owing to associated rents they offer to poor parents with children of intra-marginal abilities. Such rents do not arise in the case of public schooling since educational expenditures are made by the government instead of by parents.

6 Related Literature

6.1 Related Theoretical Models

The model of this paper is most closely related to models of human capital accumulation or occupational choice with credit market imperfections (Ray (1990, 2006), Banerjee and Newman (1993) Galor and Zeira (1993), Ljungqvist (1993), Freeman (1996), Mookherjee and Ray (2003), Mookherjee and Napel (2007)). These papers focus on the implications of credit market imperfections, and abstract from signaling distortions in labor markets or in occupational choice. Our model can be viewed as a natural extension of this literature to incorporate unobserved ability differences. In terms of results, one distinction is the lack of long run history dependence (in the sense of multiple steady states) in our model, whereas most of the previous literature emphasizes history dependence. Our focus is thus on non-steady state dynamics, which is more complicated owing to the need to keep track of both quantity and composition of the educated labor force.[14] A key distinction from the earlier literature is that in all preceding models wages equal marginal products, implying that skill premia decline in the process of development – rendering them incapable of generating co-movements of skill premia and ratios, or a Kuznets pattern.

The role of education screening for the analysis of income inequality and education policies has been explored by a number of theoretical papers, in particular Stiglitz (1975), Lang (1994), Hendel, Shapiro and Willen (2005) and Regev (2007). Stiglitz (1975) was the first to study the implications of screening for inequality and the allocation of resources to education. His paper focuses on the determinants of over-investment effects in a static

[14] Some of the earlier models focus on non-steady-state dynamics (e.g., Ray (1990, 2006), Galor and Zeira (1993), Mookherjee and Napel (2007)), where conditions for convergence to steady state are investigated. We are unable to provide convergence conditions in general, but in our simulations the dynamics have always converged.

setting. Lang (1994) discusses the implications of the human capital *vis-a-vis* signaling debate for development policy, in the context of a static signaling model.[15] Both these papers abstract from credit market imperfections and stay within a static setting.

More closely related is Hendel, Shapiro and Willen (2005), who study the effect of combining credit constraints with educational signaling for skill premia. The main point of their paper is that expanding educational subsidies can increase skill premia, since they enable high ability individuals from poor backgrounds to acquire education, which lowers the wages of the uneducated. Hence skill premia and the skill ratio in the economy can move in the same direction, one of the results of our paper. However their model cannot allow any over-investment owing to a number of restrictive assumptions: there are two ability types in the population, and low ability types cannot obtain education regardless of initial wealth. Whether high ability types acquire education depends on their family wealth. We consider a more general model where over-investment can arise owing to signaling distortions, so as to examine the interaction between signaling and credit market imperfections, and implications of these for development patterns and policy implications.

Also related is Regev (2007), who provides a static signaling model where changing composition of skilled workers can explain rising skill premia at the same time that skill ratios are rising.[16] Capital market imperfections or dynamic considerations play no role in this paper.

6.2 Related Empirical Literature on Skill Premium Dynamics

Williamson (1985) provides a comprehensive treatment of earning inequality in the UK over the course of the 19th century. Irrespective of the inequality measure used, the evidence

[15] The main point argued by this paper is that it is incorrect to argue that a greater extent of imperfect information in the labor market among employers should increase ability bias.

[16] In this model, education is equally costly for high and low ability workers. Employers are able to learn worker abilities to some extent, causing more able workers to perceive a higher return to education. But employers do not learn ability perfectly, so education still has signaling value. In this context a rise in college costs can cause (owing to strategic interactions between education decisions of different ability individuals) a rise in the proportion of individuals that acquire education, as well as a rise in the skilled wage.

shows an increase in inequality from 1827 until 1851, and a subsequent fall between 1851 and 1901. For instance, the economy-wide Gini coefficient for male earnings rose from 0.293 in 1827 to 0.358 in 1851, falling thereafter to 0.328 in 1881 and 0.331 in 1901. Decomposing these inequality changes into the role of employment shifts across sectors, changing intra-occupational inequality and changing inter-occupational inequality, the dominant source for these trends was accounted for by inter-occupational inequality. In particular the 'pay ratio' or disparity between skilled and unskilled wages displayed the Kuznets pattern and accounted for "three quarters of the rise in total earning inequality both in the economy as a whole and in non-agricultural employment" (Williamson (1985, p.43)). The pay ratio (using 'variable' weights, i.e., different census year observations) in the economy as a whole rose from 2.452 in 1815 to 3.486 in 1861, and fell thereafter to 2.483 in 1911 (Williamson (1985, Table 3.7)). Williamson (1985, Ch. 10) subsequently argued that the two key factors driving these patterns in skill premia were 'unbalanced productivity advance' and 'skills per worker growth', supplemented by changes in world market conditions.

The evolution of skill premia in 19th century experience of the United States has been the subject of some controversy. Williamson and Lindert (1980) assembled a variety of previously published evidence concerning wages of skilled artisans and unskilled workers to argue that skill premia followed an inverted U in the US case. They claimed a sharp rise in skill premia from roughly 1820 to 1860 corresponding to early industrialization, followed by a more modest rise and then plateau in the late 19th century, and then a decline in the 20th century. These findings were criticized by subsequent historians (e.g., Margo and Villaflor (1987)) who failed to find similar patterns using other sources of evidence concerning the ratio of wages of skilled artisans to unskilled workers. However Margo (2000) subsequently provided evidence that in the four decades prior to the Civil War, real wages of white-collar workers grew faster (32%) than those of unskilled workers (21%) or artisans (15%). Combining his own estimates with those of Goldin (1998), Margo argues the evidence shows that the relative wage of white-collar workers remained stationary between 1850s to the late 19th century. Since the beginning of the 20th century the work of Goldin and Katz (2007, Figure 6, p.148) indicates that the wage premiums earned by both college and high school graduates fell sharply between 1915 and 1950 (the log of both wage ratios fell from around

0.6 to below 0.35 during this period). Putting together these accounts, it appears that a Kuznets pattern characterized skill premia in the US between 1820 and 1950: rising between 1820–60, stationary until the turn of the century and falling thereafter until 1950.

The evolution of skill premia since 1950 in the US has been the subject of considerable research and discussion (e.g., see summaries in Goldin and Katz (2007) or Acemoglu (2002)). Goldin and Katz argue that an important factor underlying the rise in skill premia since the late 1970s is a slowdown in the rate of increase in supply of skills, which failed to keep up with rates of skill biased technical change. For instance, Table 1 in Goldin-Katz (2007, p. 153) shows the annual rate of change in skill supply slowed from 3.83% during 1960–80 to 2.43% during 1980–2005, while the change in relative demand for skilled workers remained stationary (3.85% in the former period, and 3.76% in the latter). The slowdown in rates of skill accumulation reflect slower growth in educational attainment among natives, which slowed from 3.83% to 2.43% across these two periods. The causes of this are not explored further by Goldin and Katz, though they argue it is unlikely to result from reaching an 'upper bound for educational attainment', since returns to further educational investments continue to be substantial (Goldin-Katz (2007, p.157)).

Most accounts of skill premia dynamics focus on the 'race between technology and education' in a traditional supply-demand framework: rises in skill premia are explained by derived demand increases in the relative demand for skilled workers owing to skill-biased technical changes that outstrip increases in supply of skilled workers. Factors explaining technical change receive considerable discussion, and is treated either as exogenous or endogenous (e.g., Acemoglu (2002) argues that such technical change is endogenous and reacts to changes in the stock of skilled workers relative to unskilled workers). The factors underlying changes in supply of skills usually receives less discussion, except for changes in public schooling or educational subsidies: e.g., the decline is skill premia between 1915-1950 in the US is explained by Goldin and Katz by an increase in educational attainment owing to reforms in public schooling. Our model emphasizes other factors such as signaling and capital market imperfections which affect the supply of skills, which have hitherto received less attention.[17]

[17] See, however, Acemoglu (2002, pp. 65–68) who dismisses the possibility that changing composition of

Direct evidence of compositional effects in skill premium dynamics in the US during the 1980s and 1990s is provided by Steinberger (2006). Using cohort level panel data he shows that college degree holders in 1999 had higher measures of pre-college unobserved skills than degree holders in 1979. For new labor market entrants, improved skill sorting accounted for 4 to 9% of the increase in the return to college education between 1979 and 1999. This accounted for one third of the observed change for males and one sixth for females.

7 Concluding Comments

The purpose of this paper was to explore the dynamic and normative consequences of co-existence of missing credit markets and unobserved ability differences. The empirical evidence on returns to schooling across a large set of countries as reviewed by Card (2001) provide persuasive evidence of heterogeneity both in ability and other determinants of school access cost such as family income. Consistent with this, we have constructed an overlapping generations economy with borrowing constraints and educational signaling, and explored the rich dynamics of such a model. Kuznets patterns naturally emerge: the education wage premium initially widens and subsequently narrows along the process of development driven by human capital accumulation. Nevertheless, the model is capable of exhibiting other dynamic patterns as well, where education premia increase and decrease across successive generations.

The effects of educational policy interventions are complex, owing to variations in rates of return to education across individuals varying in ability and family background, and pecuniary externalities that generate divergences between social and private rates of return. The social returns to education exceed the market returns in the first stage of the Kuznets

educated workers can explain the rise in 'residual inequality' in the US. His argument implicitly assumes a perfect capital market, whence there is a single threshold for unobserved ability for acquiring education. In such a context, a rising supply of skills is accompanied by lowering average ability and wage of both skilled and unskilled workers. Our model demonstrates that with capital market imperfections, there are two thresholds corresponding to whether the corresponding parent is skilled or unskilled. Upward mobility of children from unskilled backgrounds can then cause average ability of the skilled to rise, while that of the unskilled falls or remains the same.

process, and fall below in the second stage. Despite this, some general lessons for policy emerge. Social rates of return vary across families with varying incomes, which provide a welfare rationale for government educational loan programs that attract able children from poor families. The programs are funded by government borrowing from by rich families with low ability children, for whom financial bequests (in the form of government bonds) dominate investments in education. Such programs are both Pareto improving and enhance equality of educational opportunity. They result in productive efficiency improvements by altering the composition rather than the size of the educated labor force.

In order to render the dynamics of the model tractable, it was kept deliberately simple: only two occupations, education is indivisible, productivity in the traditional sector is exogenous, and in the modern sector depends multiplicatively on ability and education. Credit markets are missing entirely, and parents have a paternalistic bequest motive which incorporates only the incomes earned by their children. There is scope for extending the model in different directions to make it more realistic, which are likely to be necessary to test it empirically. The question of convergence of non-steady-state dynamics in the simple setting also remains open.

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Proofs

Proof of Proposition 1: Since F and n^P, n^R are continuous functions, (L, Q) is a continuous map, so must have a fixed point, which establishes steady state existence.

To establish steady state is unique, note that given any $w \geq En$, $L(w, \lambda)$ is a contraction map in λ alone, since:

$$L(w, \lambda) = 1 - F(n^P(w)) + \lambda[F(n^P(w)) - F(n^R(w))]$$

and $0 < F(n^P(w)) - F(n^R(w)) < 1$ as $w \geq En > v$. Hence given w the map L has a unique fixed point which we denote by $\lambda(w)$:

$$\lambda(w) = \frac{1 - F(n^P(w))}{1 - \{F(n^P(w)) - F(n^R(w))\}}. \tag{11}$$

Clearly every steady state must satisfy $\lambda = \lambda(w)$. It must also satisfy $w = q(w) \equiv Q(w, \lambda(w))$. Using the fact that $[1 - \lambda(w)][1 - F(n^P(w))] = \lambda(w) - \lambda(w)[1 - F(n^R(w))] = \lambda(w)F(n^R(w))$, we can express

$$q(w) = \int_{n^R(w)}^{\bar{n}} nf(n)dn + \frac{F(n^R(w))}{1 - F(n^P(w))} \int_{n^P(w)}^{\bar{n}} nf(n)dn. \tag{12}$$

This implies

$$q_w = -n^R f(n^R) n_w^R - n^P f(n^P) \frac{F(n^R)}{1 - F(n^P)} n_w^P + [\frac{f(n^R)}{1 - F(n^P)} n_w^R + \frac{F(n^R) f(n^P)}{[1 - F(n^P)]^2} n_w^P] \int_{n^P}^{\bar{n}} nf(n)dn$$

so

$$q_w = [m(n^P) - n^R] f(n^R) n_w^R + [m(n^P) - n^P] \frac{f(n^P) F(n^R)}{1 - F(n^P)} n_w^P$$

which is negative since $m(n^P) > n^P > n^R$ and $n_w^P, n_w^R < 0$. Hence q cannot have more than one fixed point. This concludes the proof.

Proof of Proposition 2: (a) follows from the contraction property of L in λ for given w, since $\lambda_{t+1} = L(w_t, \lambda_t)$ and $\lambda(w_t)$ solves for λ in $\lambda = L(w_t, \lambda)$.

To prove (b), recall that $Q(w, \lambda) = \alpha m(n^R(w)) + (1 - \alpha) m(n^P(w))$ where α denotes $\frac{\lambda[1-F(n^R)]}{\lambda[1-F(n^R)]+(1-\lambda)[1-F(n^P)]}$. It is easily verified that α is increasing in λ. Moreover, Q is decreasing in α since $m(n^P(w)) > m(n^R(w))$. So $Q(w, \lambda)$ is decreasing in λ, implying that $Q(w, \lambda) - w$ is decreasing in λ.

If there exists $\lambda_1 \in (0, 1)$ such that $Q(w, \lambda_1) - w = 0$, define this to be $\lambda_1(w)$. If $Q(w, \lambda) - w < 0$ for all w, set $\lambda_1(w) = 0$. If $Q(w, \lambda) - w > 0$ for all w, set $\lambda_1(w) = 1$. Note that if $w > w^*$ then $Q(w, \lambda(w)) = q(w) < w$, implying $\lambda_1(w) < \lambda(w)$. Conversely, if $w < w^*$ then $Q(w, \lambda(w)) = q(w) > w$, implying $\lambda_1(w) > \lambda(w)$. This concludes the proof.

Proof of Remark 1 Note that

$$-n_w^P = \frac{V'(w)}{U'(v - x(n^P))[-x'(n^P)]} \tag{13}$$

while

$$-n_w^R = \frac{V'(w) + U'(w - x(n^R)) - U'(w)}{U'(w - x(n^R))[-x'(n^R)]}. \tag{14}$$

By definition of n^R, n^P we have

$$U(w) - U(w - x(n^R)) = U(v) - U(v - n^P) = V(w) - V(v)$$

implying that $U(w - x(n^R)) > U(v - x(n^P))$. Therefore $U'(w - x(n^R)) < U'(v - x(n^P))$. Since $n^P > n^R$, the concavity or linearity of x implies $-x'(n^R) \leq -x'(n^P)$. Then (13, 14) imply $-n_w^R > -n_w^P$. Combined with (8) and the non-increasing hazard rate, we obtain $\alpha_w > 0$. This implies $Q_w < 0$. In the interior of the state-space $\lambda_1(w)$ is the solution to $Q(w, \lambda) = w$, so $\lambda_1'(w) = \frac{1 - Q_w}{Q_\lambda} < 0$. This concludes the proof.

Proof of Proposition 3: The rational expectations dynamics are given by

$$\lambda_{t+1} = \tilde{\lambda}(w_{t+1}; w_t, \lambda_t) = \tilde{\lambda}(Q^R(w_t, \lambda_t); w_t, \lambda_t) \equiv L^R(w_t, \lambda_t)$$

and

$$w_{t+1} = Q^R(w_t, \lambda_t).$$

Fix any $\lambda \in [0, 1]$. Then $Q^R(.; \lambda)$ is decreasing and (a.e.)C^1. Define

$$w^R(\lambda) \equiv \sup\{w | Q^R(w, \lambda) \geq w\}$$

whence (b) follows. If Q^R is continuous in w at $w^R(\lambda)$ then $w^R(\lambda)$ must be the fixed point of $Q^R(.;\lambda)$. In that case it is evident that $w^R(.)$ is decreasing at λ. If Q^R jumps downward at $w^R(\lambda)$ then $Q^R(w,\lambda) < w$ in a left neighborhood of $w^R(\lambda)$ and $Q^R(w,\lambda) > w$ in a right neighborhood of $w^R(\lambda)$. If there exist $\hat{\lambda}$ and $\tilde{\lambda} > \hat{\lambda}$ such that $\tilde{w} \equiv w^R(\tilde{\lambda}) > \hat{w} \equiv w^R(\hat{\lambda})$ then there exist $\epsilon, \delta > 0$ such that $Q^R(\tilde{w}-\epsilon, \tilde{\lambda}) > \tilde{w}-\epsilon > \hat{w}+\delta > Q^R(\hat{w}+\delta, \hat{\lambda})$, contradicting the fact that Q^R is decreasing.

Next, note that $\lambda' > \lambda$ implies $n^R(Q^R(w,\lambda'),w) > n^R(Q^R(w,\lambda),w)$ and $n^P(Q^R(w,\lambda')) > n^P(Q^R(w,\lambda))$. Therefore

$$L^R(w,\lambda') - L^R(w,\lambda) < [\lambda'-\lambda][F(n^P(Q^R(w,\lambda))) - F(n^R(Q^R(w,\lambda)),w)] < \lambda'-\lambda. \quad (15)$$

This implies $L^R(w,.)$ has at most one fixed point. Now define

$$\lambda^R(w) \equiv \sup\{\lambda | L^R(w,\lambda) \geq \lambda\}$$

Then for all $\lambda \leq \lambda^R(w)$ we have $L^R(w,\lambda) \geq \lambda$, while for λ in a right neighborhood of $\lambda^R(w)$ we have $L^R(w,\lambda) < \lambda$. Property (15) then implies that $l^R(w,\lambda) < \lambda$ for all $\lambda > \lambda^R(w)$. This establishes (a).

(c) follows (a) and (b). Finally, for (d), if $L^R(w,\lambda)$ is increasing in w then $w' > w$ implies $L^R(w',\lambda) \geq L^R(w,\lambda) \geq \lambda$ for all $\lambda < \lambda^R(w)$, implying that $\lambda^R(w') \geq \lambda^R(w)$. This concludes the proof.

Proof of Proposition 4:

(a) Note first that $\delta[w_{t+1} - v] > x(n_t^P)$ for any t. This follows from concavity of U and the definition of n_t^P:

$$x(n_t^P)U'(v) < U(v) - U(v - x(n_t^P)) = \delta[U(w_{t+1}) - U(v)] < \delta(w_{t+1} - v)U'(v).$$

For (i) note that $w_t < \tilde{n}$ implies $\delta[\tilde{n} - v] > x(n_{t-1}^P)$, or $x(\tilde{n}) > x(n_{t-1}^P)$; hence $\tilde{n} < n_{t-1}^P$. On the other hand, if $n_{t-1}^P \leq \tilde{n}$ then there is over-investment among both poor and rich at $t-1$, so the average quality of the workforce in the modern sector is at most $m(n_{t-1}^P) \leq m(\tilde{n})$. Then $w_t \leq m(\tilde{n})$. Hence $w_t > m(\tilde{n})$ must imply under-investment among the poor at $t-1$.

For (ii), suppose $\lambda_t \leq \tilde{\lambda} = 1 - F(\tilde{n})$. By definition of λ_t:

$$\lambda_{t-1}[1 - F(n^R_{t-1}) + (1 - \lambda_{t-1})[1 - F(n^P_{t-1})] \leq \tilde{\lambda} = 1 - F(\tilde{n}).$$

Since the rich always use a lower threshold it follows that $F(n^P_{t-1}) > F(\tilde{n})$.

For (b), note that w_t is an average of $m(n^P_{t-1})$ and $m(n^R_{t-1})$, so $m(n^R_{t-1}) \leq w_t$. Hence $w_t < m(\tilde{n})$ implies $n^R_{t-1} < \tilde{n}$.

Finally consider part (iii) of (a). Recalling the definition of modern sector wages, we have:

$$w_{t+1} = \frac{m(n^R_t)\lambda_t[1 - F(n^R_t)] + m(n^P_t)(1 - \lambda_t)[1 - F(n^P_t)]}{\lambda_t[1 - F(n^R_t)] + (1 - \lambda_t)[1 - F(n^P_t)]}, \tag{16}$$

and

$$w_t = \frac{m(n^R_{t-1})\lambda_{t-1}[1 - F(n^R_{t-1})] + m(n^P_{t-1})(1 - \lambda_{t-1})[1 - F(n^P_{t-1})]}{\lambda_{t-1}[1 - F(n^R_{t-1})] + (1 - \lambda_{t-1})[1 - F(n^P_{t-1})]}. \tag{17}$$

Now define

$$\tilde{w}_{t+1} = \frac{m(n^R_t)\lambda_{t-1}[1 - F(n^R_t)] + m(n^P_t)(1 - \lambda_{t-1})[1 - F(n^P_t)]}{\lambda_{t-1}[1 - F(n^R_t)] + (1 - \lambda_{t-1})[1 - F(n^P_t)]}. \tag{18}$$

Note that $w_{t+1} > w_t > w_{t-1}$ implies $n^R_t < n^R_{t-1}$, and $n^P_t < n^P_{t-1}$.

We claim that $n^P_{t-1} > w_t$, which implies under-investment among the poor at $t-1$, since $\delta[n^P_{t-1} - v] > \delta[w_t - v] > x(n^P_{t-1})$ upon using the argument in (i) above.

Suppose otherwise, that $n^P_{t-1} \leq w_t$. Then $\tilde{w}_{t+1} < w_t$, since the former is the average quality of the modern sector workforce when the poor and rich use lower thresholds n^P_t and n^R_t instead of n^P_{t-1} and n^R_{t-1} respectively, and the rich households form the same fraction λ_{t-1} of the population. The size of the workforce is larger, and all those added have ability less than $n^P_{t-1} \leq w_t$. So the average quality of the workforce must fall below w_t.

Next, note that $\lambda_t > \lambda_{t-1}$ implies $\tilde{w}_{t+1} > w_{t+1}$. $\tilde{w}_{t+1}$ is the average quality of the modern workforce when rich and poor use the same thresholds, but the rich comprise λ_{t-1} fraction of the population, rather than λ_t. Since the rich use a lower threshold, and their fraction is higher at t than $t-1$, w_{t+1} must be lower than $\tilde{w}_{t+1}$.

Therefore it follows that $w_{t+1} < \tilde{w}_{t+1} < w_t$, contradicting the hypothesis that the skill premium rises from t to $t+1$. This concludes the proof.

Proof of Proposition 5:

Take any generation $t-1$ and let the competitive equilibrium in that generation involve ability thresholds n_P, n_R for households in the traditional and modern sector respectively. Consider the following intervention at $t-1, t$, consisting of the following mechanism. It is designed to leave the competitive equilibrium wage sequence unchanged. Parents in each sector report the ability of their child to the government, and then make a decision concerning their child's education. Parents (at $t-1$) and children when they become adults (at t) receive the following transfers, conditioned on their reports and education status. The scheme is defined by parameters $\varepsilon, I_R, \beta, T, \xi$ whose choice will be explained in due course.

For households in the modern sector:

(a) if $n \in H_R = (n_R + \varepsilon, \overline{n}] \cup [\underline{n}, n_R)$ is reported, *laissez faire* is implemented i.e. the household is not subject to any taxes or transfers, and is free to decide on the education to provide to its child at its own expense.

(b) if $n \in I_R \subset [n_R, n_R + \varepsilon]$, the government will offer the following lottery. With probability $(1-\beta)$, the household should not educate its child; instead it will be required to make a financial investment in a government bond, involving a contribution of $x(n_R+\varepsilon)$, in exchange for a return $w_t - v$ paid to the child at the next date. With the remaining probability β, the household has to educate her child at her own expense and in addition pay a tax T to the government. The expected payoff to the household is then:

$$\pi_I(n, T) = (1-\beta)\underbrace{[u(w_{t-1} - x(n_R + \varepsilon)) + \delta u(w_t)]}_{\text{Financial Bond}} + \beta\underbrace{[u(w_{t-1} - x(n) - T) + \delta u(w_t)]}_{\text{self-finance + tax}} \tag{19}$$

For households in the traditional sector:

(c) if $n \in H_P = (n_P, \overline{n}] \cup [\underline{n}, n_P - \xi)$ is reported, *laissez faire* is implemented.

(d) if $n \in I_P = [n_P - \xi, n_P]$ is reported, the child has to be enrolled in a public school in which the government spends $x(n_P - \xi)$ per child, and the child will be required to pay back $w_t - v$ to the government at time t.

The proof consists in showing that the parameters of this scheme: $\varepsilon > 0, \beta \in (0,1), I_R \subset [n_R, n_R+\varepsilon]$ and $T \in (0, w_{t-1} - x(n_R))$ can be chosen to ensure incentive compatibility, budget balance and a Pareto improvement, while leaving undisturbed the competitive equilibrium wage at every date. Given any $T \in (0, w - x(n_r))$ we shall construct all the other parameters as a function of T. Then we shall show that for T sufficiently close to $w_{t-1} - x(n_R)$ the scheme will have all the desired properties.

For parents in the traditional sector, parents with children of abilities below $n_P - \xi$ will not want to report I_P since their children will not be able to complete an education, and will thus remain unskilled at t, while having to pay back $w_t - v$ to the government. This is dominated by the laissez faire option where they do not educate their child. By construction, reporting a type in I_P by any parent with a child with ability at least $n_P - \xi$ will generate a payoff equal to what would obtain under laissez faire where the child is not educated. So those with types in I_P will not benefit by deviating to H_P. And those with children of ability greater than n_P were better off educating their children under laissez faire, so they would not benefit from deviating to I_P. Hence the mechanism is incentive compatible for parents in the traditional sector.

We now turn to incentives for parents in the modern sector. Define the function

$$\underline{\beta}(T) \equiv \frac{u'(w_{t-1} - x(n_R))}{u'(w_{t-1} - x(n_R) - T)} \tag{20}$$

for $T \in (0, \bar{T} \equiv w_{t-1} - x(n_R))$, which maps into $(0,1)$. As $T \longrightarrow \bar{T}$, we have $\underline{\beta}(T) \longrightarrow 0$ since $u'(c)$ tends to ∞ as $c \to 0$. Then select any smooth strictly decreasing function $\beta(T) \in (\underline{\beta}(T), 1)$ such that $\beta(T) \longrightarrow 0$ as $T \longrightarrow \bar{T}$.

Next define the function $\varepsilon(T)$ by the following condition

$$[1-\beta(T)][u(w_{t-1}-x(n_R+\varepsilon(T)))-u(w_{t-1}-x(n_R))] = \beta(T)[u(w_{t-1}-x(n_R))-u(w_{t-1}-x(n_R)-T)] \tag{21}$$

which ensures that type n_R is indifferent between H_R and I_R, i.e., $\pi_I(n_R, T) = \pi_H(n_R)$ for any positive $T < \bar{T}$, where $\pi_H(n) \equiv \max\{u(w_{t-1} - x(n)) + \delta u(w_t), u(w_{t-1}) + \delta u(v)\}$ denotes expected payoff in *laissez faire*.

Now $\beta(T) > \underline{\beta}(T)$ ensures that $[\pi(n,T) - \pi_H(n)]$ is strictly increasing in n at n_R for any $T \in (0, \bar{T})$. Hence types in a right neighborhood of n_R strictly prefer I_R to H_R.

Reporting a type in I_R is feasible only for those with $n > n(T)$, where $n(T)$ solves $w_{t-1} - x(n) = T$. As $T \longrightarrow \bar{T}$, $n(T) \longrightarrow n_R$ from below.

Note also that any type $n > n_R + \varepsilon(T)$ strictly prefers to report truthfully rather than reporting a type in I_R. Moreover $\varepsilon(T) \longrightarrow 0$ as $T \longrightarrow \bar{T}$, using (21) and the fact that $u(0)$ is finite.

For any $T \in (0, \bar{T})$, define I_R as the set of types between n_R and $n_R + \varepsilon(T)$ that strictly prefer I_R to H_R. Here we drop the dependence of the sets I_R and H_R on T to simplify the notation.

By construction, the mechanism is incentive compatible for all parents in the modern sector. It induces a reduction in the fraction of their children with types in I_R who get educated from 1 to $\beta(T)$. Let $\mu(T)$ denote the measure of the set I_R, given any $T \in (0, \bar{T})$). Then the fall in the measure of children in the modern sector who get educated is given by $\psi(T) \equiv \mu(T)[1 - \beta(T)]\lambda_{t-1}$, which is strictly positive and approaching 0 as $T \longrightarrow \bar{T}$.

Now select $\xi(T)$ such that $(1 - \lambda_{t-1})[F(n_P) - F(n_P - \xi(T))] = \psi(T)$, so $\xi(T)$ is positive and approaches 0 as $T \longrightarrow \bar{T}$. In a competitive equilibrium $n_P > n_R$. For T close enough to $\bar{T}$, we must therefore have $n_P - \xi(T) > n_R + \varepsilon(T)$. Since the scheme is incentive compatible for parents in the traditional sector, we have ensured an increase in education of children from the traditional sector that exactly offsets the decrease in education of children from the modern sector. Hence the proportion of skilled agents in the economy at t is exactly the same as before.

And the quality of skilled people at t is now higher, since the children from the traditional sector replaced children from the modern sector with lower abilities. This would raise the modern sector wage at t. This increase will be taxed away by the government, to ensure that the modern sector wage after taxes remains the same.

Parents in the modern sector at date $t-1$ with children abilities in I_R are strictly better off, while all other parents at $t-1$ are as well off as before. And payoffs from t onwards for all agents are unaffected. Hence the scheme is Pareto improving.

It remains to check that the scheme is financially feasible for the government. At date t the government collects a tax on modern sector wages, while redistributing the money paid by recipients of public schooling to the children of parents in the modern sector who contributed to the government bond at the previous date. Hence the government runs a surplus at t. At date $t-1$, a fraction $(1-\beta(T))$ of modern sector households with children abilities in I_R do not educate their children, and instead contribute $x(n_R+\varepsilon(T))$ to the government. The remaining fraction $\beta(T)$ educate their child at their own expense and pay taxes T. Hence the scheme raises revenues per modern sector household at $t-1$ of

$$[(1-\beta(T))x(n_R+\varepsilon(T))+\beta(T)T]\mu(T).$$

The expenses incurred in paying for public schools equals $(1-\lambda_{t-1})[F(n_P)-F(n_P-\xi(T))]x(n_P-\xi(T))$. Hence budget balance at $t-1$ is ensured if

$$(1-\lambda_{t-1})[F(n_P)-F(n_P-\xi(T))]x(n_P-\xi(T))\leq\lambda_{t-1}\mu(T)[(1-\beta(T))x(n_R+\varepsilon(T))+\beta(T)T]$$

or

$$(1-\beta(T))x(n_P-\xi(T))\leq[(1-\beta(T))x(n_R+\varepsilon(T))+\beta(T)T].$$

This is ensured as long as $x(n_P-\xi(T))\leq x(n_R+\varepsilon(T))$, which we saw will be the case for T sufficiently close to $\bar{T}$.

This concludes the proof. □

Appendix A: Example of Multiple Steady States

Consider a simple example with diminishing returns in the traditional sector, and where ability which matters only in the modern sector takes three possible values: $n \in \{I, N, G\}$ with $I < N < G$. One can find similar examples with continuously distributed abilities which are 'close' to this discrete distribution. Education costs are given by $x(I) = \infty$, $x(N) = X$ and $x(G) = 0$. Assume $N > X$. The distribution of abilities is given by: $p(I) = \varepsilon = p(G)$, $p(N) = 1 - 2\varepsilon$. Decreasing returns in the traditional sector induce the following wage formation process:

$$v = \begin{cases} v_2 & \text{if } \lambda > \lambda^* \\ v_1 & \text{if } \lambda < \lambda^* \end{cases}$$

where $\lambda^* > 1/2$ and $v_2 > X > v_1 > 0$. In the modern sector the wage formation process is the usual one, $w = E[n|e = 1]$. In the following we will show that for N large enough there exist at least two steady states.

A 'Poverty Trap' Steady State

Consider $\lambda < \lambda^*$, so the wage in the traditional sector is $v_1 < X$ and only G kids in the traditional sector can receive education. Upward mobility is $U = \varepsilon(1 - \lambda)$; downward mobility depends on the wage in the modern sector which we compute below. In the households from the modern sector I kids do not receive education, G kids always receive education, N kids will be educated provided $w^e - v$ is large enough. For the moment we suppose it is, we verify this later. Hence, in this case, downward mobility is given by $D = \varepsilon\lambda$. The steady state skill ratio is obtained by equating upward and downward mobility: $\varepsilon(1 - \lambda) = \varepsilon\lambda$ i.e., $\lambda_1^{ss} = 1/2 < \lambda^*$. The wage in the modern sector is given by

$$\begin{aligned} w_1^{SS} &= \frac{\lambda_1^{ss}[(1 - 2\varepsilon)N + \varepsilon G] + \varepsilon(1 - \lambda_1^{ss})G}{\lambda_1^{ss}} \\ &= \frac{\lambda_1^{ss}(1 - 2\varepsilon)N + \varepsilon G}{\lambda_1^{ss}} \\ &= (1 - 2\varepsilon)N + \frac{\varepsilon G}{\lambda_1^{ss}}. \end{aligned}$$

A necessary condition for $\lambda_1^{ss} = 1/2$ to be an equilibrium is $w_1^{SS} > X$ which is easily verified: $(1-2\varepsilon)N + \frac{\varepsilon G}{\lambda} > X$ for $N > X$. Then a sufficient condition for $\lambda_1^{ss} = 1/2$ and w_1^{SS} to form an equilibrium is:

$$U(w_1^{SS}) - U(w_1^{SS} - X) < V(w_1^{SS}) - V(v_1)$$

With $U = V = \ln(.)$ this reduces to

$$\begin{aligned} \frac{w_1^{SS}}{w_1^{SS} - X} &< \frac{w_1^{SS}}{v_1} \\ w_1^{SS} - X &> v_1 \end{aligned}$$

which is verified for $N > X + v_1$, since:

$$\begin{aligned} (1-2\varepsilon)N + \frac{\varepsilon G}{\lambda} &> v_1 + X \\ (1-2\varepsilon)N + 2\varepsilon G &> v_1 + X \end{aligned}$$

and $G > N > X + v_1$.

Developed Steady State

Consider $\lambda > \lambda^*$, then the wage in the traditional sector is $v_2 > X$, so N kids in the traditional sector can also receive education, provided their parents have incentives to invest, which we assume for the moment and verify later. In this case N and G kids in both sectors receive education.

Hence, upward mobility is given by: $U = (1-\varepsilon)(1-\lambda)$ whereas downward mobility is given by: $D = \varepsilon\lambda$ The steady state skill ratio is given by $(1-\varepsilon)(1-\lambda) = \varepsilon\lambda$, or $\lambda_2^{ss} = 1 - \varepsilon > \lambda^*$, for $\varepsilon < 0.5$.

Remember that at $\lambda_2^{ss} > \lambda^*$, we have $v = v_2 > X$, whereas the wage in the modern sector is given by:

$$\begin{aligned} w_2^{SS} &= \frac{\lambda[(1-2\varepsilon)N + \varepsilon G] + (1-\lambda)[(1-2\varepsilon)N + \varepsilon G]}{\lambda} \\ &= \frac{(1-2\varepsilon)N + \varepsilon G}{1-\varepsilon} \\ &= \frac{(1-2\varepsilon)}{1-\varepsilon}N + \frac{\varepsilon}{1-\varepsilon}G \end{aligned}$$

Notice that $\frac{(1-2\varepsilon)}{1-\varepsilon}N + \frac{\varepsilon}{1-\varepsilon}G < (1-2\varepsilon)N + 2\varepsilon G$ for $N < G$ and $\varepsilon < 1/2$, therefore $w_2^{SS} < w_1^{SS}$.

So $w_2^{SS} > v_2 (> X)$ whenever:

$$\begin{aligned} \frac{(1-2\varepsilon)}{1-\varepsilon}N + \frac{\varepsilon}{1-\varepsilon}G &> v_2 \\ (1-2\varepsilon)N + \varepsilon G &> (1-\varepsilon)v_2 \end{aligned}$$

which is satisfied for $N > v_2$.

In order for λ_2^{ss}, w_2^{SS} to form an equilibrium we verify incentives to invest in households in the modern sector:

$$U(w_2^{SS}) - U(w_2^{SS} - X) < V(w_2^{SS}) - V(v_2)$$

and in the traditional sector:

$$U(v_2) - U(v_2 - X) < V(w_2^{SS}) - V(v_2)$$

Consider $U = V = \ln(.)$, then incentives to invest in households in the modern sector are satisfied whenever:

$$\begin{aligned} \frac{w_2^{SS}}{w_2^{SS} - X} &< \frac{w_2^{SS}}{v_2} \\ v_2 + X &< w_2^{SS} \\ v_2 + X &< \frac{(1-2\varepsilon)}{1-\varepsilon}N + \frac{\varepsilon}{1-\varepsilon}G \end{aligned}$$

and this is satisfied for $N > v_2 + X$.

In households in the traditional sectors incentives to invest exist whenever:

$$\begin{aligned} \frac{v_2}{v_2 - X} &< \frac{w_2^{SS}}{v_2} \\ (1-\varepsilon)(v_2)^2 &< (v_2 - X)[(1-2\varepsilon)N + \varepsilon G] \end{aligned}$$

Since $G > N$ the inequality above is preserved if:

$$(v_2)^2 < (v_2 - X)N$$

Hence for N sufficiently large, i.e. $N > \tilde{N} = (v_2)^2/(v_2 - X) > v_2 + X\lambda_2^{ss}$, w_2^{SS} does form a steady state.

Appendix B: Parental Status Observable by Private Employers

In the following we explore the implications of allowing employers in the modern sector to observe the occupation of the parent, and condition wage offers on this.

If a parent with income y anticipates that her child if educated will receive a wage offer of w^e in the modern sector, she will decide to educate her child if and only if the ability of the latter exceeds the threshold $n * (w^e; y)$ which solves

$$U(y) + V(v) = U(y - x(n)) + V(w^e)$$

Clearly the threshold n^* is decreasing in parental income y, implying that a high parental income is a negative signal to an employer about the ability of an educated job applicant. Incorporating this, employers will offer a wage equal to the expected ability of the applicant, which is in turn is a function of the wage offered. In equilibrium with correctly anticipated wages, the wage $w(y)$ will solve $w = E[n|n \geq n^*(w; y)]$. It is evident then that the wage will be decreasing in parental income.

This implies that the equilibrium ability thresholds used by parents to make the education decision will be decreasing in income. It will still be the case that there will be a misallocation across traditional and modern sectors with respect to education decisions, with children from traditional sector households smarter on average than those from modern sector households. The key inefficiency in the market equilibrium in the case where employers cannot condition on parental backgrounds therefore extends to this case.

The nature of income distribution dynamics will be qualitatively different, however, in some respects. It can be shown that there will be a unique steady state, with a non-degenerate wage distribution within the modern sector. There will be wage dispersion in the modern sector as it will include educated workers with disparate parental backgrounds. For instance there will be some whose parents were in the traditional sector, who were smart enough to exceed the high threshold in that sector, who received an education. And there will also be those whose parents were in the modern sector, whose abilities exceed the threshold used by their parents. The latter will receive a lower wage offer than the former.

A major distinction from our model is that here there is no interdependence of wages across households: the equilibrium wage for anyone in the modern sector depends only on the income of the parent of the worker, which is observed by the employer. Conditioning on this information, wages of other modern sector workers in the economy does not matter for the determination of employers' assessments of ability. The key pecuniary externality in our model – wherein employers use the single economy-wide modern sector wage prevailing in the previous generation to form their expectations of ability of educated people in the current generation – therefore no longer obtains.

We do not provide a formal account of the dynamics in this case. Under weak assumptions on the ability distribution (viz. that it is dispersed enough at the bottom end), the dynamic process over the wage distribution is ergodic, as it satisfies condition M of Stokey and Lucas (1989): at any stage there is a probability bounded away from zero that a child will end up working in the traditional sector and hence receiving wage v as an adult. The lower endpoint of the support of the distribution is v, and the upper endpoint is $w(v)$. An educated person whose parents were from the traditional sector will receive the highest wage $w(v)$ in the economy. Those whose parents were from the modern sector will receive a wage which is decreasing in the wage that their parents received. Hence the model predicts that *conditional on two successive generations of the same family remaining in the modern sector, there will be a negative correlation between wages of parents and children.* The sign of the unconditional correlation is ambiguous, as the probability of children going to the modern sector is higher for families with parents in the modern sector. If the proportion of agents in the modern sector is high enough, the unconditional correlation will be negative, as it will then be close to the conditional correlation. In contrast in our model where wages cannot be conditioned on parental incomes, the conditional correlation will be positive during phases where modern sector wages are rising, and the unconditional correlation will always be positive. It therefore appears that the version corresponding to the assumption of inability of employers to conditional wages on parental background, is more plausible empirically.

Appendix C: Linear Utility

Suppose $U(c) = c$ and $V = \delta U$, whence credit constraints do not affect education decisions. In this case parental income has no effect on the ability threshold $n^*(w)$ corresponding to anticipated modern sector wage w, the former solving $x(n) = \delta[w - v]$. Competitive equilibrium then involves a stationary wage w^* which solves $w = m(n^*(w))$. It is evident that such a wage is uniquely defined.

Such a model therefore displays no dynamics at all, and predicts a zero intergenerational parent-child correlation in incomes and occupations. Moreover, the key inefficiency of our model disappears, as all parents make education decisions in the same way, so there is no misallocation between households in different sectors.

With regard to normative properties, the capital market imperfection plays no role at all, and only the signaling distortion applies. Accordingly, the model exhibits over-investment, as the marginal entrant to the modern sector has an ability $n^*(w^*)$ which solves $x(n) = \delta[m(n) - v] > \delta[n - v]$.

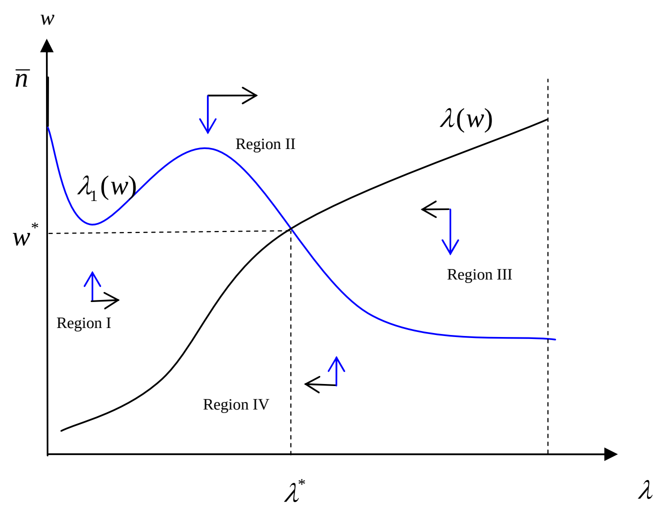


Figure 1.a: Static Expectations Dynamics

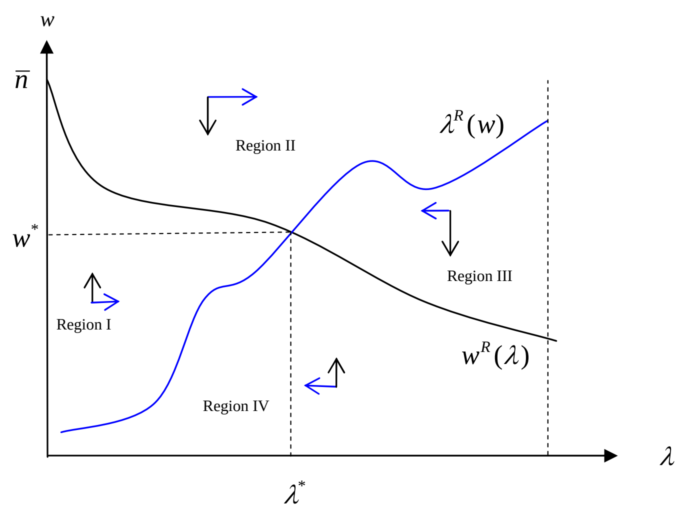
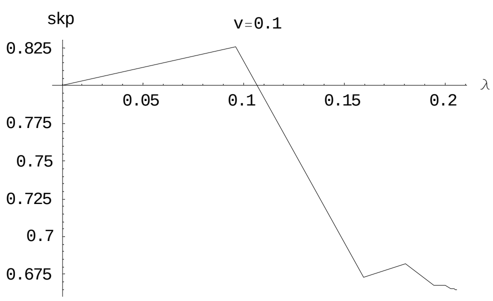
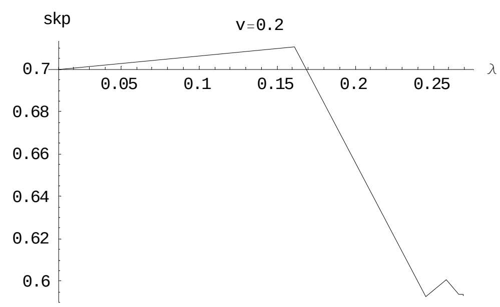


Figure 1.b: Rational Expectations Dynamics

Figure 2.Simulation, baseline model.

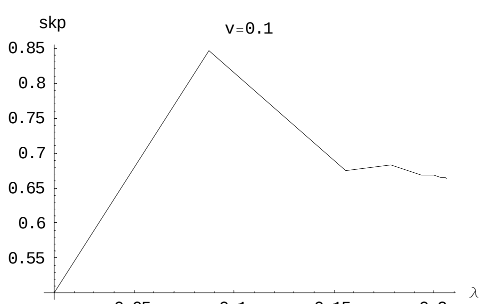


2.a

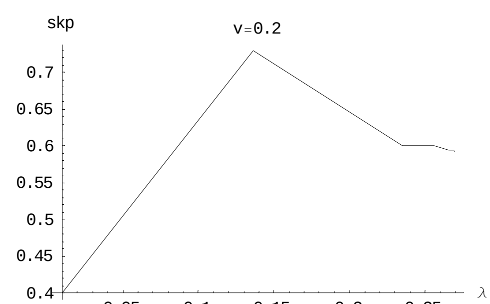


2.b

Uniform distribution on [0,1], log utility, CRS; $w(0) = 0.9, \lambda(0) = 0.01$, 3.a: v=0.1, 3.b: v=0.2.



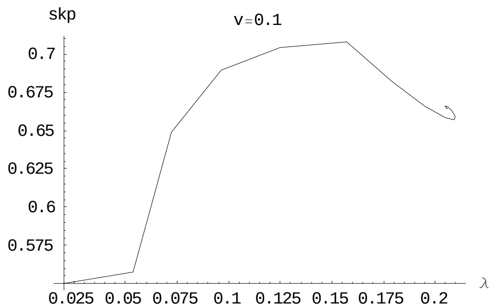
2.c



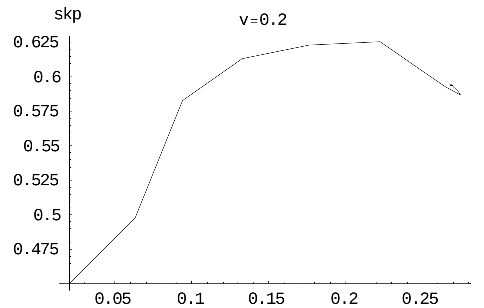
2.d

Uniform distribution on [0,1], log utility, CRS; $w(0) = 0.6, \lambda(0) = 0.01$; 3.c v=0.1, 3.d v=0.2.

Figure 3. Simulation, extensions.

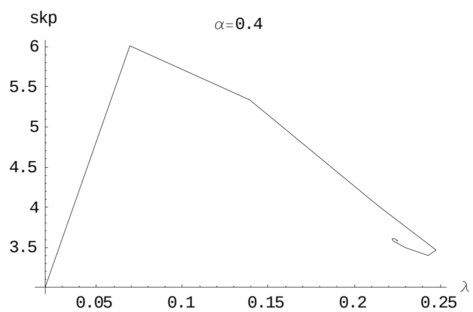


3.a

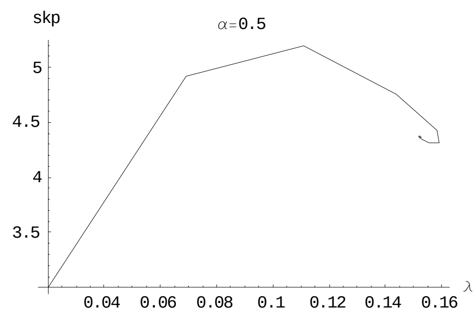


3.b

Slowed down dynamics with 5 periods of working life per cohort, static expectations, Uniform distribution on [0,1], log utility, CRS; $w(0) = ... = w(4) = 0.65$, $\lambda_c(0) = ... = \lambda_c(4) = 0.041$, $\lambda(0) = ... = \lambda(4) = 0.1$; 3.a: v=0.1, 3.b: v=0.2.



3.c



3.d

CES production function $y = [(A_h H)^\alpha + (A_l L)^\alpha]^{1/\alpha}$. Static expectations, Uniform distribution on [0,1], log utility, CRS; A_h: average ability in the skilled sector, A_l=0.1, $w(0) = 0.3$, $v(0)$=0.1, $\lambda(0) = 0.01$; 3.c: α=0.4, 3.d: α=0.5.