On the impossibility of informationally efficient markets

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Basic idea

- It is costly to become informed about underlying economic fundamentals. Hence, not all indvidiuals will choose to acquire information: the return to being fully informed (in terms of better trading returns) must just balance the cost of information for the marginal informed investor.
- Hence, prices will not reflect all available information, so long as there are limits on the positions taken by informed traders.

Model elements

- Individuals can invest their wealth in one of two assets, a riskless asset with return R and a risky asset with return u.
- The risky return u = $\rho+\theta+\varepsilon$, where ρ is the unconditional expected return, θ can be learned at a cost and "ε" is unknown. That is, even the informed face risk. Both and "θ" and "ε" are independent, zero mean normal random variables, so that "u" is also a normal random variable. [Notation differs slightly from GS in ways that will make some analysis easier below. However, we will assume that all random variables are normal, as they do].
- An individual (trader) must decide how much wealth to place in the risky security (call that amount x) and how much to place in the riskless security (wealth – x).

GS use a particular utility function

- Called constant absolute risk aversion, it specifies that utility of wealth (W) is

$$V(W_i) = -e^{-aW_i}$$

$$V'(W_i) = ae^{-aW_i}$$

$$V''(W_i) = -a^2 e^{-aW_i}$$

V is increasing and concave in wealth

A convenient implication

- For any trader, the demand for the risky asset arising from this utility function is

$$x = \frac{Eu \mid D - RP}{a \, \mathrm{var}(u \mid D)}$$

- In this expression Eu|D and var(u|D) mean the expected return given information (data) that the individual has. The parameter “a” indexes (absolute) risk aversion. The higher is “a”, the lower is the demand for risky assets

Informed demand and uninformed demand

- Informed guys have paid to see (θ) so their demand is low or high based on this variable.
- Uniformed guys can only learn from prices

$$x_I = \frac{\rho + \theta - RP}{a\sigma_\varepsilon^2} \qquad \text{and} \qquad x_U = \frac{Eu \mid P - RP}{a\,\mathrm{var}(u \mid P)}$$

- Prices clear markets at a supply x that is random, x=μ+z, with z being a zero mean random variable

$$\lambda x_I + (1-\lambda) x_U = x$$

What do the uninformed learn from price?

- These agents understand the structure of the economy. Prices clear markets, so

$$\lambda x_I + (1-\lambda) x_U = x$$

$$\Rightarrow \lambda[\frac{\rho + \theta - RP}{a\sigma_e^2}] + (1\text{-}\lambda)\frac{Eu \mid P - RP}{a\,\mathrm{var}(u \mid P)} = x = \mu + z$$

- Thus the equilibrium price must depend on
  - The random variables θ and z
  - The other variables in this expression, which are assumed will turn out to be constants

What do the uniformed learn from price?

- Suppose that there is a linear price function of the form, $P = \pi_0 + [\pi_\theta \theta + \pi_z z]$

  $= \text{ prior mean + [new info]}$

- This captures the idea that price depends on θ and z, but does not capture the exact form without restrictions on the π coefficients.
- Before seeing the price or buying the signal, the expected price is π_0 because the random variables θ and z have mean zero.

Regression and expectation revision with normal random variables

- General concept

Suppose that w and v are multivariate normal, with mean m_w and m_v Then, the conditional expectation takes a familar regression form

$$Ew\,|\,v = m_w + \frac{\mathrm{cov}(w,v)}{\mathrm{var}(v)}(v - m_v)$$

- Application here

$$w = u = \rho + \theta + \varepsilon \quad \text{v=P=}\pi_0 + [\pi_\theta \theta + \pi_z z]$$

$$Eu\,|\,P = \rho + \frac{\pi_\theta \sigma_\theta^2}{\pi_\theta^2 \sigma_\theta^2 + \pi_z^2 \sigma_z^2}[P - \pi_0]$$

$$= \rho + \beta[\pi_\theta \theta + \pi_z z]$$

How do beliefs respond to price?

- If the price is very informative about θ, because there is no z, then beliefs move strongly with price and there full revelation of information
- Mathematically, if we drop the terms with bars,

$$Eu \mid P = \rho + \frac{\pi_\theta \sigma_\theta^2}{\pi_\theta^2 \sigma_\theta^2 \overline{+ \pi_z^2 \sigma_z^2}} [\pi_\theta \theta + \overline{\pi_z z}]$$

$$= \rho + \theta$$

Variance of expectation errors

$$\mathrm{var}(u \mid P) = E(u - Eu \mid P)^2$$

$$= \sigma_\theta^2 + \sigma_e^2 - \beta^2[\pi_\theta^2\sigma_\theta^2 + \pi_z^2\sigma_z^2]$$

$$= \sigma_\theta^2 + \sigma_e^2 - \frac{\pi_\theta^2\sigma_\theta^2}{\pi_\theta^2\sigma_\theta^2 + \pi_z^2\sigma_z^2}\sigma_\theta^2$$

constant that depends on information content of market price. If no noise, then price fully reflects "θ" and uncertainty only from "ε". If large noise, no reduction in uncertainty from seeing price

Solving for price

- Find π coefficients that are consistent with market equilibrium for all realizations of shocks

$$\lambda[\frac{\rho+\theta-RP}{a\sigma_{\varepsilon}^{2}}]+(1\text{-}\lambda)\frac{Eu\,|\,P-RP}{a\,\mathrm{var}(u\,|\,P)}=x=\mu+z$$

$$\Rightarrow A[\rho+\theta-RP]+B[Eu\,|\,P-RP]=C\mu+Cz$$

$$\Rightarrow A[\rho+\theta-R\pi_0-R\pi_\theta\theta-R\pi_z z]+B[\rho-R\pi_0+(\beta-R)(\pi_\theta\theta+\pi_z z)]=C\mu+Cz$$

$$A=\lambda a\,\mathrm{var}(u\,|\,P);B=(1\text{-}\lambda)a\sigma_{\varepsilon}^{2};C=[a\,\mathrm{var}(u\,|\,P)][a\sigma_{\varepsilon}^{2}]$$

Solving for price (cont'd)

$$A[\rho + \theta - R\pi_0 - R\pi_\theta \theta - R\pi_z z] + B[\rho - R\pi_0 + (\beta - R)(\pi_\theta \theta + \pi_z z)] = C\mu + Cz$$

$$\Rightarrow A[\rho - R\pi_0] + B[\rho - R\pi_0] = C\mu \qquad \Rightarrow \pi_0 = \frac{\rho}{R} - \frac{C\mu}{(A+B)R}$$

$$\Rightarrow A[1 - R\pi_\theta] + B[(\beta - R)\pi_\theta] = 0 \qquad \Rightarrow \pi_\theta = \frac{A}{AR + B(R-\beta)}$$

$$\Rightarrow A[-R\pi_z z] + B[(\beta - R)\pi_z] = C \qquad \Rightarrow \pi_z = \frac{-C}{AR + B(R-\beta)}$$

$$P = [\frac{\rho}{R} - \frac{C\mu}{(A+B)R}] + [\frac{A}{AR + B(R-\beta)}][\theta - \frac{C}{A} z]; \quad \frac{C}{A} = \frac{[a \operatorname{var}(u \mid P)][a\sigma_\varepsilon^2]}{\lambda a \operatorname{var}(u \mid P)} = \frac{a\sigma_\varepsilon^2}{\lambda}$$

Price is a noisy signal

- The supply shift z "obscures" the signal (θ) that future profits are high

$$P = [\frac{\rho}{R} - \frac{C\mu}{(A+B)R}] + [\frac{A}{AR + B(R-\beta)}][\theta - \frac{a\sigma_{\varepsilon}^{2}}{\lambda} z]$$

- The more informed traders there are (higher λ), the lower the variability of this noise (also if informed traders are less risk averse (lower a) or face less uncertainty about returns.

GS's summary

- "If λ>0, the price system conveys information about θ, but it does so imperfectly"
- In terms of paper's title, the market is not fully efficient even if λ is close to 1, as it does not reflect the information about θ as it would if it were common knowledge

$$[\frac{\rho+\theta-RP}{a\sigma_{\varepsilon}^{2}}]=\mu+z \Rightarrow P=\frac{1}{R}[\rho+\theta-(a\sigma_{\varepsilon}^{2})(\mu+z)]$$

Informational equilibrium

- Suppose that there is a cost to becoming uniformed (some expenditure of wealth).
- Then, at the time of deciding to purchase information (before knowing what the information will be) individuals must be indifferent between
  – Becoming informed at a cost
  – Remaining uninformed

Informational equilibrium

- GS compute the welfare for individuals under this assumption and find the fraction (λ) that is an equilibrium in the market for information.
- The derivations are somewhat involved, so that we do not present these. However, they obtain a number of intuitive results, including:

Some implications

1. An increase in the quality of information (var(q)/var(e)) increases the informativeness of the price system
2. A decrease in the cost of information increases the informativeness of the price system.
3. A decrease in risk aversion leads to larger positions by informed individuals, so increasing price system informativeness.

Information externality

- Some benefit is conferred on the uniformed by each purchaser of information, as the price becomes a sharper signal.
- Individuals do not take this into account in deciding whether to become informed.
- Thus, plausibly, there is too little information production relative to the socially optimal level.

Links to Efficient markets

- GS: We are attempting to redefine the Efficient Markets notion, not destroy it. We have shown that when information is very inexpensive, or when informed traders get very precise information, then equilibrium exists and the market price will reveal most of the informed traders' information. How- ever, it was argued in Section III that such markets are likely to be thin because traders have almost homogeneous beliefs.

Links to efficient markets

- GS: We have argued that because information is costly, prices cannot perfectly reflect the information which is available, since if it did, those who spent resources to obtain it would receive no compensation. There is a fundamental conflict between the efficiency with which markets spread information and the incentives to acquire information.