On Measuring Vulnerability to Poverty[*]

Indranil Dutta
University of Manchester, UK

James Foster
George Washington University, USA

Ajit Mishra
University of Bath, UK

ABSTRACT

There is a growing interest on dynamic and broader concepts of deprivation such as vulnerability, which takes in to account the destitution of individuals from future shocks. We use the framework of decision making under uncertainty to arrive at a new measure of vulnerability to poverty. We highlight the importance of current standard of living to better capture the notion of vulnerability. In conceptualizing the new class of measures of vulnerability we thus move beyond the standard expected poverty measures that is commonly found in the literature. We also axiomatically characterize the new class of measure and discuss some of it's properties.

April, 2010.

1 Introduction

In recent years development policy has increasingly been linked to poverty reduction. While it is important to focus on poverty, there is a growing recognition that reducing just the level of poverty may not be a wholly satisfactory approach to development. According to Amartya Sen (Asia Week, October 1999), "..the challenge of development includes not only the elimination of persistent and endemic deprivation, but also the removal of vulnerability to sudden and severe destitution." In a similar vein the World Bank (1998) states that, "Protecting vulnerable groups during episodes of macroeconomic contraction is vital to poverty reductions in developing countries." Although the new emphasis has lead to an increased attention on vulnerability, important questions about what we exactly mean by vulnerability, and how we should measure vulnerability remains open. In this paper we conceptualize and characterize new class of vulnerability indices.

Vulnerability is widely used in a variety of contexts from climate change to food insecurity. For most purposes, however, vulnerability measures are composite indices mainly at the country level, which combine factors that captures a country's proneness to shocks and its' ability to recover from shocks.[1] While this approach may have its merits, especially given that aggregate information is readily available, we follow a more micro-theoretic approach where vulnerability of each individual is first calculated and then individual vulnerabilities are aggregated to form the society or country's vulnerability. The latter approach is very similar to the measurement of poverty, where a society's poverty is the aggregate sum of individual poverty levels (see Sen 1976).

An individual can be vulnerable to falling below a threshold across several dimensions, such as health, food consumption and income, and across

---

[1]See for instance the Commonwealth Vulnerability Index in Atkins et al (2000) or the Economic Vulnerability Index in Bruguglio and Galea (2003).

different time periods. For simplicity, we restrict ourselves to conceptualizing vulnerability along a single dimension which we consider to be income.[2] An important aspect of vulnerability where one has to ex-ante estimate what happens in the future adds a layer of complexity to the concept. While it is straightforward to calibrate individual's poverty level (i.e. shortfall from the poverty line), to measure an individual's income vulnerability we need to know the possible states of the world in the future and the probability of their occurrence. The information on the different states of the world becomes significantly more complicated and difficult to obtain as the length of the future increases. Thus, as in other studies such as Kamanou and Morduch (2002) and Lighon and Schecter (2003) we restrict ourselves to measuring vulnerability just one period ahead.

An early study which attempted to empirically estimate vulnerability was by Pritchett et al (2000). Vulnerability was defined as the probability of falling below the poverty line in any of three consecutive time periods in the future. Other papers such as Christianensen and Boisvert (2000), Chaudhuri et al (2002) and Chaudhuri (2003) followed along similar lines to measure vulnerability. A major drawback of these papers is that they fail to consider the depth of the fall below the poverty line. More recently several papers including Kamanou and Morduch (2002), Lighon and Schecter (2003) and Christianensen and Subbarao (2005) extends this framework to include the depth of the loss but the analysis is confined to only one time period ahead. In particular Lighon and Schecter (2003) employ a slightly different measure where they assume a specific individual utility function through which they include a relative risk aversion parameter and base their analysis on the expected short fall in utility in the future. Building on these work, Calvo and Deron (2006) axiomatically characterize a new measure of vulnerability which is sensitive to the size of the loss – increasing loss

---

[2]While income vulnerability has an obvious policy importance, our analysis can be applied to other dimensions such as food consumption.

increases vulnerability at an increasing rate. The common thread across all these different measures is that they can broadly be classed as expected poverty measures. So for instance the Lighon and Schecter measure is the expected poverty gap, whereas Calvo and Dercon's measure is the expected Chakravarty index and Kamanou and Morduch (2002) employs the expected Foster-Greer-Thorbecke (FGT) index. A survey of the expected poverty measures of vulnerability can be found in Hoddinott and Quisumbing (2003).

One implication of the expected poverty measures would be that vulnerability apart from accounting the poor will also include people living on the edge. As a consequence it will always indicate a higher percentage of people who are vulnerable than who are poor.[3] In other words the set of poor will always be a subset within the broader set of the vulnerable.[4] Therefore, it is not a surprise that Ersado (2008), which adopts the methodology of Lighon and Schecter (2003), finds the "factors determining poverty and vulnerability are quite similar."

The other broad approach to measuring vulnerability is to consider the variations around a given level of income which is different from the poverty line. Morduch (1994) suggests deviations from the permanent income line as a measure of vulnerability. More specifically he suggests considering the inability to smooth consumption as a component of poverty. Consumption smoothness as a method of risk sharing and reduction of vulnerability has also been studied by Dercon and Krishnan (2000). This concept of relating the lack of consumption or income smoothing to vulnerability, has serious drawbacks including the fact that standard deviations around a given consumption path may not be a good indicator of the vulnerability that indi-

---

[3]One exception to this is the paper by Basu and Nolen (2005) where as the risk of falling in to unemployment spreads across greater proportion of the population, there is a decrease in overall societal vulnerability.

[4]Chaudhuri et al (2002) finds that the set of poor and the set of vulnerable are distinct within the expected poverty framework. This result, however, holds only under the assumption that individuals with expected poverty less than 0.5 are not vulnerable.

viduals may face with uncertain future income.[5] This method, however, has the advantage of conceptually distinguishing poverty from vulnerability and thus may yield separate sets of policy prescriptions to reduce vulnerability and poverty.

In this paper we develop a new measure of vulnerability which is distinct from expected poverty measures yet does not have the drawbacks of the consumption smoothing approach. We draw on the two broad approaches to measuring vulnerability to put forth a hybrid measure which includes the shortfalls as in the expected poverty measures but it also imbibes the individualistic aspect of the consumption smoothing approach where individuals may have different minimum income levels (or standard of living) which they strive to maintain in future periods. Unlike most of the current literature on vulnerability we provide a full axiomatic characterization of our proposed measure. The plan of the paper is as follows: the next section demonstrates that vulnerability to poverty is not just expected poverty but is a distinct concept from poverty; hence the set of poor or expected poor will not necessarily be a subset of those who are vulnerable. In the following sections we introduce and motivate the axioms and characterize a class of vulnerability measure. We then go on to discuss a specific example of the measure. We conclude by highlighting future directions of research.

2   The concept of Vulnerability measure

In general vulnerability at the individual level can be thought in terms of the uncertainty in the outcomes of different indicators such as income and consumption that the individual faces in the future. When it comes to conceptualizing vulnerability, we start with some broad characteristics that we expect a reasonable vulnerability measure to satisfy.

---

[5]For a discussion of the shortcomings of this approach see Christianensen and Subbarao (2005).

4

First, a measure of vulnerability has to be an ex-ante measure in the sense it should inform us about potential deprivations in the future. Vulnerability is different from other measures of ill-being in essence for being a dynamic concept that anticipates the loss of future income today. Second, typically vulnerability is associated with a negative outcome. A reasonable measure of vulnerability thus have to focus on downside risk. In other words, we are interested in the shortfalls (from a given a reference point) rather than the gains. The literature so far have mainly considered the short falls from the poverty line – a notion that we shall question later. Third, vulnerability is an individual specific concept since each individual views risk differently and therefore same shortfalls in income may reflect different levels of vulnerability. This difference is also reflected in the fact that for same levels risk households do undertake different coping strategies. A one size fits all framework may not be appropriate in this context.

While we agree that vulnerability should be about downside risk, one important way in which our conceptualization of vulnerability differs from the literature is by abandoning the assumption that the shortfall in income is essentially the shortfall from a given poverty line. In a detailed study across four communities in different parts of the world Moser (1996) finds that "individuals and households ... mobilise their assets to protect their standard of living in the face of economic crisis." Similarly, Ersado (2008) in the context of calibrating vulnerability in rural Siberia argues that "..in measuring vulnerability not only should current income and consumption should be taken in to account but their assets and changes in assets over time...." Our methodology explicitly accounts for individual's current standard of living since it may convey important elements about a person's vulnerability. In this context the standard of living represents a broad set of factors including individual's assets and income along with other dimensions of well-being such as health.

Individuals may be vulnerable if they are unable to maintain in the future a certain minimum standard of living which may be different from the poverty line. The current standard of living, especially if it is low, also may indicate the severity of a future fall in to poverty. In other words, individuals with low current standard of living may suffer more severely from a down turn in the future than some one with a higher current standard of living. Just pegging vulnerability to current standard of living, however, would make it very individualistic and we may end up declaring a person whose annual income may reduce from million dollars to half a million as more vulnerable compared to one whose may decrease from $300 to $200. Therefore, we consider a reference line for each individual which is composed of their current standard of living and the poverty line. The shortfall from the reference line then represents vulnerability. The poverty line, which indicates the minimum level of income below which individuals suffer from absolute deprivation, is the same for everyone. The reference line on the other hand may differ for each individual because it also takes in to account their current living standards. The merits of this type of hybrid reference lines with a relative and absolute component have been discussed by Foster (1998).

As is the case with most of the literature, we presume that if the poverty line increases, so should the reference income line. It is quite reasonable that the minimum standard of living that individuals would want to maintain in the future should be positively tied to the poverty line, since income below the poverty line indicate absolute deprivation. When it comes to the link between reference line and the standard of living, we keep it open and assume that they could either be positively or negatively correlated. Both these possibilities are plausible.

Standard of living can be positively correlated to the reference line because an individual with a higher current standard of living may want to

6

maintain a similar level of living in the future too and for any deviation from that could consider themselves vulnerable. A richer person, who has never experienced poverty, may also find it much harder to cope once they are in poverty than someone who has experienced poverty before. Hence for similar levels of future income below the poverty line, a richer person may be relatively more vulnerable. It implies that the reference line from which the future shortfall is calculated is higher for the richer individual. This becomes apparent in an interesting study of the unemployed in Northern Ireland (NI) during 1983-1984 (see Evanson 1985). Two thirds of the NI population were Catholic and the rest Protestant. The majority of the unemployed were Catholics. The study considered two groups: one Protestant and another Catholic and questioned them about their lived experience under unemployment. On an average Protestant sample had a higher income compared to the Catholic sample before unemployment. On the question of "impact of unemployment on living standard" close to 90 percent of the Protestant sample said they were worse under unemployment compared to around 74 percent of the Catholic sample. Around 80 percent of the Protestant sample reported "loss of status due to unemployment" compared to around 54 percent of Catholic sample. Similarly a lot more percentage of people from the Protestant group considered themselves to be depressed most of the time compared to the Catholics. Although the Protestant sample on an average came from a higher income background, they also were worse-off compared to the Catholics under unemployment which implies a higher level of discomfort associated with falling in to poverty from a higher standard of living. It thus gives some credence to the notion that higher current living standard may indicate higher vulnerability for similar levels of below poverty future income.

On the other hand higher standard of living can imply a lower vulnerability in the sense that higher living standards today would reduce the

minimum income needed in the future. In other words, we would see a lower reference line associated with higher current income. Individuals with lower current living standard, may not have as much assets and networks, to help them cope once they are in poverty in the future and hence the severity of a fall in income below poverty would be much higher. In the context of the Bangladesh famine of 1974, Sen (1981, p145) states: "It is the landless end of village spectrum that is caught firmly in the *langarkhanas.* The average chance of ending up in *langarkhanas* for those with less than half an acre of land was $4\frac{1}{2}$ times that of those owning between half an acre and one acre of land, and 165 times that of those with 5 acres or more."[6] More strikingly Sen (1981) finds that the landless labourers were the worst effected in terms of the intensity of destitution and mortality during the famine. What it implies is that people with no or very little asset are significantly effected when it comes to sudden shocks to future income as happened in the Bangladesh famine when there was a sudden collapse of their exchange entitlements (Sen 1981). Their assetlessness perhaps makes them unable to develop coping mechanisms to overcome future income shocks. Lower the current assets or income, the more likely are the people going to be vulnerable to poverty from future income shocks. Thus the current levels of income, or assets does play an important role in understanding vulnerability.

To illustrate our conceptual framework consider two individuals, A and B, with A having a higher current income relative to B. Suppose the current income y_t, which for simplicity is a proxy for current standard of living, is positively correlated with the reference line. Let the reference line be a simple average of the poverty line z, and the current income. Thus for $y_t > z$, the reference line will lie below the current income and above the poverty line and for $y_t < z$, then the reference line will lie above the current income but below the poverty line. Assume that the current income of both

---

[6]Langarkhanas are soup kitchens which are opened temporarily to feed the famine stricken.

A and B are above the poverty line.

Suppose individuals face the following two state lottery: with probability half in the first state both A and B receive $y_{t+1} < z$, and with probability half in the second state their future income remain the same as the current income. In the latter state since both receive income equal to their current income which is above the poverty line, there is no shortfall and will thus not matter for vulnerability. Therefore, we represent only the first state in a schematic diagram below.

Insert Figure 1

As evident, for both individuals the fall in future income is the same from the poverty line but different when considered from their respective current income. In an expected poverty framework, both A and B would be considered to be equally vulnerable but in our approach they will have different levels of vulnerability. If higher standard of living reflects a higher reference line, then as shown in the figure above, the reference line will be some where between the poverty line and the current income. Thus the expected fall from the reference line will be higher for A (shown as V^A) than B (shown as V^B), since A has a higher current income. Thus A has higher vulnerability than B. Note that we consider vulnerability only if future income falls below the poverty line, otherwise we may end up declaring as vulnerable very rich individuals whose income might fall below their reference line but still may be substantially higher than the poverty line or for that matter the income of most of the population.

Now let us examine another situation where both individuals have the same current income but different future incomes. Consider a two state lottery with equal probabilities of occurrence where in state 1 both A and B receive income greater than the poverty line and in state 2 A's future income is below the poverty line, while B's will be above the poverty line.

Thus, whatever the state, B will be above the poverty line, whereas A has a 50 percent chance of being poor in the future. As earlier we represent the state where individuals have a possibility of falling below the poverty line through the following diagram.

Insert Figure 2

As shown in the figure above, in state 2 for both individuals their future incomes are above their reference line. Note that in state 1, both receive an income above the poverty line and thus would not be deemed vulnerable at all. In our methodology none of the individuals fall short of their reference line and hence will not be vulnerable, although clearly one of them has a high probability to remaining poor in the future. Echoing a similar view World Bank (1997) state, "The poor are not necessarily vulnerable; for example, subsistence farmers in remote areas are almost always poor but are not particularly vulnerable to macroeconomic shocks." However under an expected poverty framework, individual A would definitely be vulnerable since he with a positive probability he will remain below the poverty line in the future. As earlier it is presumed here that the current income and the reference line are positively correlated. If they were negatively correlated, however, the reference line would be greater than the poverty line. Thus any income below the poverty line, would be considered a positive vulnerability. In the next section we develop these concepts in a formal framework.

2.1   Notation

Suppose an individual faces a finite set of future incomes $\{y_{t+1}^1, y_{t+1}^2, .., y_{t+1}^n\}$ with probability $p = \{p_1, p_2, .., p_n\} \in P$ where P is the set of probability distributions such that for all $s = 1, ..i, j, .., n$, $p_s \geq 0$, and $\sum_{s=1}^{n} p_s = 1$. Let the poverty line $z \in \mathbb{R}_{++}$ indicates some minimum income that we

10

expect individuals to have, below which they are considered deprived. Since our emphasis is on measuring vulnerability of the individual to fall below poverty, we consider those individuals who have positive probability of falling below the poverty line z. Thus by avoiding all n-tuple of future incomes where none of the incomes will be below the given poverty line we will be focussing only on individuals who has a chance of falling in to poverty in the future. Individuals whose futute income in all states equal zero are also not considered. These are the most deprived, but since maximal deprivation is a certainty vulnerability, which essentially is about the uncertainty in the future, may not be a relevant issue for such individuals.

We represent the shortfall that the individual faces for each different level of future income y_{t+1}^s through a deprivation function d^s which depends not only on the level of income in future income y_{t+1}^s and the given poverty line z, but also on an indicator of the standard of living $y_t \in \mathbb{R}_+$ of the individual. Therefore, with any future income y_{t+1}, the associated shortfall is given by the function $d(z, y_t, y_{t+1})$ where: $d : \mathbb{R}_{++} \times \mathbb{R}_+ \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ and has the following three properties:

- (P1) $d(z, y_t, y_{t+1}) \in \mathbb{R}_+$,

- (P2) $d(z, y_t, y_{t+1})$ is continuous in y_t, y_{t+1}, and z, and

- (P3) for $\theta, \lambda > 0$ and $d(z, y_t, y_{t+1}) > 0$, $d(z+\theta, y_t, y_{t+1}) > d(z, y_t, y_{t+1})$, and $d(z, y_t, y_{t+1}) > d(z, y_t, y_{t+1} + \lambda)$.

Property (P1) implies that the level of deprivation will not be negative. The lowest level of deprivation in any state therefore is, zero. Property (P2) captures the intuition that small changes in y_t, y_{t+1}, and z will not lead to big changes in the level of deprivation. The third property (P3) implies that if the poverty line z increases the deprivation level will increase too and if the level of future income y_{t+1} decreases then vulnerability will decrease. Note that property P3 holds only when the deprivation is positive.

The individual faces a simple deprivation lottery $L = (p_1, d(z, y_t, y_{t+1}^1);$ $...; p_n, d(z, y_t, y_{t+1}^n))$ which is the source of his vulnerability. For notational convenience d^s shall represent $d(z, y_t, y_{t+1}^s)$.

Let $\mathcal{F}$ be the set of all deprivation lotteries. Our vulnerability measure for an individual is a function $V : \mathcal{F} \to \mathbb{R}_+$. The vulnerability to poverty depends only on the level of deprivation in each state and the associated level of probability. A permutation of the states, therefore, will not change the overall level of vulnerability. In otherwords, it is state independent.

An alternative interpretation, which we will use more extensively, would be that there are n different future states of nature and associated with each state of nature s is an income y_{t+1}^s and a deprivation function $d(z, y_t, y_{t+1}^s)$. The level of income in each of these states reflect the final amount that the individual receives in the future. So if the individual is unemployed in one of the future states, then the final income that we consider here will take in to account any insurance that he may have and any benefits that he may receive. Therefore, the coping strategies that individuals may have under different states are built in to the final income levels.[7]

One departure from previous axiomatization of measures of vulnerability is that we do not start with any pre-specified functional form of the deprivation function, d^s, associated with each state s. The reason is that instead of comparing the future income in state s from a given benchmark z, we now also have to take in to consideration the current standard of living y_t. Hence it is not a priori obvious how these three elements will interact to provide a level of deprivation in each state.

2.2 The Measure

Consider a lottery $L = (p_1, d(z, y_t, y_{t+1}^1); p_2, d(z, y_t, y_{t+1}^2); ...; p_n, d(z, y_t, y_{t+1}^n))$. The general structure of a vulnerability measure V associated with lottery

---

[7]Coping strategies can also include changing consumption patterns which we do not consider in this paper.

L would be

$$V(L) = \sum_{s=1}^{n} p_s h\left(z, y_t, y_{t+1}^s\right)$$

where h is continuous in its arguments.[8]

However, we can take a step further and untangle the deprivation that individuals may face in each state of the world. The deprivation in each state measures the income shortfall in that state from a reference line which is dependent on both z and y_t. Let the shortfall in income in each future state be: $d^s(z, y_t, y_{t+1}^s) = R(z, y_t) - y_{t+1}^s$, $s = 1, .., n$. A class of vulnerability measures that only considers the level of absolute deprivation in each state will be

$$\widehat{V}(L) = \sum_{s=1}^{n} p_s f\left(R(z, y_t) - y_{t+1}^s\right). \tag{1}$$

The reference line $R(z, y_t)$, reflects the fact that when it comes to vulnerability the individual considers both his current living standard (represented here using y_t) and also the poverty line as important. It should be noted that the above equations represents a class of vulnerability measures at the individual level. To measure the societal vulnerability level one can aggregate the level of vulnerability across all individuals in the society. Our axiomatization, however, will focus on characterizing a class of measures at the individual level as represented in (1).

3   Axioms

Our first axiom captures the intuition that the vulnerability measure should be decomposable. In other words, the vulnerability of a convex combination of lotteries should be the same as the convex combination of the vulnerability of each of the lotteries. The implication of this axiom would be to make the vulnerability measure linear in probabilities. It will thus generate the

---

[8]h can also be considered to reflect the risk preference of individuals over future income.

von Nueman-Morgenstern expected utility structure for the vulnerability measure.

Axiom 1 *Axiom of Decomposability (A1): Consider any two deprivation lotteries L and L'. Then for $0 \leq \alpha \leq 1$, $V(\alpha L + (1-\alpha)L') = \alpha V(L) + (1-\alpha)V(L')$.*

As in the case of the von Nueman-Morgenstern expected utility the above axiom can be derived from more fundamental axioms on the lottery space. Although this is more of a technical axiom, an intuitive interpretation of the axiom can be provided along the following lines. Consider the case where a farmer faces two states (say, normal rainfall and very low rainfall) in the future but the outcome in those states depends on whether the government undertakes a policy (such as of providing extra subsidy if it turns out to be low rainfall and charging higher taxes if it is normal rainfall) which ex-ante is uncertain. Thus, if the government undertakes the policy the farmer faces the vulnerability $V(L)$ from lottery L and if it does not then he faces vulnerability $V(L')$ from L'. In such case according to Axiom 1 overall vulnerability should be the 'expected' vulnerability. It ignores the fact that uncertainty in government policy may actually make the overall vulnerability worse. In this sense the axiom implies that vulnerability is not effected by higher order uncertainty per se. Broadly what the axiom argues for is that, if suppose, in the worst case the farmer faces vulnerability $V(L')$ and in the best case he faces $V(L)$, then given the uncertainty about whether it is going to be $V(L)$ or $V(L')$, it is reasonable to expect that ex-ante overall vulnerability will be somewhere between $V(L)$ and $V(L')$. If this is the case then one natural expectation may be that where exactly between $V(L)$ and $V(L')$ the overall vulnerability will lie, should depend on the probability with which L and L' takes place. It is this intuition that Axiom 1 captures.

The intuition for our next axiom comes from Sen (1981) who in the context of Sahelian farmers diversifying in to cash crops notes that although

14

they may have more income, they may be more vulnerable in the sense that they are more prone to sudden collapse of their entitlement than previously. Thus in some states of the world the farmers are better off while in other states they are worse off compared to pre-diversification in to cash crops. In terms of the distribution it means that although overall the expected income of the Sahelian farmer may have increased, the expected increase in inequality between the states outweighs that benefit and hence leads to higher vulnerability. A reasonable measure of vulnerability thus should incorporate the general intuition that as the 'distribution' of income becomes worse, vulnerability will increase.

Consider two lotteries L and $L^{'}$ such that $L^{'}$ is derived from L through a transfer where the future income in a worse-off state have decreased, whereas the future income in a better off state has increased. Thus the vulnerability from $L^{'}$ should be higher than the vulnerability from L.

Axiom 2 *Axiom of Transfer (A2): Consider two lotteries $L = (p_1, d(z, y_t, y_{t+1}^1);$ $..; p_i, d(z, y_t, y_{t+1}^i); p_j, d(z, y_t, y_{t+1}^j); ..; p_n, d(z, y_t, y_{t+1}^n))$ and $L^{'} = (p_1, d(z, y_t, y_{t+1}^1);$ $..; p_i, d(z, y_t, y_{t+1}^i - t); p_j, d(z, y_t, y_{t+1}^j + t); ..; p_n, d(z, y_t, y_{t+1}^n)),$ such that $p_j = p_i > 0$, and $d^i > d^j \geq 0$. Then $V(L') > V(L)$.*

Since the probability of the worse off state from which income is transferred is the same as than the probability of the better off state where income is transferred to, the overall expected future income remains the same, yet the individual becomes more vulnerable. This axiom is similar to the transfer axiom in the poverty literature (see Sen 1976) but instead of the transfer of income between individuals, there is transfer of deprivation levels between states of the world. In our framework the transfer axiom would imply that the vulnerability measure be convex in the deprivation levels. Standard measures of vulnerability such as those, which just considers the probability of having a shortfall does not satisfy the Transfer axiom. Vulnerability measures based on expected loss will also violate the Transfer axiom.

15

Most poverty measures satisfies the property of monotonicity where by decrease in income leads to an increase in poverty. A similar intuition guides our next axiom of monotonicity which captures the notion that increase in deprivation in any state should increase vulnerability. Measures of vulnerability which solely rely on the probability of falling below a certain reference line, as in Pritchett et al (2000), violates the monotonicity axiom, since there will not be any changes in vulnerability so long as the probability remains same.

Before we proceed to the axiom, let us introduce the following definition of a lottery which is derived through an incremental increase in future income in any one state of a lottery L.

Definition 1 *Suppose $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$. Then the lottery derived from L by an incremental increase in future income in state i is $L^{\delta_i} = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i + \delta), .., p_n, d(z, y_t, y_{t+1}^n))$, $\delta > 0$.*

A formal exposition of the monotonicity axiom is as follows:

Axiom 3 *Axiom of Monotonicity of Future Income (A3): Consider any two lotteries L and L^{δ_i} where $p_i > 0$, and $d(z, y_t, y_{t+1}^i) > 0$. Then $V(L) > V(L^{\delta_i})$..*

As described in the previous section, one important feature of the vulnerability measure that we propose is that standard of living as an integral part it. We also discussed that higher current standard of living can reflect a higher level of vulnerability or it can also mean a lower level of vulnerability. Whatever the direction of the link between vulnerability and current standard of living, we want it to be monotonic. Therefore we rule out the possibility that for some levels of standard of living the link is positive and for other levels it is negative. We capture this intuition by the following axiom:

Axiom 4 *Axiom of Monotonicity of Current Standard of Living (A4): Consider two lotteries $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$ and $L^\delta = (p_1, d(z, y_t+\delta, y_{t+1}^1); ..; p_s, d(z, y_t+\delta, y_{t+1}^i); ..; p_n, d(z, y_t+\delta, y_{t+1}^n)), \delta > 0$. Then either for all δ and L, $V(L^\delta) \geq V(L)$ or for all δ and L, $V(L^\delta) \leq V(L)$.*

If there is an increase in current living standard by δ amount, then either vulnerability will increase compared to the status-quo or it will decrease.

In a significant departure from the standard characterization of poverty measures we have not explicitly assumed the deprivation function, d, to have any specific functional form. In the poverty literature, deprivation measures the shortfall in income from a given income level which typically is the poverty line and is thus linear in income. In the next axiom we shall capture this intuition but since we are interested in vulnerability our focus will be on deprivation in future states. Hence we will be concerned with future income.

Before we introduce the axiom, let us define the concept of an *equally distributed deprivation* lottery associated with any lottery L.

Definition 2 *For any lottery $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$ the equally distributed deprivation lottery of L is represented as $L_E = (p_1, \overline{d}; ..., p_k, \overline{d}; ...; p_n, \overline{d})$, where $\overline{d} = (d(z, y_t, y_{t+1}^1) + .. + d(z, y_t, y_{t+1}^n))/n$.*

Thus for any lottery L there is an equally distributed deprivation lottery L_E where the deprivation is the same in every state. Note that the deprivation is equally distributed in all the states including those states where there was no deprivation to begin with.

Under this axiom, the vulnerability rankings of two equally distributed lotteries are preserved under incremental changes in future income. The axiom focuses on equally distributed deprivation thus ruling out any kind of distributional issues regarding deprivation in different states within the

lotteries. Consider an individual facing two situations A and B. Under both situations, suppose the individual suffers from deprivation in the future with certainty. Further let the vulnerability emanating from the deprivation is higher in A relative to B. Now if future income increases by the same amount in both situations (such that deprivation in both situations is not completely eliminated), it is reasonable to expect that the individual will continue to have higher level of vulnerability associated with situation A. Similarly, if initially the individual faced the same level of vulnerability in both A and B, then a small equal increase in income in both situations should reasonably still leave the vulnerability in A and B the same. Our next axiom captures this intuition in the context of uncertainty with multiple states.

Axiom 5 *Axiom of Independence (A5): Let $L = (p_1, d(z, y_t, y^1_{t+1}); ..; p_i, d(z, y_t, y^i_{t+1}); ..; p_n, d(z, y_t, y^n_{t+1}))$ and $L' = (p_1, d(z', y'_t, y'^1_{t+1}); ..; p_i, d(z', y'_t, y'^i_{t+1}); ..; p_n, d(z', y'_t, y'^n_{t+1}))$. If $V(L_E) \geqq V(L'_E)$ and for some $\delta > 0$, $d(z, y_t, y^i_{t+1} + \delta) > 0$, $d(z', y'_t, y'^i_{t+1} + \delta) > 0$ and $p_i > 0$, then $V\left(L^{\delta_i}_E\right) \geqq V(L'^{\delta_i}_E)$.*

4 Characterization of the Measure

In this section we characterize the vulnerability measure presented in Section 2. We first characterize the measure in (1). To do so, we start with a set of definitions.

In Axiom 1, we propose that the vulnerability from a convex combination of two lotteries is the same as a convex combination of the vulnerabilities from two lotteries. For any two lotteries L' and L'' a convex combination of two lotteries is represented as $\alpha L' + (1 - \alpha)L''$ where $\alpha \in (0, 1)$. A precise definition of what we mean by convex combination of two lotteries is as follows:

Definition 3 *Suppose $L' = (p_1, d^1; p_2, d^2; ...; p_n, d^n)$ and $L'' = (q_1, d^1; q_2, d^2; ...; q_n, d^n)$.*

Then $\alpha L' + (1-\alpha)L''$ is represented by $L = (\alpha p_1 + (1-\alpha)q_1, d^1; \alpha p_2 + (1-\alpha)q_2, d^2; ...; \alpha p_n + (1-\alpha)q_n, d^n).$

Next we use Axiom 1 to derive the expected utility form of the vulnerability function, V. Here we follow the approach of Kreps (1988, p.50) closely.

Lemma 1 *If a vulnerability index V satisfies axiom A1 and A3 then it can be written as $V(L) = \sum_{s=1}^{n} p_s f(d^s)$ where f is monotonic.*

Proof: We first show by induction that $V(L) = \sum_{s=1}^{n} p_s f(d^s)$ then we demonstrate f is monotonic.

Let m represent the number of states with non-zero probabilities. Let $\widetilde{L}_i = (0, d^1; ...; 1, d^i; ...; 0, d^m)$ be the degenerate lottery where $p_i = 1$ and $p_{j \neq i} = 0$. Clearly, for such a degenerate lottery $m = 1$. Since V is not state dependent, for the degenerate lottery we can write $V(\widetilde{L}_i) = f(d^i)$. Hence the claim in Lemma 1 is trivially true for $m = 1$.

When $m = 2$, without loss of generality, consider $L2 = (p_1, d^1; (1-p_1), d^2; 0, d^3; ....; 0, d^n)$. Using Definition 3, $L2 = p_1\widetilde{L}_1 + (1-p_1)\widetilde{L}_2$. Then applying Axiom A1 we can write $V(p_1\widetilde{L}_1 + (1-p_1)\widetilde{L}_2) = p_1 f(d_1) + (1-p_1)f(d_2)$. Hence the claim is true for $m = 2$.

Now suppose it is true for any $m = n - 1$.

We shall show that it will be true for $m = n$. Since it is true for $m = n - 1$, we know that $V(L_{n-1}) = p_1 f(d_1) + p_2 f(d_2) + ... + p_{n-1} f(d^{n-1})$

Without any loss of generality let $L_n = (p_1, d^1; p_2, d^2; ...p_{n-1}, d^{n-1}; 0, d^n)$.

Consider $\widehat{L}_{n-1} = (p_1/(1-p_n), d^1; p_2/(1-p_n), d^2; ...; p_{n-1}/(1-p_n), d^{n-1}; 0, d^n)$. Then we can write (using A3 and Definition 1)

$$V(L_n) = V(p_n\widetilde{L}_n + (1-p_{n-1})\widehat{L}_n) = p_n V(\widetilde{L}_n) + (1-p_{n-1})V(\widehat{L}_{n-1}).$$

Since $V(\widehat{L}_n) = \sum p_i/(1-p_n)f(d_i)$ it can be easily shown that

$$V(L_n) = \sum_{s=1}^{n} p_s f(d^s). \tag{2}$$

To show that f is monotonic in d, consider two lotteries such that
$L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_k, d(z, y_t, y_{t+1}^k), ..; p_n, d(z, y_t, y_{t+1}^n))$ and
$L^{\delta_i} = (p_1, d(z, y_t, y_{t+1}^1); ..; p_k, d(z, y_t, y_{t+1}^k + \delta); ..; p_n, d(z, y_t, y_{t+1}^n)), d(z, y_t, y_{t+1}^k) >$
0. From property P3 of d (where deprivation d is monotonically decreasing
in y_{t+1}^s) we know $d(z, y_t, y_{t+1}^k) > d(z, y_t, y_{t+1}^k + \delta)$. Therefore, given P1, we
can write $d(z, y_t, y_{t+1}^k) = d(z, y_t, y_{t+1}^k + \delta) + \gamma$, $\gamma > 0$.

Then using Axiom 3 and equation (**??**)we can demonstrate that for all
$\gamma > 0$, $f(d(z, y_t, \widehat{y}_{t+1}^k) + \gamma) > f(d(z, y_t, \widehat{y}_{t+1}^k))$, where $\widehat{y}_{t+1}^k = y_{t+1}^k + \delta$. Thus
f is monotonic in $d(z, y_t, y_{t+1})$.■

Next, we show that for any lottery L we can find a suitably defined
lottery L' (where future incomes are different in atleast two states) such that
$V(L_E) = V(L'_E)$. This would allow us to later on establish the additive
separability between z, y_t, and y_{t+1}.

Lemma 2 *Let* $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$.
Given A1, A3, there exists another lottery $L' = (p_1, d(z, y_t, y_{t+1}'^1); ..; p_i, d(z, y_t, y_{t+1}^i + \epsilon); ..; p_n, d(z, y_t, y_{t+1}'^n))$, $\epsilon > 0$, *such that* $V(L_E) = V(L'_E)$.

Proof: Let $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$.
Suppose $\forall s \leq n, p_s > 0$ and $d(z, y_t, y_{t+1}^s) > 0$. Let $y_{t+1}^i = \min(y_{t+1}^1, .., y_{t+1}^i, .., y_{t+1}^n)$.
Given the continuity and montonicity of $d(z, y_t, y_{t+1})$ in y_{t+1}, (property P2
and P3), we shall be able to find another lottery $L' = (p_1, d(z, y_t, y_{t+1}'^1); ..; p_i, d(z, y_t, y_{t+1}^i + \epsilon); ..; p_n, d(z, y_t, y_{t+1}'^n))$, such that $d(z, y_t, y_{t+1}^i) - d(z, y_t, y_{t+1}^i + \epsilon) =$
$\max\left(\sum_{s \neq i}(d(z, y_t, y_{t+1}'^s) - d(z, y_t, y_{t+1}^s))\right)$ and $d(z, y_t, y_{t+1}^i + \epsilon) > 0$. Hence
by definition of L', $V(L_E) = V(L'_E)$.■

Based on the above definitions and lemma we characterize a class of

absolute measures of vulnerability.

Theorem 1 *For any lottery L the measure of vulnerability $V(L)$, satisfies axioms A1-A5 iff*

$$V(L) = \sum_{s=1}^{n} p_s f(R(z, y_t) - y_{t+1}^s),$$

where f is convex.

Proof: That V satisfies A1-A5 can easily be checked. Here we prove the necessary condition. Using Lemma 1 and property P1 of deprivation function d, for any deprivation lottery L, vulnerability can be represented by :

$$V(L) = \sum_{s=1}^{n} p_s h\left(d(z, y_t, y_{t+1}^s)\right) \tag{3}$$

where $d(z, y_t, y_{t+1}^s) \geq 0$ and h is monotonic .

Consider any lottery $L = (p_1, d(z, y_t, y_{t+1}^1); ..; p_i, d(z, y_t, y_{t+1}^i); ..; p_n, d(z, y_t, y_{t+1}^n))$, where $\forall s = 1, .., k \geq 1$, $d(z, y_t, y_{t+1}^s) > 0$ and $p_s > 0$. For a suitably defined $\delta > 0$ we know from Lemma 2 there exists $L' = (p_1, d(z, y_t, y_{t+1}'^1); ..; p_i, d(z, y_t, y_{t+1}'^i); ..; p_n, d(z, y_t, y_{t+1}'^n))$ where, $y_{t+1}'^i = y_{t+1}^i + \delta$ and $V(L_E) = V(L_E')$. Thus

$$V(L_E) = \sum_{s=1}^{n} p_s h\left(\overline{d}(z, y_t, y_{t+1}^s)\right) = \sum_{s=1}^{n} p_s h\left(\overline{d}(z, y_t, y_{t+1}'^s)\right) = V(L_E') \tag{4}$$

where $\overline{d}(z, y_t, y_{t+1}^s) = (d(z, y_t, y_{t+1}^1) + .. + d(z, y_t, y_{t+1}^i) + .. + d(z, y_t, y_{t+1}^n))/n$ and $\overline{d}(z, y_t, y_{t+1}'^s) = (d(z, y_t, y_{t+1}'^1) + .. + d(z, y_t, y_{t+1}^i + \delta) + .. + d(z, y_t, y_{t+1}'^n))/n$. Then applying Axiom 4 for the case where $z' = z$, and $y_t' = y_t$, we will be able to derive

$$V(L_E^{\delta_i}) = \sum_{s=1}^{n} p_s h\left(\overline{d}_\delta(z, y_t, y_{t+1}^s)\right) = \sum_{s=1}^{n} p_s h\left(\overline{d}_\delta(z, y_t, y_{t+1}'^s)\right) = V(L_E'^{\delta_i}) \tag{5}$$

where $\overline{d}_\delta(z, y_t, y_{t+1}^s) = (d(z, y_t, y_{t+1}^1) + .. + d(z, y_t, y_{t+1}^i + \delta) + .. + d(z, y_t, y_{t+1}^n))/n$

21

and $\overline{d}_\delta(z, y_t, y'^s_{t+1}) = (d(z, y_t, y'^1_{t+1}) + .. + d(z, y_t, y^i_{t+1} + 2\delta) + .. + d(z, y_t, y'^n_{t+1}))/n$.

Subtracting equations (5) from (4) and interchanging terms we get

$$d(z, y_t, y^i_{t+1}) + d(z, y_t, y^i_{t+1} + 2\delta) = 2d(z, y_t, y^i_{t+1} + \delta).$$

Since this holds for all possible lotteries, using Aczel (1966, p.43), we can derive

$$d(z, y_t, y^i_{t+1}) = A(z, y_t)y^i_{t+1} + \beta(z, y_t) \qquad (6)$$

where $A(z, y_t)$ and $\beta(z, y_t)$ are independent of y^i_{t+1}. When $y^i_{t+1} = 0$, it implies $\beta = d(z, y_t, 0) \geq 0$.

Further consider a lottery $\widetilde{L} = (p_1, d(\widetilde{z}, \widetilde{y}_t, y^1_{t+1}); ..; p_i, d(\widetilde{z}, \widetilde{y}_t, y^i_{t+1}); ..; p_n, d(\widetilde{z}, \widetilde{y}_t, y^n_{t+1}))$ where $y^i_{t+1} > 0$, $p_i > 0$, $d(\widetilde{z}, \widetilde{y}_t, y^i_{t+1}) > 0$ and $\forall s \neq i$, $d(\widetilde{z}, \widetilde{y}_t, y^s_{t+1}) = 0$. It can be shown that given properties P2, and P3 of d and Axioms 3 and 4, we can suitably choose a neighbourhood around $\widetilde{z}$ and $\widetilde{y}_t$ such that for each z and y_t within that neighbourhood we will be able to find another lottery $L' = (p_1, d(z, y_t, y'^1_{t+1}); ..; p_i, d(z, y_t, y'^i_{t+1}); ..; p_n, d(z, y_t, y'^n_{t+1}))$, where $\forall s, d(z, y_t, y'^s_{t+1}) = d(\widetilde{z}, \widetilde{y}_t, y^s_{t+1})$. It implies that for all z and y_t within that neighbourhood, $V(\widetilde{L}_E) = V(L'_E)$. Thus

$$V(\widetilde{L}_E) = \sum_{s=1}^{n} p_s h\left(\overline{d}(\widetilde{z}, \widetilde{y}_t, y^s_{t+1})\right) = \sum_{s=1}^{n} p_s h\left(\overline{d}(z, y_t, y'^s_{t+1})\right) = V(L'_E), \quad (7)$$

where $\overline{d}(\widetilde{z}, \widetilde{y}_t, y^s_{t+1}) = d(\widetilde{z}, \widetilde{y}_t, y^i_{t+1})/n$ and $\overline{d}(z, y_t, y'^s_{t+1}) = d(z, y_t, y'^i_{t+1})/n$.

From Axiom 5 and using a suitably chosen $\delta > 0$ we can write

$$V(\widetilde{L}^{\delta_i}_E) = \sum_{s=1}^{n} p_s h\left(\overline{d}_\delta(\widetilde{z}, \widetilde{y}_t, y^s_{t+1})\right) = \sum_{s=1}^{n} p_s h\left(\overline{d}_\delta(z, y_t, y'^s_{t+1})\right) = V(L'^{\delta_i}_E) \quad (8)$$

where $\overline{d}_\delta(\widetilde{z}, \widetilde{y}_t, y^s_{t+1}) = (d(\widetilde{z}, \widetilde{y}_t, y^i_{t+1} + \delta)/n) > 0$ and $\overline{d}_\delta(z, y_t, y'^s_{t+1}) = (d(z, y_t, y'^i_{t+1} +$

$\delta)/n) > 0$. Subtracting (7) from (8) and using (6), we can show

$$A(\widetilde{z}, \widetilde{y}_t) = A(z, y_t).$$

Since this must hold for all values of z and y_t within the suitably defined neighbourhood, it can only be true if $A(z, y_t) = A(z, y_t) = w$, where w is a constant. Thus

$$d(z, y_t, y^i_{t+1}) = wy^i_{t+1} + \beta(z, y_t) \qquad (9)$$

Next consider any lottery L and the associated lottery L^{δ_i}, where $p_i > 0$. Using axiom A3 and (9) one can show $w < 0$. Further $d(z, y_t, y^i_{t+1}) \in \mathbb{R}_+$ (from property P1 of $d(z, y_t, y^i_{t+1})$). Thus (6) can be written as

$$d(z, y_t, y^i_{t+1}) = \begin{cases} \theta(R(z, y_t) - y^i_{t+1}) & \text{if } R(z, y_t) \geq y^i_{t+1} \\ 0 & \text{otherwise} \end{cases} , \qquad (10)$$

where $R(z, y_t) = (\beta(z, y_t)/\theta)$ and $\theta = |w|$.

We next demonstrate that h is convex. Consider two lotteries $L = (p_1, d(z, y_t, y^1_{t+1}); ..; p_i, d(z, y_t, y^i_{t+1}); p_j, d(z, y_t, y^j_{t+1}); ..; p_n, d(z, y_t, y^n_{t+1}))$ and $L' = (p_1, d(z, y_t, y^1_{t+1}); ..; p_i, d(z, y_t, y^i_{t+1}-t); p_j, d(z, y_t, y^j_{t+1}+t); ..; p_n, d(z, y_t, y^n_{t+1}))$ where $p_j = p_i > 0$, and $d(z, y_t, y^i_{t+1}) > d(z, y_t, y^j_{t+1}) > 0$. Using axiom A2, (3) and cancelling terms we get

$$((h(d(z, y_t, y^i_{t+1}-t)) - h(d(z, y_t, y^i_{t+1})) > h(d(z, y_t, y^j_{t+1})) - h(d(z, y_t, y^j_{t+1}+t))$$

Using (10), from the above equation one can derive

$$(h(d^i(z, y_t, y^i_{t+1}) + \lambda) - h(d^i(z, y_t, y^i_{t+1})) > h(d^j(z, y_t, y^j_{t+1})) - h(d^j(z, y_t, y^j_{t+1}) - \lambda)$$
$$(11)$$

where $\lambda = \theta t$. Thus h is a convex function of the deprivation level, d (Royden 1988).

From (3), (10) and (11), $V(L) = \sum p_s h(\theta(R(z, y_t) - y^s_{t+1}))$ where h is

convex. Since $\theta > 0$ is a constant, we can write $V(L) = \sum p_s f(R(z, y_t) - y_{t+1}^s)$, where f is convex. $\blacksquare$

5 Discussion

So far we have characterized a broad class of vulnerability measure. However, for empirical applications specific functional forms are required, which we shall elaborate on in this section. In (1) both the reference income $R(z, y_t)$, and vulnerability under a degenerate lottery, represented by f, have general structures that can be made more specific.

There are several functional forms that can be used for representing $R(z, y_t)$ depending on how z and y_t are related and whether y_t is positively or negatively correlated with $R(z, y_t)$. One interpretation of $R(z, y_t)$ is that it is the minimum living standard that people should maintain in the future to be not considered as vulnerable. It is, however, quite reasonable to expect that for every doubling of the current income, the minimum living standard $R(z, y_t)$ will not double too. In other words, reasonable $R(z, y_t)$ should satisfy the condition that changes in current income does not translate to equivalent changes in the reference line.

If y_t and $R(z, y_t)$ are positively correlated, an interesting functional form of the reference line is $R(z, y_t) = z^{1-\alpha} y_t^{\alpha}$ where $0 \le \alpha \le 1$ (see Foster 1998). It is homogenous of degree 1 (HD-1) and satisfies the property that a percent change in y_t leads to a $\alpha \le 1$ percent change in $R(z, y_t)$. This implies that the current income elasticity of the reference line is not greater than one which concurs with our intuition that reference income should not vary too much with changes in current income. Another functional representation of the reference line is $R(z, y_t) = (1 - \delta)z + \delta y_t$ where $0 \le \delta \le 1$. Clearly the rate of change in reference income with respect to the current income is less than one. So long as $(1 - \delta)/\delta > z/y_t$, it is also the case that the current income elasticity of the reference line is less or equal to one.

24

On the other hand if y_t and $R(z, y_t)$ are negatively correlated, the reference line can be represented by $R(z, y_t) = z^{1+\alpha}/y_t^\alpha$ where $0 \leq \alpha \leq 1$. Clearly as y_t increases, $R(z, y_t)$ decreases indicating that a currently richer person would suffer less vulnerability compared a poorer person for same level of future income. Here again the absolute current income elasticity of the reference line is less than one. The linear representation of the reference line is $R(z, y_t) = (1 - \delta)z - \delta y_t$ where $0 \leq \delta \leq 1$. Under this functional form, with sufficiently high y_t it is possible to generate $R(z, y_t) < 0$, which implies that those with high levels of income in the current period would not be vulnerable what ever their future income is.

Given a particular functional form of the reference line and depending on the choice of f, specific classes of measures of vulnerability can be derived. For example, we can generate the Foster-Greer-Thorbecke (FGT) class of absolute vulnerability measures which is

$$V(L) = \sum_{s=1}^{n} p_s \left(A z^{1-\alpha} y_t^\alpha - y_{t+1}^s \right)^\gamma \tag{12}$$

where $\gamma > 1$. Note that if $\gamma \leq 1$, then the vulnerability function for each state of the world will not be convex thus violating axiom A2. When $\alpha = 0$, the measure would be the standard expected FGT poverty index. When $\alpha = 1$, the measure is completely relative in the sense that it depends on the current and future income. Thus by varying α we can get a whole set of values of vulnerability ranging from the expected poverty measures to the relative measures. From this perspective, the class of measure that we have proposed is very general and incorporates the expected poverty measures as a special case.

6 Conclusion

In this paper we have attempted to conceptualize and characterize a new class of vulnerability measure. Detailed case studies indicate that existing measures of vulnerability based on the expected poverty framework may be unable to fully capture the different facets of vulnerability. The studies also find that individual's current levels of wealth and income impact their income vulnerability by affecting their ability to build up coping mechanisms for future income shocks and also by their willingness to use current levels of income and wealth as benchmarks for the future living standards. Our proposed measure, by taking into account current assets and income, is thus closer to the broader notion of vulnerability. It has to be noted that although our exposition of the measure in this paper is based on income, it can be applied for calibrating vulnerability along other dimensions of well-being. Thus if we are interested in food insecurity, we can use food consumption instead of income and arrive at a measure of vulnerability to food deprivation.

We use the standard framework of decision making under uncertainty to characterize a class of absolute measure of vulnerability. The measure that we have characterized extremely general. For instance the functional form of our reference line which combines the poverty line and some indicator of the living standard (such as income) is left quite unrestricted. Thus, unlike other measures we are able to consider two opposing view points: (a) where current living standard reduces future vulnerability and (b) where current living standards exacerbates future vulnerability, within one unified framework. We also provide specific examples of our measure by indicating how the FGT indices can be adopted for our measures. Despite the generality, our axioms rule out some obvious measures of vulnerability such as those belonging to FGT class of vulnerability measures (12) with $\gamma = 0,1$. Although we have provided an example of our measure, depending on func-

tional structures many more vulnerability measures can be developed.

There are, however, some shortcomings in our analysis. In particular we have considered vulnerability just one period ahead. Although analyzing vulnerability too far in the future may be meaningless, vulnerability over multiple time periods would considerably enrich the analysis. We have also assumed a probability distribution over future states of the world. But in a completely uncertain world we may not have those information and thus may not be able to apply the standard von-Nueman Morgenstern framework. Finally it is quite probable that some specific measures of the proposed class of measures may not always satisfy reasonable properties. For instance Menezes et al (1980) considers a downward shift of a distribution, keeping the mean and the variance the same, as reflecting higher down side risk and thus higher vulnerability. It can, however, be easily shown that the FGT class (12) for $\gamma = 2$ does not satisfy the property of higher vulnerability emanating from a downward movement of the distribution. What would be the most parsimonious structure that will capture all the different facets of vulnerability remains a topic of further research.

References

Atkins, J. P., S. Mazzi and C.D. Easter (2000), A Commonwealth Vulnerability Index for Developing Countries: The Position of Small States, Economic Paper 40, Common Wealth Secretariat.

Basu, K. and P. Nolen (2005), Vulnerability, Unemployment and Poverty: A New Class of Measures, Its Axiomatic Properties and Applications, Mimeo.

Bruguglio, L. and W. Galea (2003). Updating and Augmenting the Economic Vulnerability Index. Occasional paper, University of Malta.

Calvo, C. and S. Dercon (2006), Measuring Individual Vulnerability, Mimeo.

Chaudhuri, S., J. Jalan and A. Suryahadi (2002), Assessing Household Vulnerability to Poverty from Cross-Sectional Data: A Methodology and Estimates from Indonesia, Working Paper, Columbia University.

Chaudhuri, S. (2003), Assessing Vulnerability to Poverty: Concepts, Empirical Methods and Illustrative Examples,Working Paper, Columbia University.

Christianensen, L. and R. N. Boisvert (2000) On Measuring Household Food Vulnerability: Case Evidence from Nothern Mali, Working Paper, World Bank.

Dercon, S. and P. Krishnan (2000), In Sickness and In Health: Risk Sharing within Households in Rural Ethiopia, Journal of Political Economy, 108, 688-727.

Evanson, E. (1985), On the Edge: A study of Poverty and Long term Unemployment in Northern Ireland, Child Poverty Action Group.

Ersado, L. (2008), Rural Vulnerability in Serbia, Working Paper 4010, World Bank.

Foster, J (1998), Absolute versus Relative Poverty, American Economic Review Papers and Proceedings, 88, 335-341.

Hoddinott, J. and A. Quisumbing (2003), Methods of Microeconometric Risk and Vulnerability Assessments, SPDP Series 0324, World Bank

Kamanou, G., and J. Morduch (2002) Measuring Vulnerability to Poverty, WIDER Discussion Paper, 2002/58

Kreps, D. (1988), Notes on the Theory of Choice, Westview Press, Boulder.

Ligon, E. and L. Schecter (2003), Measuring Vulnerability, Economic Journal, 113, C95-C102.

Menezes, C., C. Geiss and J. Tressler (1980), Increasing Downside Risk, American Economic Review, 921-932.

Morduch, J. (1994), Poverty and Vulnerability, American Economic Review Papers and Proceedings, 84, 221-225.

Moser, C. O. N. (1996) Confronting Crisis: A Comparative Study of Households Response to Poverty and Vulnerability in Four Poor Urbane Communities, Working Paper, ESD No. 8, World Bank.

Niculescu, C.P. and L-E, Persson (2006) Convex Functions and Their Applications: A Contemporary Approach, Springer, New York.

Pritchett, L., A. Suryahadi and S. Sumarto (2000), Quantifying Vulnerability to Poverty, World Bank, Policy Research Working Paper 2437.

Royden, H. L. (1988), Real Analysis: 3rd Edition, McMillan, US.

Sen, A. (1976), Poverty: An Ordinal Approach to Measurement, Econometrica, 44, 219-231.

Sen, A (1981), Poverty and Famines, Oxford University Press, Oxford.

Sen, A. (1999), A Plan for Asia's Growth, Asia Week, October 8, 25. Available at: http://www.asiaweek.com/asiaweek/magazine/99/1008/viewpoint.html

Simon, C. P. and L. Blume, (1994), Mathematics for Economists, W.W.Norton & Co., London

World Bank (1997), Poverty Lines, Policy Research and Poverty and Social Policy Department World Bank, No. 6. Available at http://www.worldbank.org/html/ prdph/lsms/research/povline/pl_n06.pdf.
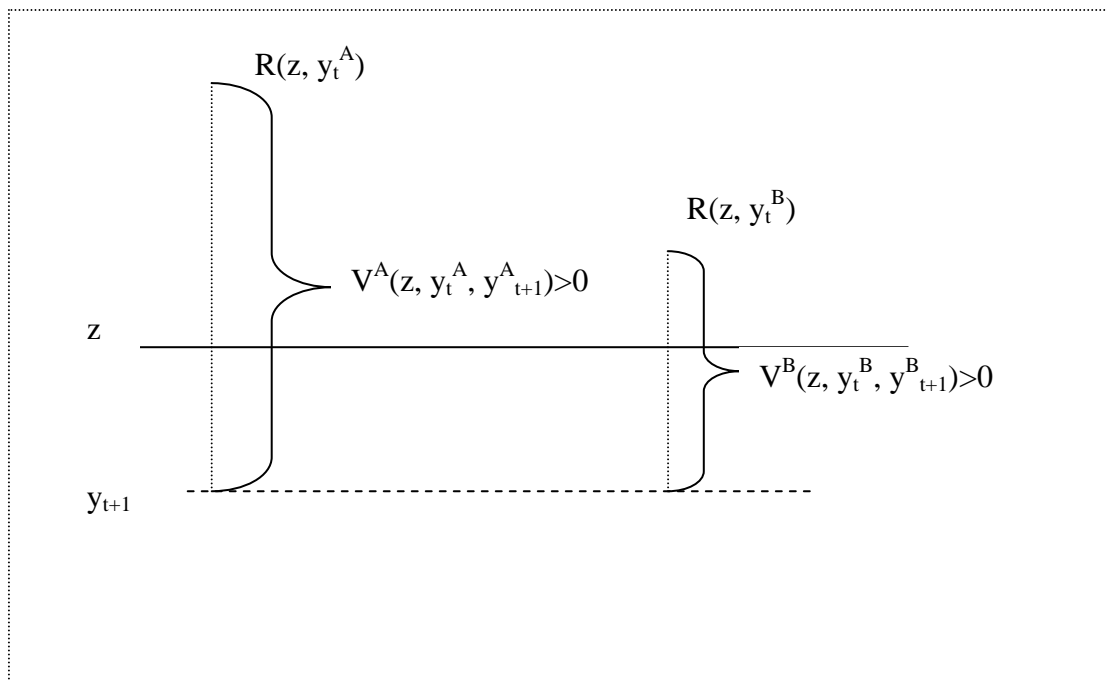
Figure 1: Schematic diagram of a two state lottery where both A and B receive the same future income below the poverty line with probability half and the status quo with probability half. Person A, starting from a higher living standard, has higher vulnerability.
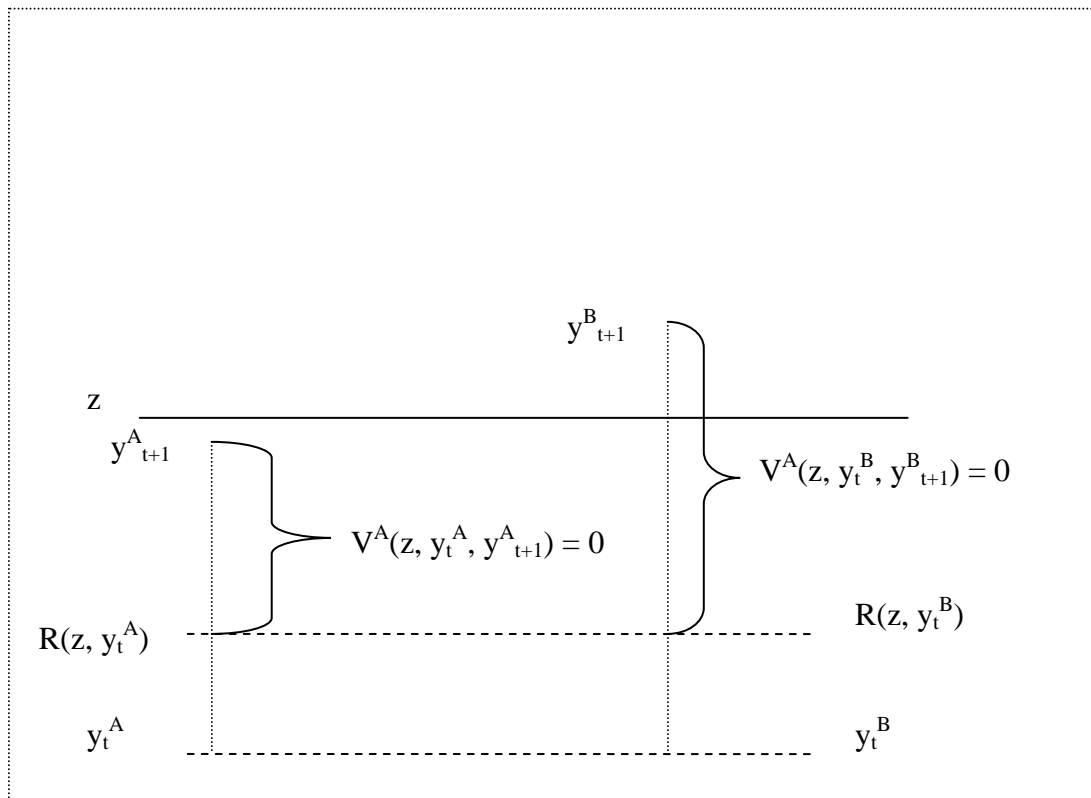
Figure 2: Schematic diagram of a two state lottery where both A and B receive z with probability half and in another state A receives a future income below the poverty line and B receives a future income above the poverty line. Both A and B are not vulnerable.