

# DAY LABORATORY EXERCISE #5: PARALLAX & TRIANGULATION

## Goals:

- To use trigonometry to determine distances and sizes of objects
- To gain experience in measuring angles and distances
- To measure a stellar parallax

**Equipment:** rulers, magnetic compass, trigonometric calculator

## Methods:

- Observe how far your finger appears to jump when you open and close either eye
- Observe angles to the Hyatt hotel from two different locations
- Measure the parallax of Barnard's star from simulated HST images

**Introduction** - How can we find the distances to stars? Direct probes such as laser ranging use the time delay between a light pulse sent to an object and the return of the reflected light pulse plus the finite speed of light to measure distances. However, stars are so far away and such poor reflectors that this approach is impractical (though it is used to determine distances to planets in our solar system). The question of measuring star distances has occupied astronomers at least since the time of the ancient Greek astronomer Hipparchus. He realized that if the Earth were to move around the Sun, then the stars should show the effects of **parallax**. This occurs whenever our viewpoint to nearby and distant objects changes, leading to an apparent angular change which is different for nearby objects, relative to more distant ones. The effect diminishes with distance so that even for the nearest stars, the parallax is so small that extremely precise measurements are needed just to detect the effect.

In this laboratory exercise, you will use the parallax technique to measure the length of your arm and to become familiar with the necessary geometry. Next, a variant of parallax geometry, called triangulation, will be used to measure the distance and height of the Hyatt hotel (or some other visible building) across the Charles River. Finally, you will use simulated HST images containing one of the nearest stars, Barnard's star, taken on four different dates to measure its stellar parallax and thereby determine its distance.

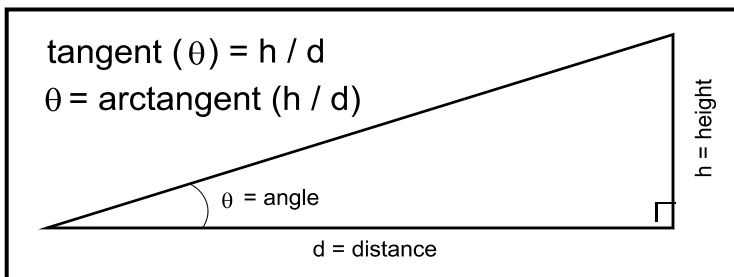


Figure 1: Right triangle showing the relation between the tangent of the opening angle and the ratio of the sides of the triangle.

## Astronomical Trigonometry -

Astronomers use trigonometry to solve many problems involving distances and angles. Most of these problems can be simplified to a single triangle, as shown in Figure 1. Triangles in general have three sides and three angles, but often right triangles (ones with a 90 degree angle) make problem solving much simpler. In this figure, our

triangle has two marked sides (**d** and **h**) and one marked angle ( $\theta$ ). For such a right triangle, the trigonometric function known as the **tangent** of the angle is exactly equal to the length of the **h** side

divided by lengths of the  $d$  side. We can also state that the angle  $\theta$  is equal to the **arctangent** (the inverse tangent) of the  $h/d$  ratio. With just the tangent trigonometric function, we can solve many astronomical geometry problems such as those arising in parallax and triangulation.

**Parallax Trigonometry** - Figure 2 shows how parallax is used to measure the length of your arm. With a finger held out at arm's length, if you close one eye and look, your finger is seen in projection against a distant object, like a wall of the lab. For example, if your left eye is open ("L" on the figure), you will see your finger ("F") projected onto the wall at some location ("WL"). When you close your left eye and open the right ("R"), your finger ("F") will appear to move its location ("WR").

The parallax angle  $\pi$  is defined to be one-half of the angle between the projected locations of your finger on the wall, so in the figure a dotted line (from "CE" to "CW") is shown separating the two  $\pi$  angles. The angle  $\beta$  also measures the angle between the projected locations of your finger on the wall. If the length of your arm is small compared to the distance you stand from the wall, then the angle  $\beta$  is equivalent to twice the  $\pi$  angles.

In that case,  $\beta/2 = \pi = \arctangent(P/2 / [W-A])$ , where  $P$  is the measured distance between locations on the wall where your finger was projected (measured in cm) and  $[W-A]$  is the distance from your finger to the wall (measured in cm). We can rewrite this as:  $P = 2 * [W-A] * \arctangent(\pi)$ .

Your arm distance is also related to  $\pi$ , through the angle made by the lines CE-F and F-L. Here,  $B = 2 * A * \arctangent(\pi)$ , where  $B$  is the distance between your eyes (the "baseline") and  $A$  is the distance from your eyes to your finger at the end of your arm.

Seeing that we have  $\arctangent(\pi)$  in common in the wall and arm equations, we can substitute one into the other to obtain a simpler relation:  $A = B * ([W-A] / P)$ , which relates arm length to eye separation and the apparent angular jump of projected finger locations on the distant lab wall. This can be solved alone for  $A$  to be:

$$A = (B / (B+P)) * W$$

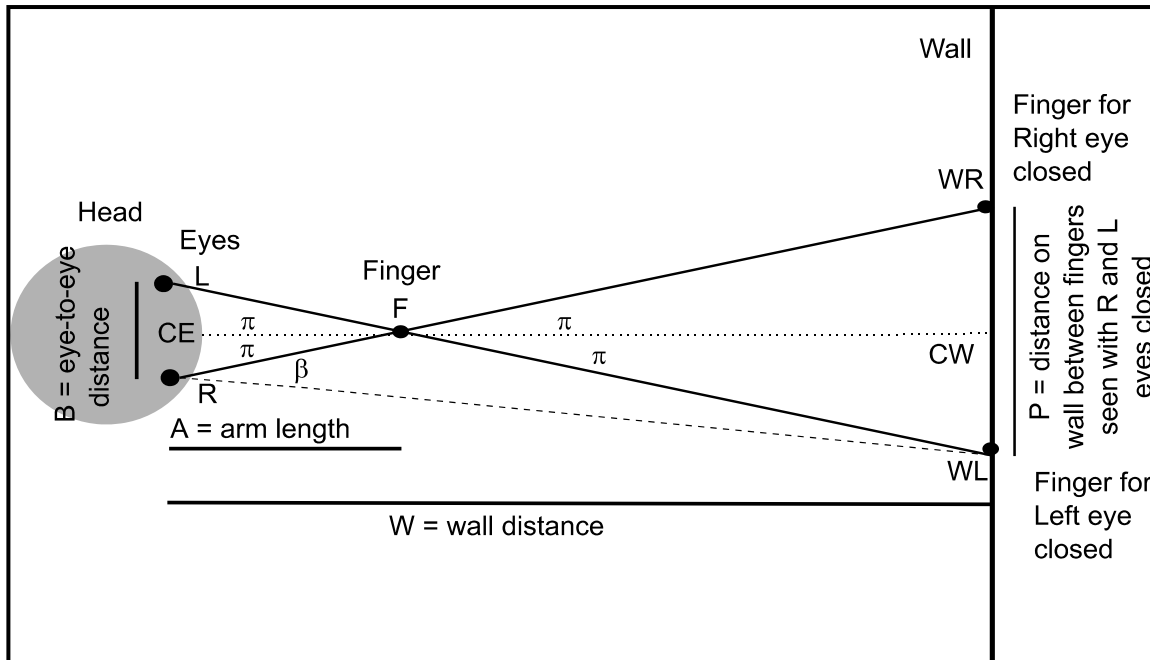


Figure 2: Schematic top view of the configuration used to measure the length of your arm using parallax. The angles  $\pi$  are the parallax angles.

**Triangulation to distant objects** - A terrestrial version of parallax which is practiced frequently is triangulation. Here we measure two angles and one baseline, and use trigonometry to determine object distances and heights.

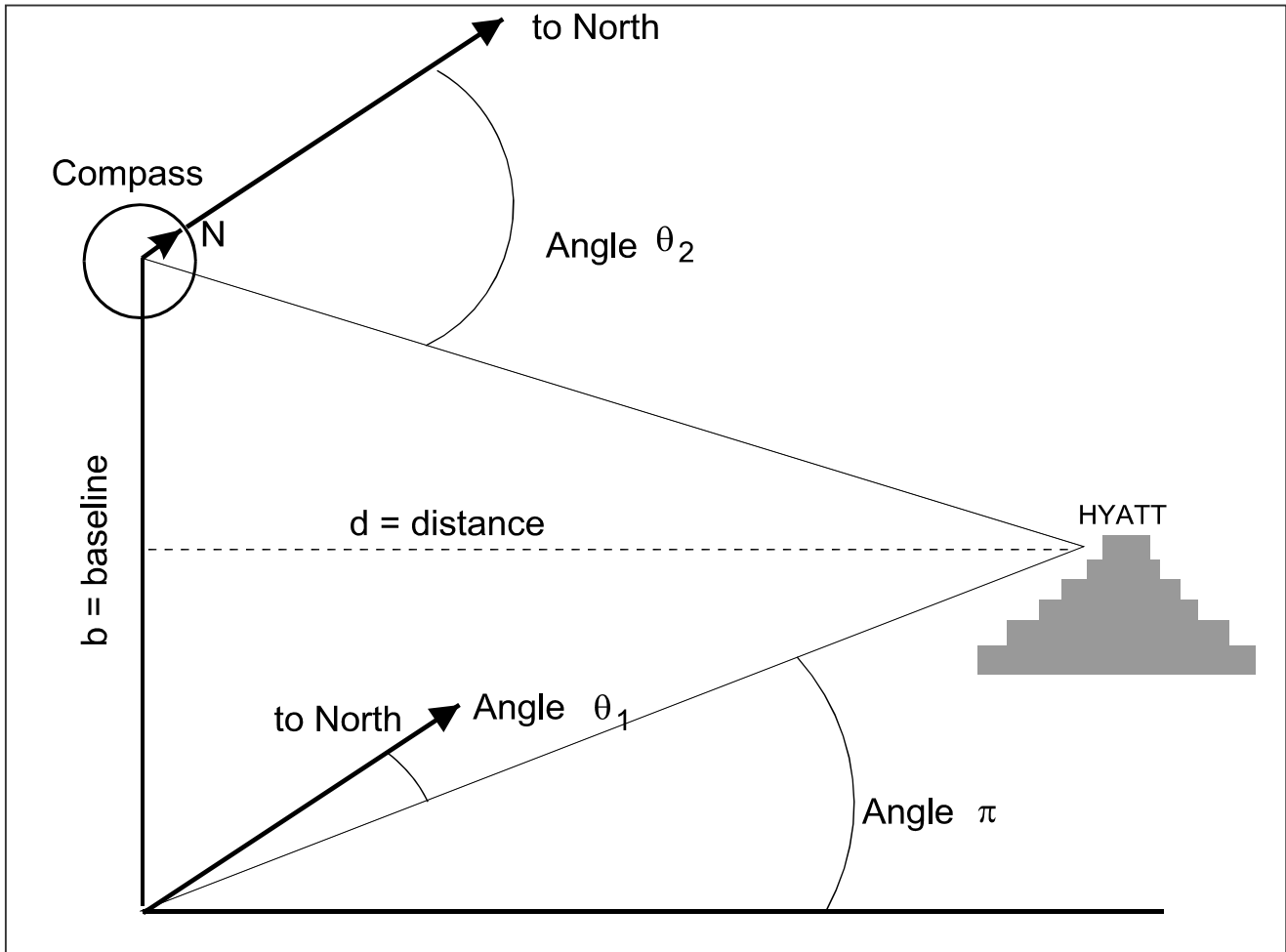


Figure 3: Triangulation geometry for measuring the distance to the Hyatt Hotel across the Charles River. The baseline  $b$  connects the two points from which sightings of the Hyatt are taken. Each sighting consists of a measurement of the angle between the North magnetic pole and the direction to the Hyatt.

The geometry is similar to the lab parallax case. Here the distance to the Hyatt,  $d$ , is given by:

$$d = (b/2) / \text{tangent}(\pi).$$

And, we can estimate angle  $\pi$  using the following relation:

$$\pi = (1/2) * (\theta_1 - \theta_2).$$

**Parallax to Barnard's Star** - If we take photographs or images of nearby stars at different times of the year, then the motion of the Earth about the Sun causes our observing location to be different and the nearby stars will show parallax motions. Barnard's star is one such nearby star. But, in addition to showing parallax effects on images taken 6 months apart, this star is also moving in the plane of the sky at a rapid angular rate, meaning it is moving through the Milky Way, ALSO. This **proper motion** is seen as that motion of the star's position against the fixed background stars which does not return to its original value one year later. Recall that on photographs taken exactly one year apart, stars will show no parallax effects. A series of images, taken over more than a year, can be analyzed to measure the star's parallax, and therefore distance. These same images can be used to measure the star's proper motion and, combined with the distance value, can be used to determine the speed of the star as it moves across the sky.

Sensing the small parallax of Barnard's star requires a large telescope with either exceptionally good seeing or a location in orbit about the Earth. And, while HST has not studied Barnard's star, we have calculated some simulated images for this star and its neighbors and will use these to deduce the star's distance and speed.

## DAY LABORATORY EXERCISE #5: PARALLAX

Name/ID\# \_\_\_\_\_

TA's Initials: \_\_\_\_\_

Class/Section: AS102/ \_\_\_\_\_

Date due: \_\_\_\_\_

### Procedure -

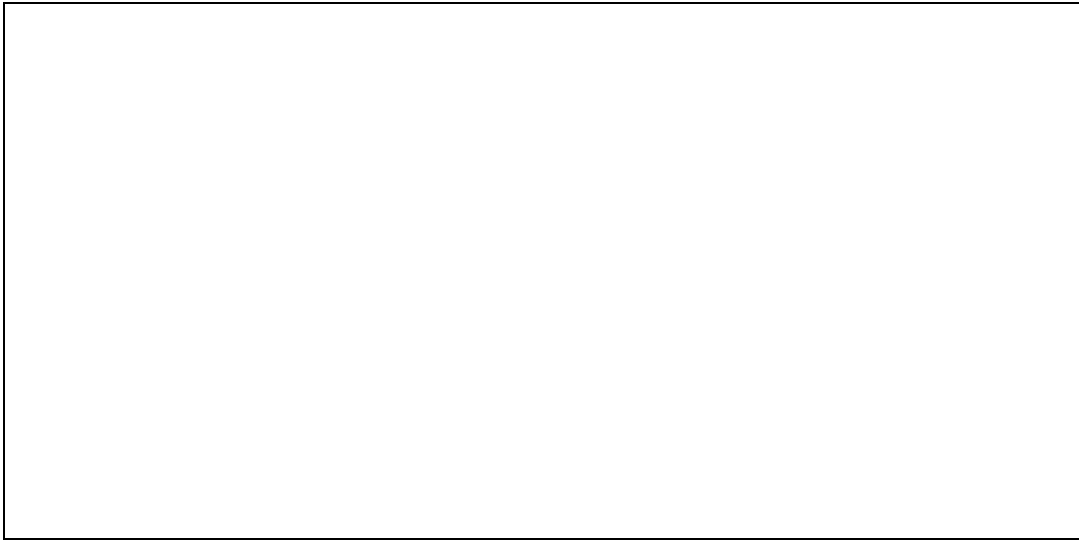
1. Lab: Hold your index finger at arm's length in front of you and close your left eye. Have your partner place a mark on the far lab wall to indicate where your finger appears in projection against the wall. Open your left eye and close your right, and have your partner place a new mark on the wall. Measure the distance between the marks. Measure the distance from you to the wall. Perform the trigonometric calculations indicated.
2. Hyatt: Measure the angle between magnetic north and the sign for the Hyatt hotel from two locations (or "stations") on our side of the river. If the weather is good, go outside and do this from the easternmost and westernmost ends of the "BU beach." Be sure to measure the distance between your measuring locations. If the weather is poor, perform the measurements from room CAS 500 and the TA's office near the elevator on the 5<sup>th</sup> floor. Remember to measure the distance between your measuring stations.
3. Barnard's star: Use the four simulated HST images. Determine which part of the motion of the star is due to proper motion and which is due to parallax.

### 1. Parallax in the Laboratory

- Sketch where you and your partner need to stand.



- Draw a triangle to show what must be measured.



- What values did you measure?

W= P=
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- What value do you calculate for your arm length?

A=
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## **2. Triangulation across the Charles River**

- Establish a Station 1 measuring position. Align the magnetic compass so that the needle points to magnetic north and the dial reads 0 degrees in that direction. Be careful not to take a compass reading close to the windowsills in the building, because the metal in the sills will alter the compass reading.
  - What is the dial reading for the direction to the Hyatt sign? Repeat 5 times.

- Establish a Station 2. Again, align the compass with magnetic north.
  - What is the dial reading for the direction to the Hyatt sign? Repeat 5 times.

- What is the distance between Stations 1 and 2?

- What distance do you calculate to the Hyatt?

- How could you establish the height of the Hyatt?

### **3. Parallax of Barnard’s Star - Stellar Distances**

- Examine the simulated HST images and identify Barnard’s star (the one which moves the most).
- Trace the location of Barnard’s star onto a single sheet of paper.
- How might you separate the star’s proper motion from its parallax motion?

- Using the images, what is the value of the star’s proper motion, in units of mm (on the images) per year of time?

- Similarly, what is the value of the star’s parallax, in units of mm (on the images).

- The focal length of the HST camera is 72 meters, which leads to a “plate scale” of 0.119 arcseconds per mm on your paper. Using this factor, what is the parallax of Barnard’s star in arcseconds?



- What is the distance to Barnard's star in parsecs?

- What is the proper motion of Barnard's star in parsecs per year?

- What is the speed of Barnard's star across the sky, in units of km/s?

**Summary Questions:**

1. What parallax would you expect for a star at a distance of 100pc? Of 1000pc? Of 1,000,000pc? Would these be easy or difficult parallax values to measure? Why?

2. How might you improve your distance estimate to the Hyatt hotel? What tools or procedure would you change?

