Embodying Computation at Higher Types

S. Barry Cooper, *TURING 100*, University of Boston, 12 November, 2012

Articles

UBIQUITY SYMPOSIUM 'WHAT IS COMPUTATION?' COMPUTATION IS PROCESS

November 2010 | BY **DENNIS J. FRAILEY**

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Various authors define forms of computation as specialized types of processes. As the scope of computation widens, the range of such specialties increases. Dennis J. Frailey posits that the essence of computation can be found in any form of process, hence the title and the thesis of this paper in the Ubiquity symposium discussion what is computation. --Editor

The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century. Denning [2010], **in his opening statement**, describes how *computation* was originally defined in the 1930s and how that definition has progressed through the ensuing decades. Church, Gödel, and Turing defined it in terms of mathematical functions, which they divided into the decidable (can be evaluated by algorithms) and the un-

"What Is Computation?"

- **ACM *UBIQUITY* SYMPOSIUM: *Computation Is Process* by Dennis J Frailey:**

- The concept of computation is arguably the most dramatic advance in mathematical thinking of the past century

- Church, Gödel, and Turing defined it in terms of mathematical functions ... They were inclined to the view that only the algorithmic functions constituted computation

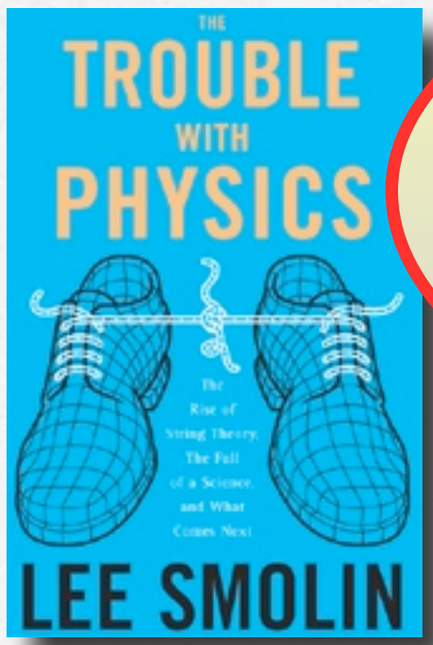
I'll call this the "mathematician's bias" because I believe it limits our thinking and prevent us from fully appreciating the power of computation

Causal Character of Emergence ?

… Causality as Basic

- In modelling the physical universe –

… causality itself is fundamental

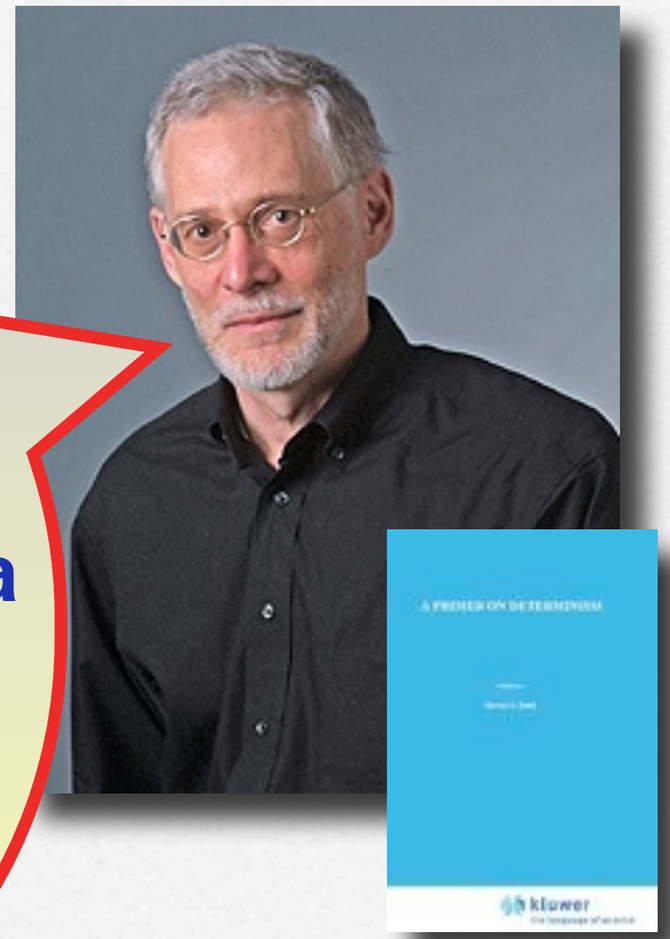


Lee Smolin, 'The Trouble With Physics', p.241

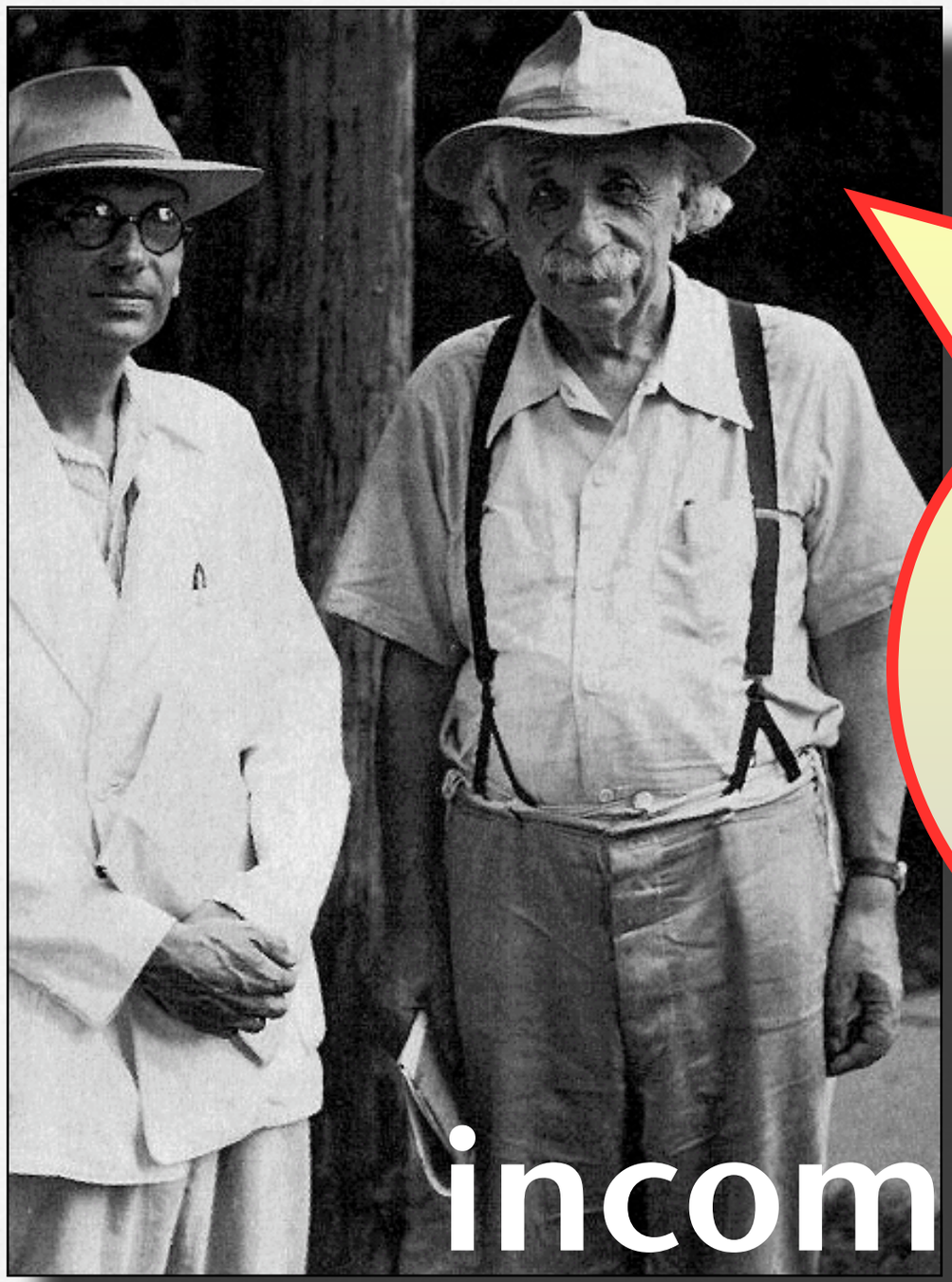
Causality as a 'truly obscure concept' ...

John Earman in "A Primer On Determinism", D. Reidel/Kluwer, 1986, p.5:

... the most venerable of all the philosophical definitions [of determinism] holds that the world is deterministic just in case every event has a cause. The most immediate objection to this approach is that it seeks to explain a vague concept - determinism - in terms of a truly obscure one - causation.



Causality ...



From A. Einstein: "Autobiographical Notes", in "Albert Einstein: Philosopher-Scientist" (P. Schilpp, ed.), Open Court Publishing, 1969, p.63

... I would like to state a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e. intelligibility, of nature ... nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory) ...

incomplete ...

Causality Incomplete

If the creation of the universe can be described as a quantum process, we would be left with one deep mystery of existence: What is it that determined the laws of physics?

A.H. Guth, *The Inflationary Universe - The Quest for a New Theory of Cosmic Origins*, Addison-Wesley, 1997

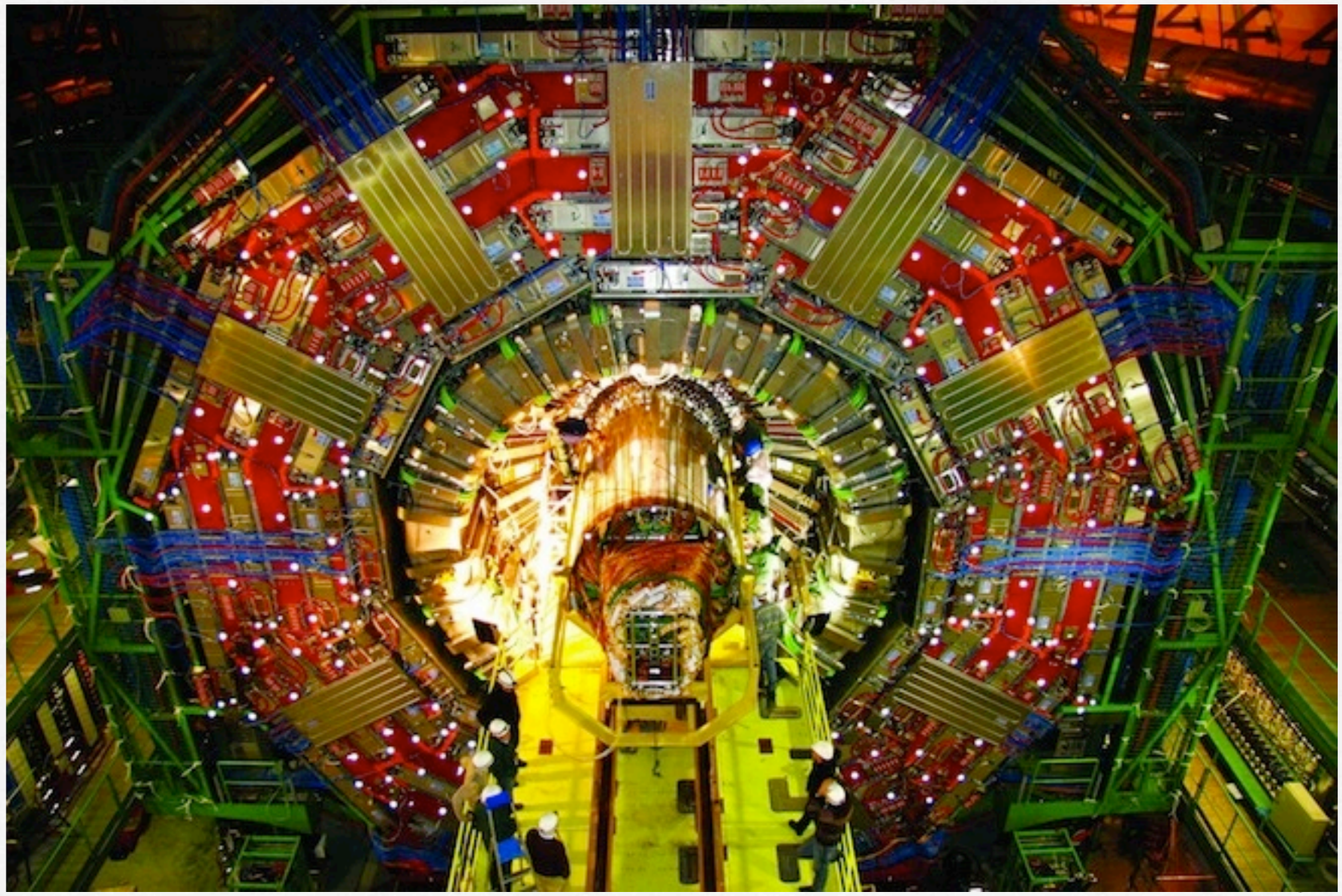
One way of thinking about what is unsatisfactory about the standard model is that it leaves seventeen non-trivial numbers still to be explained, ...

Peter Woit: *Not Even Wrong - The Failure of String Theory and the Continuing Challenge to Unify the Laws of Physics*, Jonathan Cape, 2006

Causality Incomplete

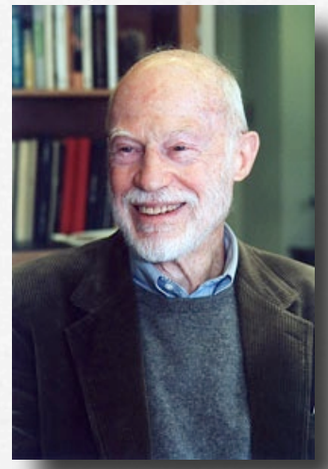
ABC/ BBC News:

Supersymmetry running out of places to hide



Causality Abrogated ...

Bryce DeWitt

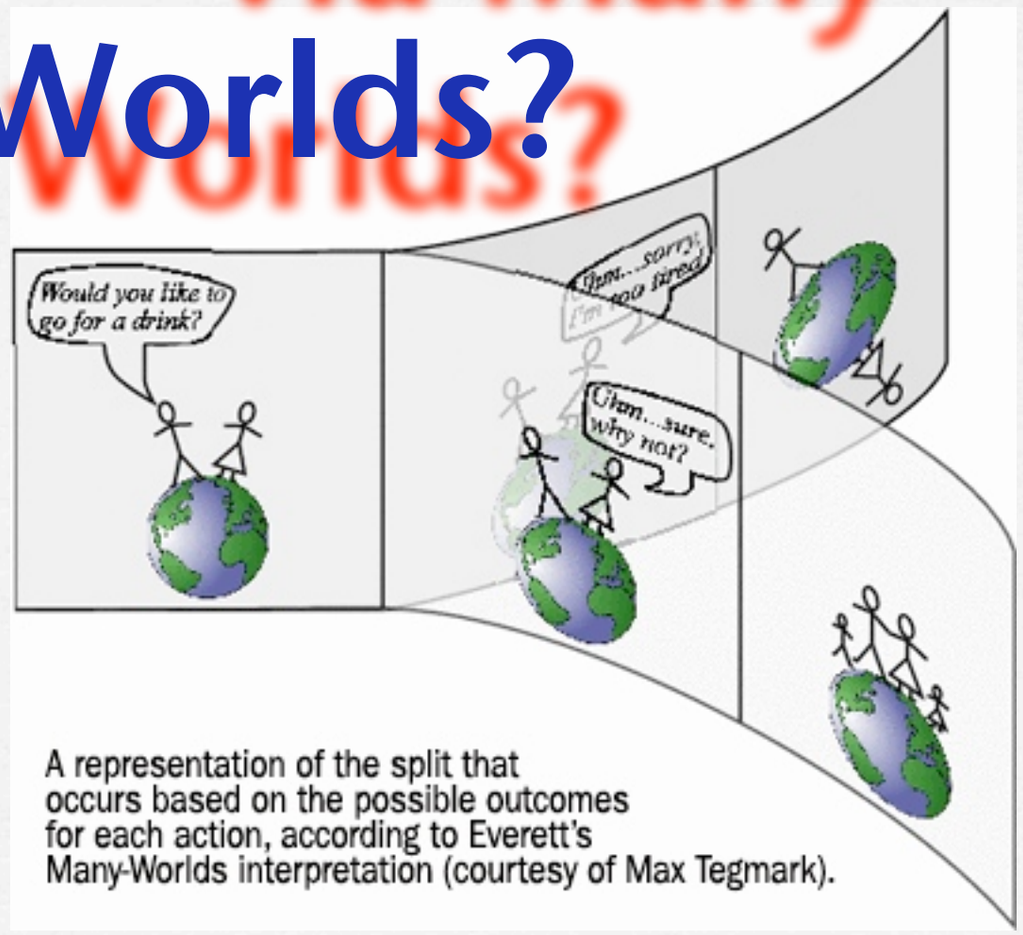


... via Many Worlds?

Hugh Everett III

(Nov.11, 1930- July 19, 1982)

John Wheeler



A representation of the split that occurs based on the possible outcomes for each action, according to Everett's Many-Worlds interpretation (courtesy of Max Tegmark).

By Max Tegmark

Parallel Universes

Not just a staple of science fiction, other universes are a direct implication of cosmological observations

Is there a copy of you

reading this article? A person who is not you but who lives on a planet called Earth, with misty mountains, fertile fields and sprawling cities, in a solar system with eight other planets? The life of this person has been identical to yours in every respect. But perhaps he or she now decides to put down this article without finishing it, while you read on.

The idea of such an alter ego seems strange and implausible, but it looks as if we will just have to live with it, because it is supported by astronomical observations. The simplest and most popular cosmological model today predicts that you have a twin in a galaxy about 10 to the 10^{28} meters from here. This distance is so large that it is beyond astronomical, but that does not make your doppelgänger any less real. The estimate is derived from elementary probability and does not even assume speculative modern physics, merely that space is infinite (or at least sufficiently large) in size and almost uniformly filled with matter, as observations indicate. In infinite space, even the most unlikely events must take place somewhere. There are infinitely many other inhabited planets, including not just one but infinitely many that have people with the same appearance, name and memories as you, who play out every possible permutation of your life choices.

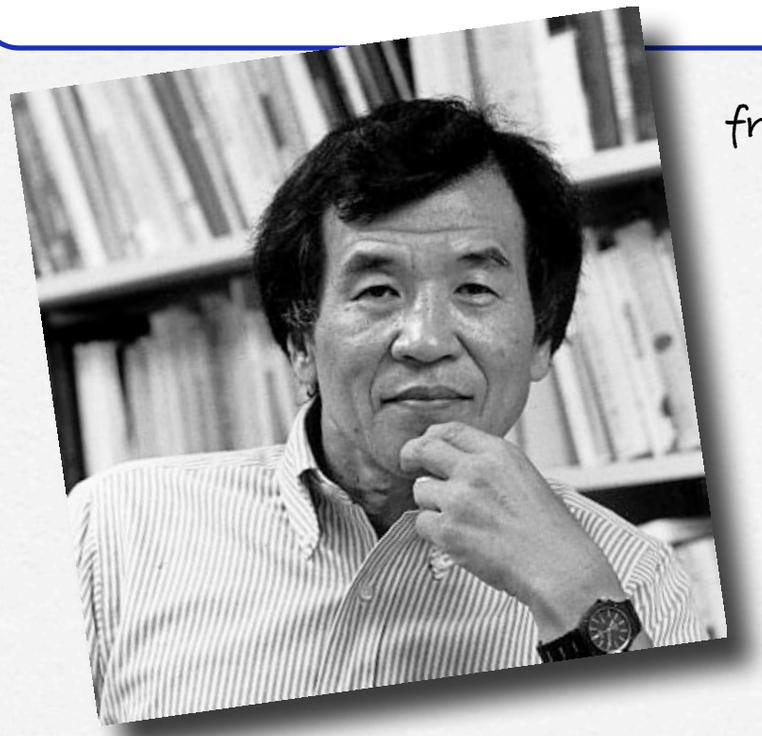
You will probably never see your other selves. The farthest you can observe is the distance that light has been able to travel during the 14 billion years since the big bang expansion began. The most distant visible objects are now about 4×10^{26} meters away—a distance that defines our observable universe, also called our Hubble volume, our horizon volume or simply our universe. Likewise, the universes of your other selves are spheres of the same size centered on their planets. They are the most straightforward example of parallel universes. Each universe is merely a small part of a larger "multiverse."

By this very definition of "universe," one might expect the notion of a multiverse to be forever in the domain of metaphysics. Yet the borderline between physics and metaphysics is defined by whether a theory is experimentally testable, not by whether it is weird or involves unobservable entities. The frontiers of physics have gradually expanded to incorporate ever more abstract (and once metaphysical) concepts such as a round Earth, invisible electromagnetic fields, time slowdown at high speeds, quantum superpositions, curved space, and black holes. Over the past several years the concept of a multiverse has joined this list. It is grounded in well-tested theories such as relativity and quantum mechanics, and it fulfills both of the basic criteria

ALFRED T. KAMAJIAN

… Causality Fragmented

Supervenience 'represents the idea that mentality is at bottom physically based, and that there is no free-floating mentality unanchored in the physical nature of objects and events in which it is manifested'



from Jaegwon Kim: "Mind in a Physical World", MIT Press, 1998, pp.14-15

"A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties."

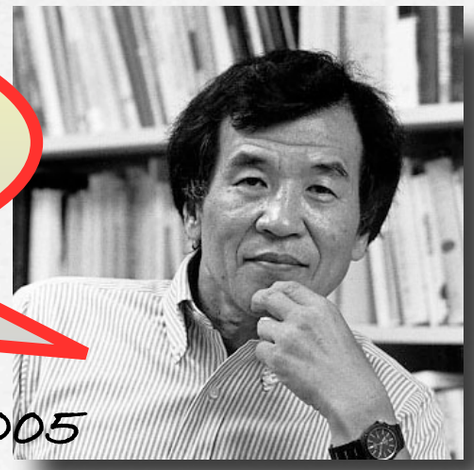
Stanford Encyclopedia of Philosophy

… Causality Fragmented

- How can mentality have a computational role in a world that is fundamentally physical?

- And what about 'overdetermination' - the problem of phenomena having both mental and physical causes?

… the problem of mental causation is solvable only if mentality is physically reducible; however, phenomenal consciousness resists physical reduction, putting its causal efficacy in peril.



-Jaegwon Kim: Physicalism, or Something Near Enough, Princeton, 2005

Mental Causality Hard to Simulate

- Solomon Feferman, 1988: "Turing in the Land of O(z)", in "The Universal Turing Machine: A Half-Century Survey" (Herken R., ed.), Oxford University Press, pp.131-2:

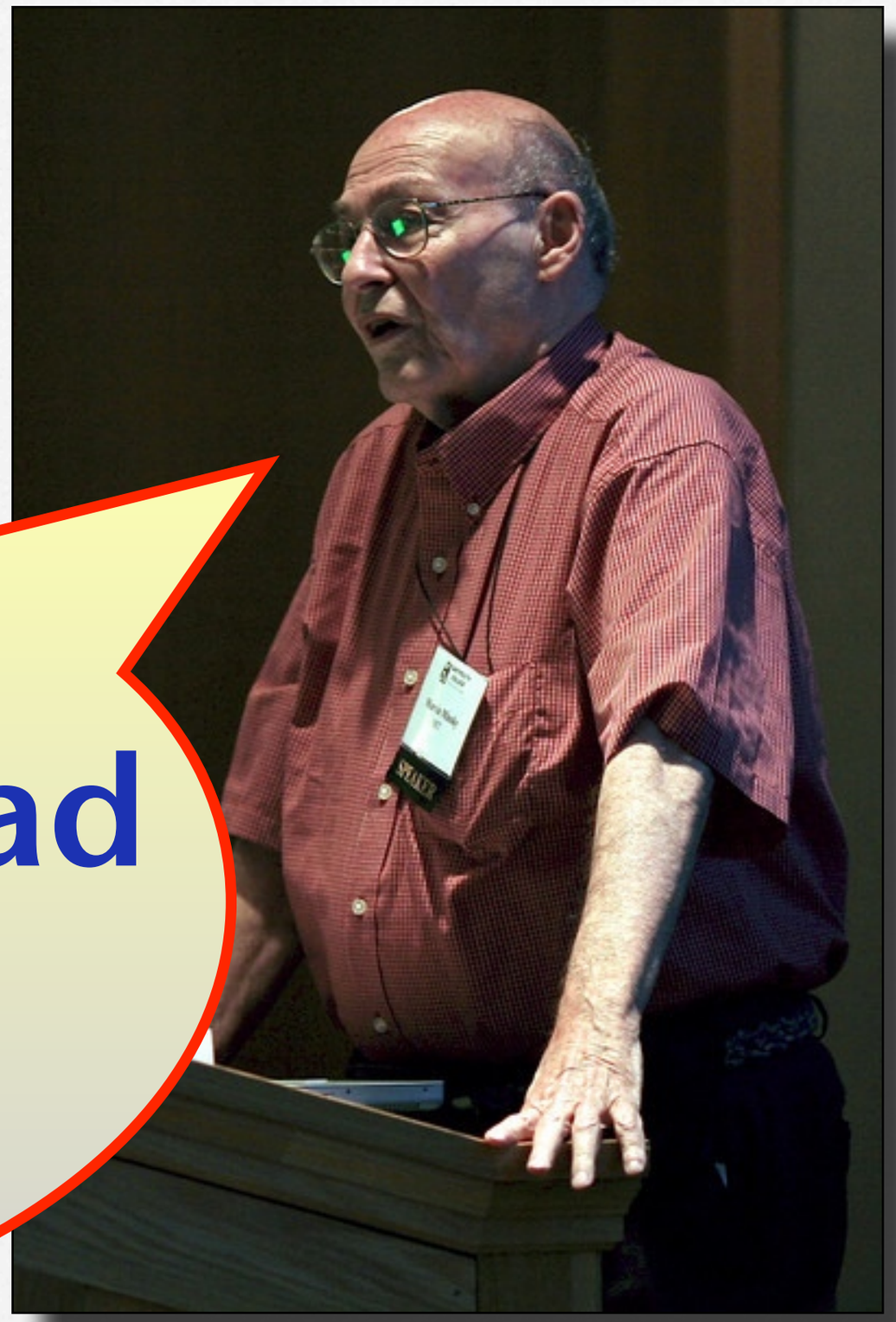
> **"Turing, as is well known, had a mechanistic conception of mind, and that conviction led him to have faith in the possibility of machines exhibiting intelligent behavior."**

- Rodney Brooks in Nature, 2001:

… neither AI nor Alife has produced artifacts that could be confused with a living organism for more than an instant

- Marvin Minsky at Boston University, May 2003:

AI has been brain-dead since the 1970s



- **But what is intelligence? "... if a machine is expected to be infallible, it cannot also be intelligent. There are several theorems which say almost exactly that."**

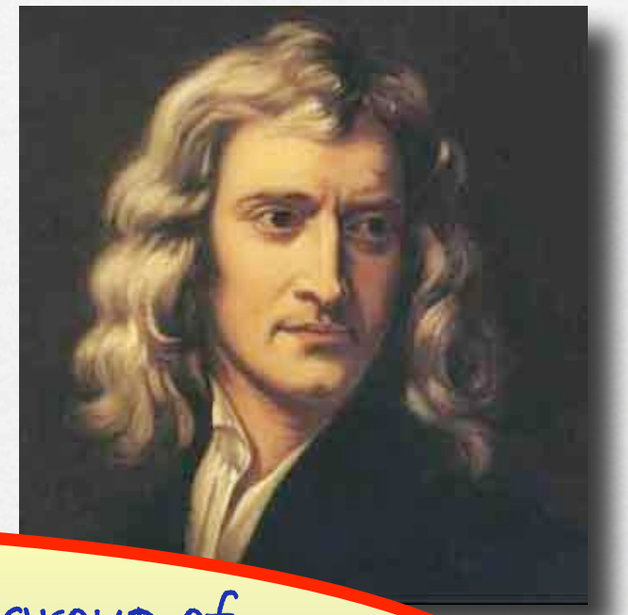
– A.M. Turing, talk to the London Mathematical Society, February 20, 1947, quoted in Andrew Hodges, p.361

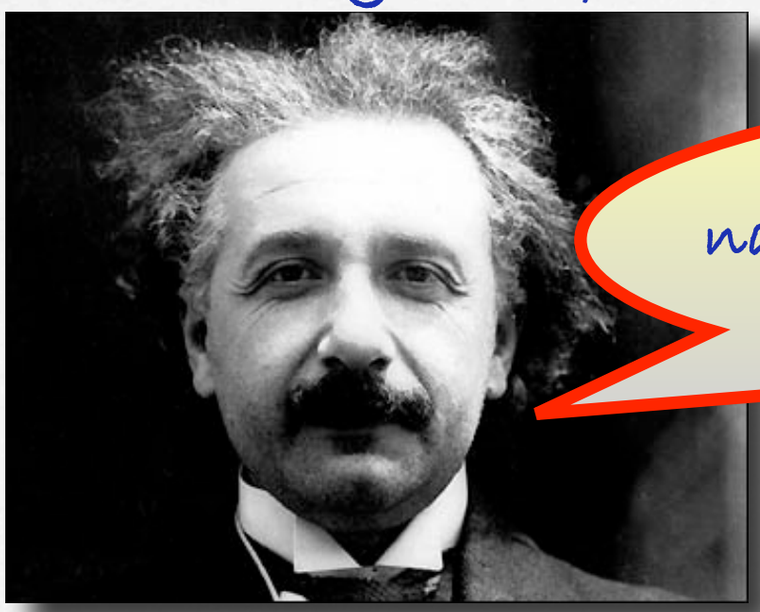
- **"The results which have been described in this article are mainly of a negative character, setting certain bounds to what we can hope to achieve purely by reasoning. These, and some other results of mathematical logic may be regarded as going some way towards a demonstration, within mathematics itself, of the inadequacy of 'reason' unsupported by common sense."**

– final paragraph of Alan Turing, Solvable and Unsolvable Problems, Penguin Science News 31, 1954, p.23

… Causality as Computation

- Newton onwards – mathematics rules science
  - we look for computable natural laws …
  - theories which computably predict …
  - try to capture truth via proofs …





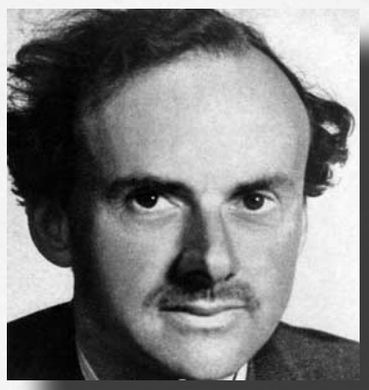
When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them

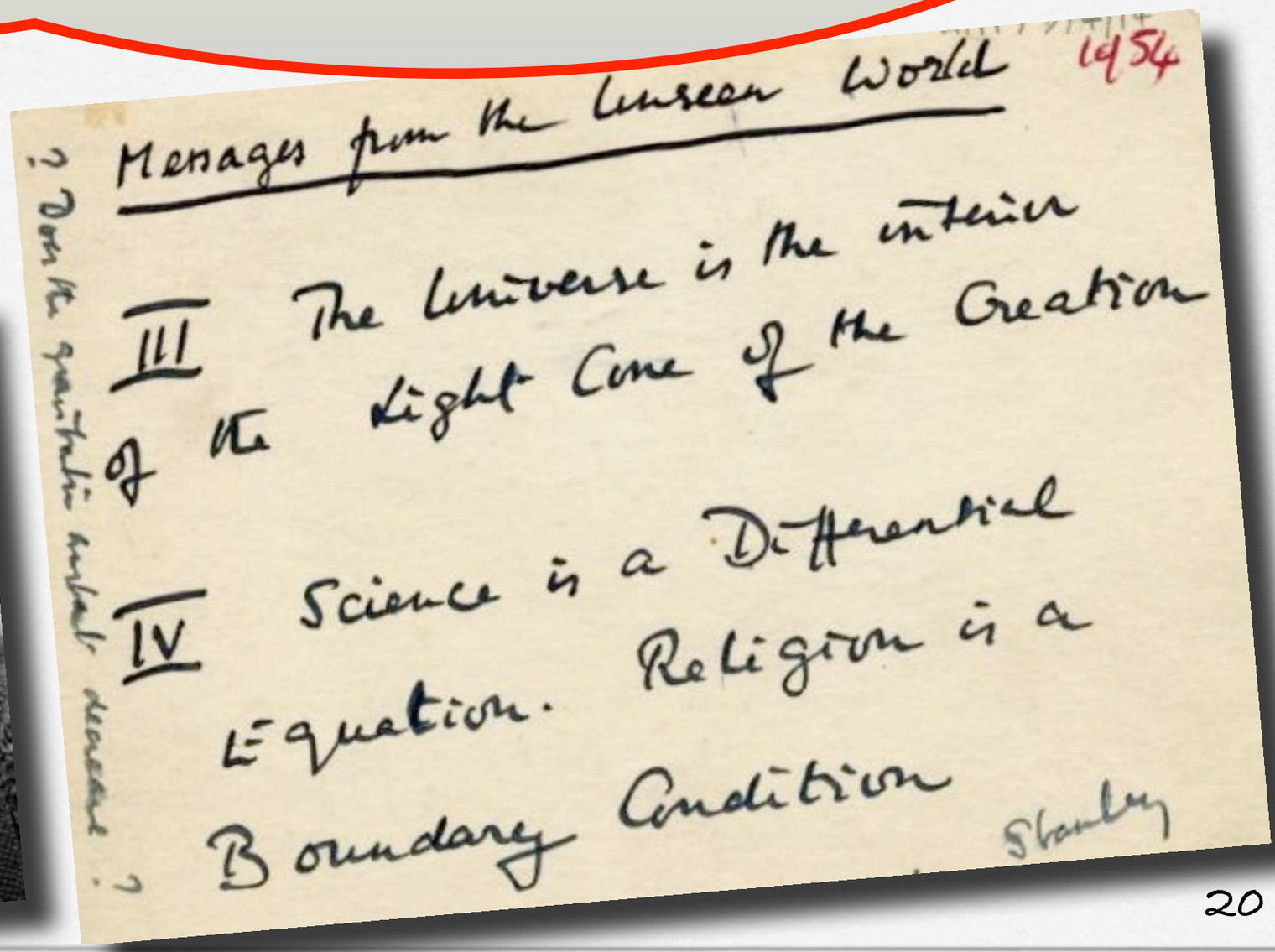
Albert Einstein: P.54, `Out of My Later Years', 1950

The TURING Approach to COMPUTATION

... at heart more of an applied than a pure mathematician

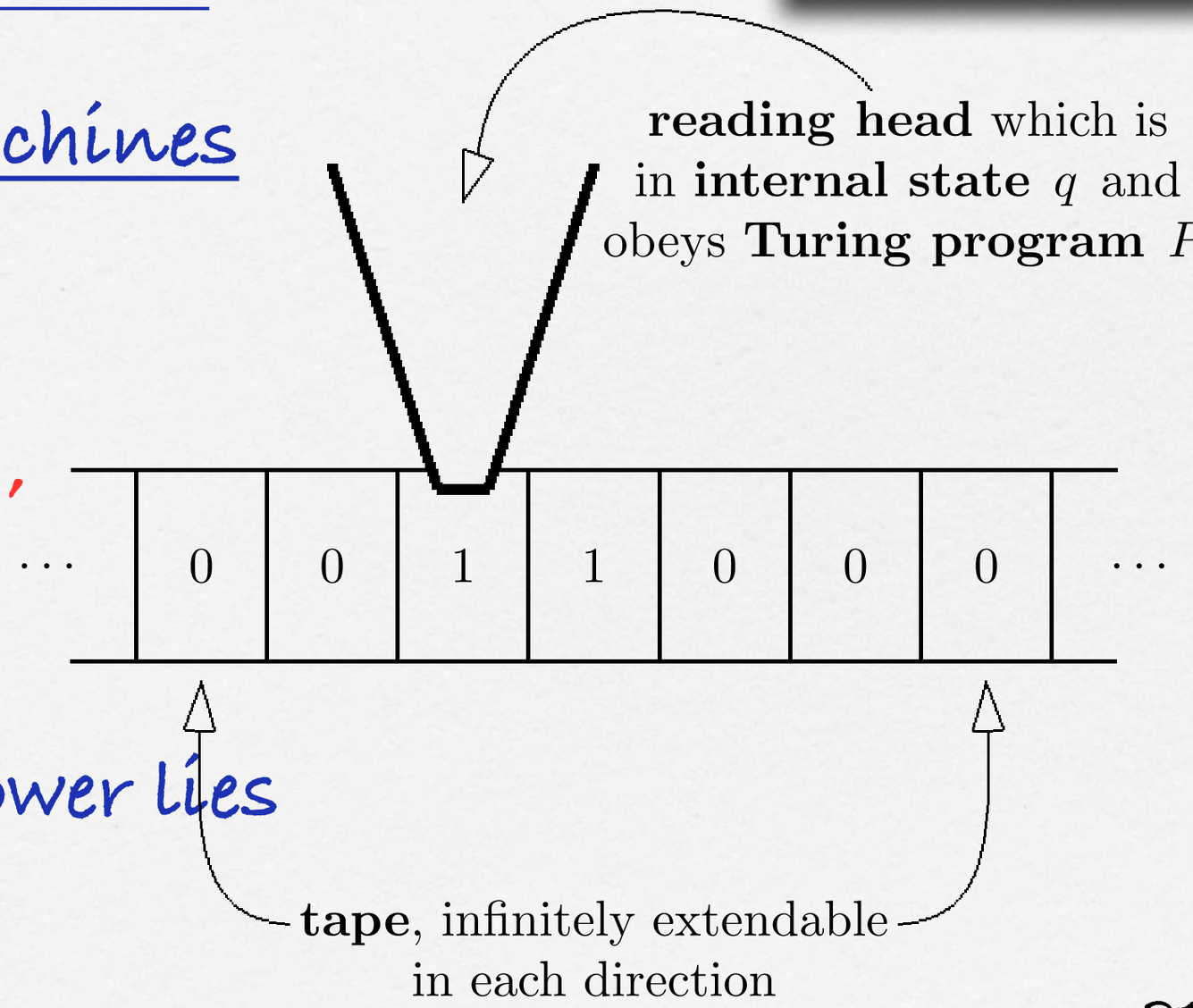
[See A. Hodges in 'Collected Works']





Computation seen as DISEMBODIED

- 1936 - Turing's machines
- Hardware trivial
- Computes 'anything' computable
- All the computing power lies in the program ...



Universality, and Programs as Data

Turing, 1936: Build a UNIVERSAL TURING MACHINE, which can simulate ANY other machine

Depends on TYPE 0 description of e^{th} Turing machine

There exists a Turing machine U — the **Universal Turing Machine** — *which if given input (e, x) simulates the e^{th} Turing machine with input x.*

That is, $\varphi_U^{(2)}(e, x) = \varphi_e(x)$.

A New Computing Paradigm ...

The omnipotent computer -

Functionalism and AI - stress what a computer does as something realisable in different hardware - Hilary Putnam: "Minds and Machines", 1960

I am building a brain ...

Hilary Putnam

A New Computing Paradigm ...

- Successful reduction of "natural" examples to the Turing model - e.g. quantum computation (David Deutsch)

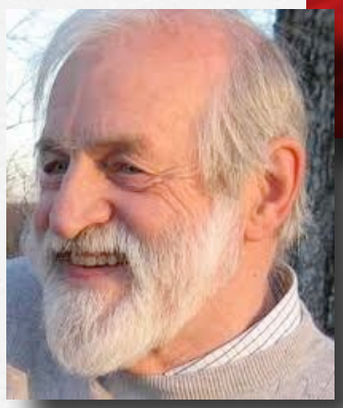
I am sure we will have [conscious computers], I expect they will be purely classical, and I expect that it will be a long time in the future.

Question and Answers with David Deutsch, on New.Scientist.com News Service, December, 2006

Complexity, Emergence, and Descriptions

Emergence of patterns in Nature

1950s - Alan Turing proposes a simple reaction-diffusion system describing chemical reactions and diffusion to account for morphogenesis



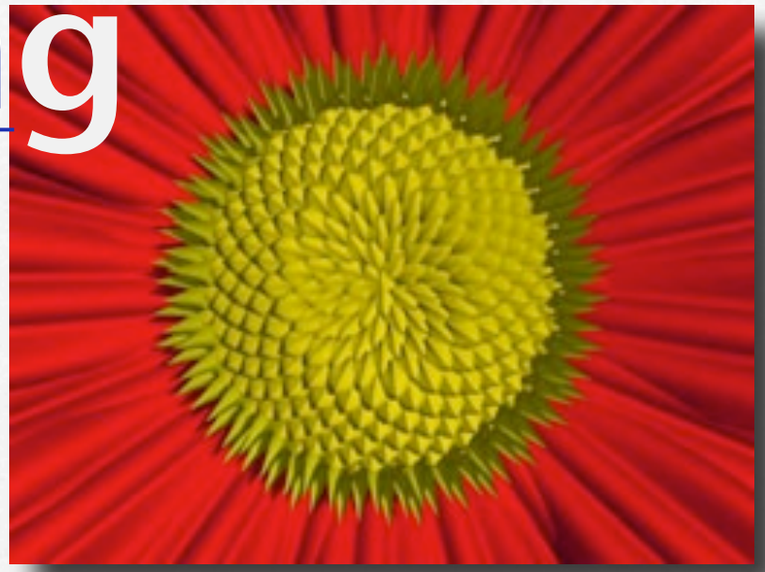
James D. Murray

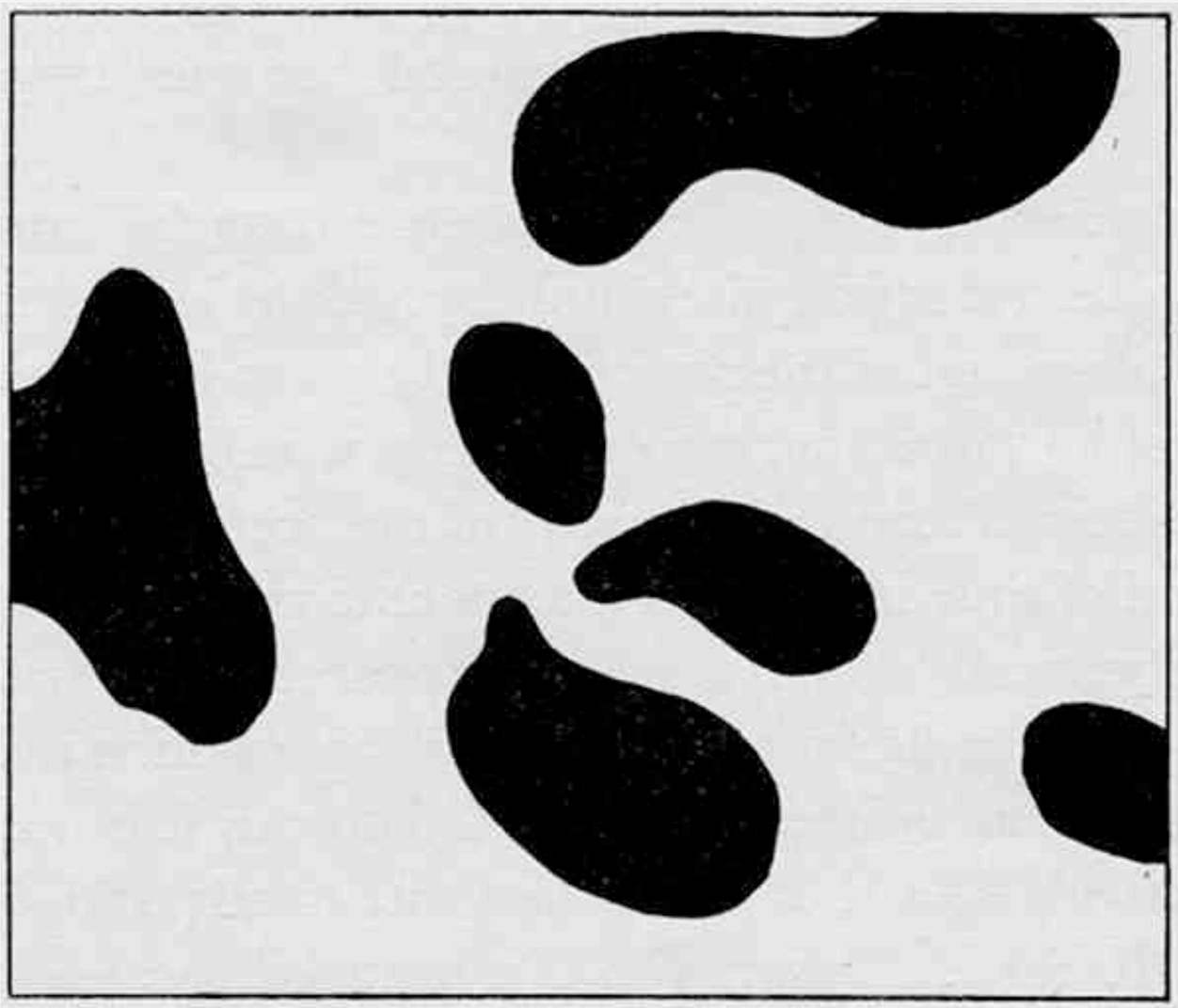
Philip Maini

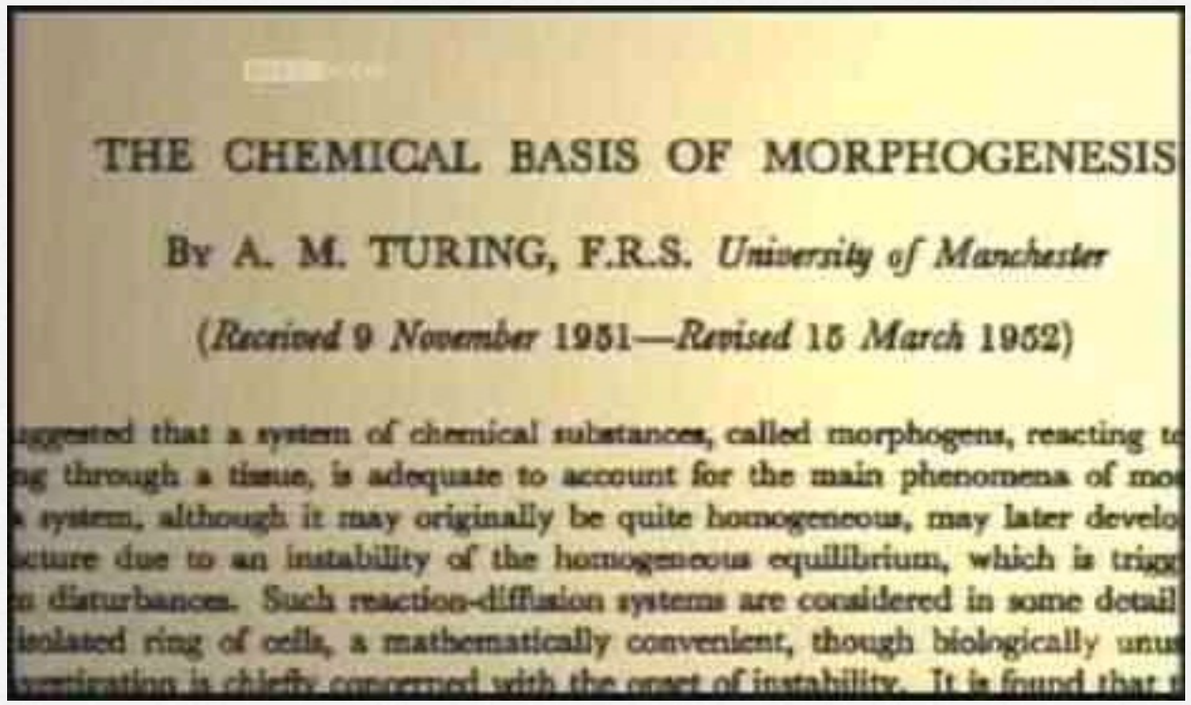
From website of the Biological Modeling and Visualization research group, Department of Computer Science at the University of Calgary

See http://www.swintons.net/jonathan/turing.htm

WHAT IS interesting about ...







THE CHEMICAL BASIS OF MORPHOGENESIS

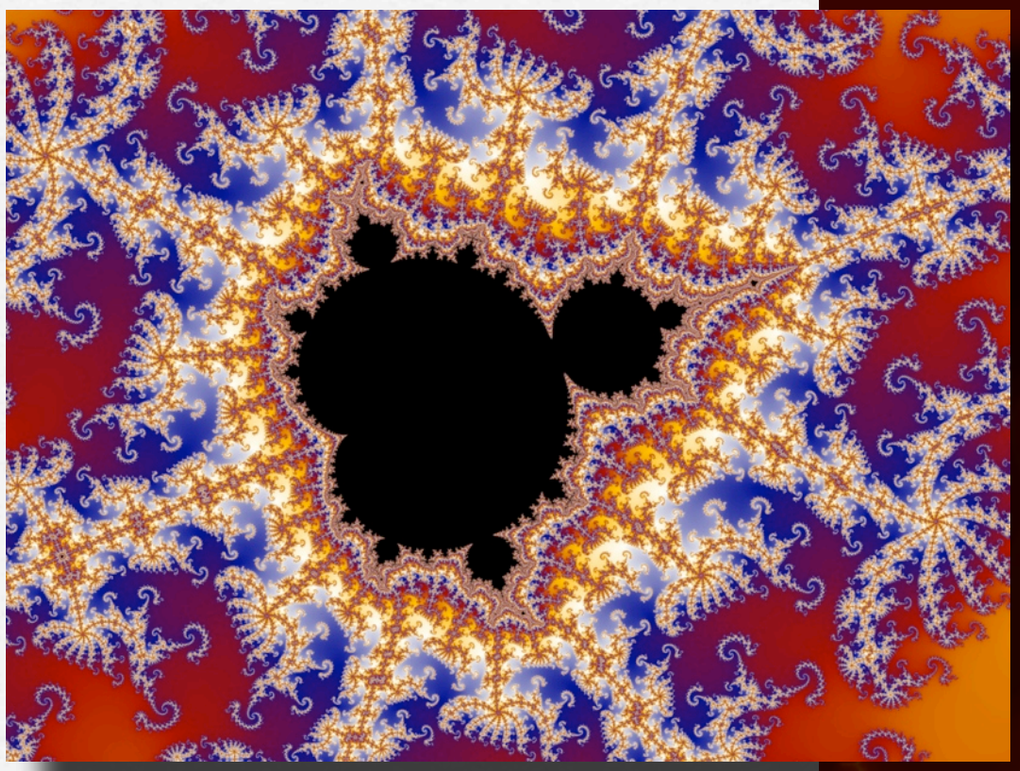
By A. M. TURING, F.R.S. *University of Manchester*

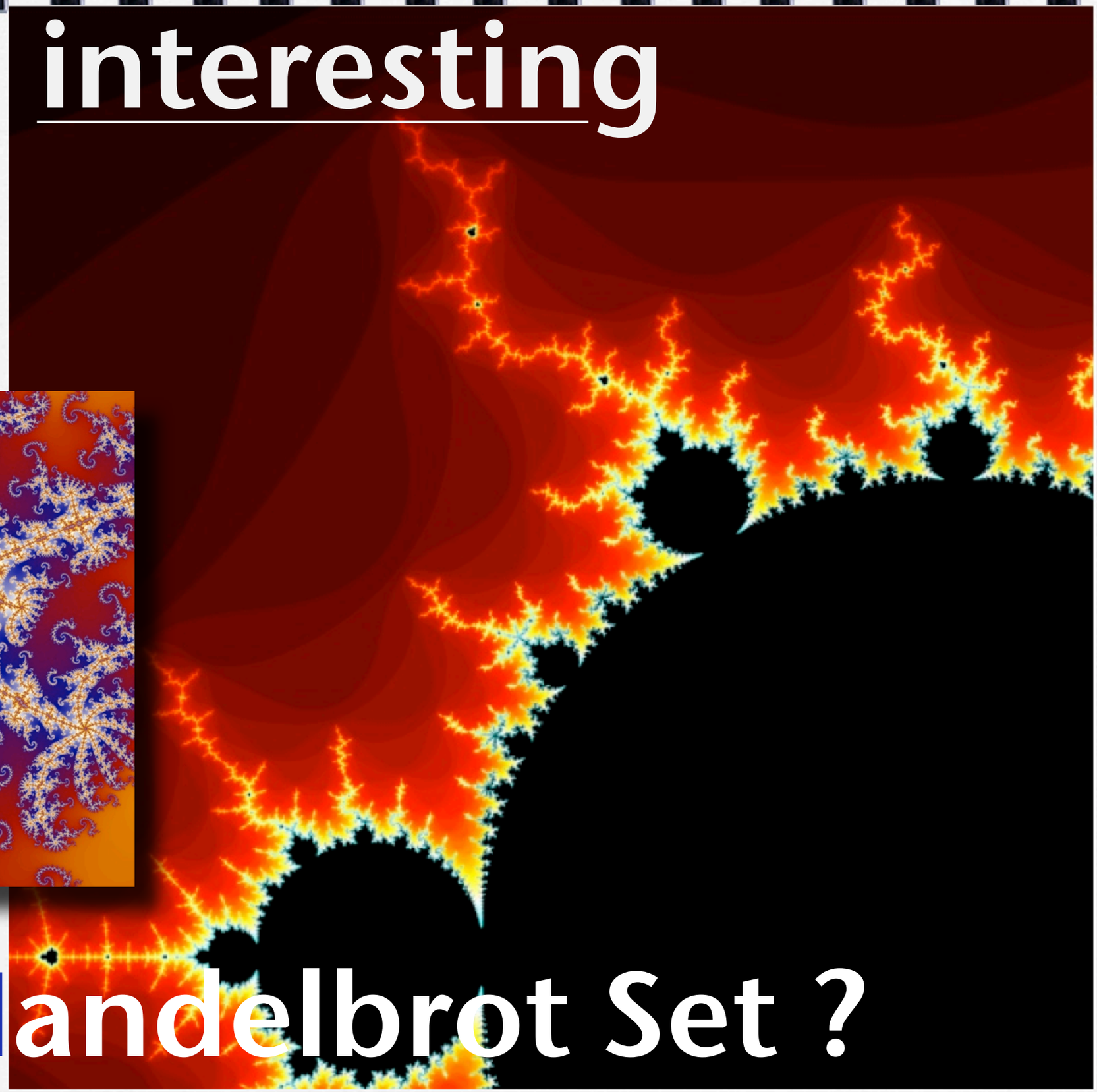
(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

ggested that a system of chemical substances, called morphogens, reacting t
ng through a tissue, is adequate to account for the main phenomena of mo
system, although it may originally be quite homogeneous, may later develo
cture due to an instability of the homogeneous equilibrium, which is trigg
disturbances. Such reaction-diffusion systems are considered in some detail
isolated ring of cells, a mathematically convenient, though biologically unu
vestigation is chiefly concerned with the onset of instability. It is found that

... Morphogenesis ?

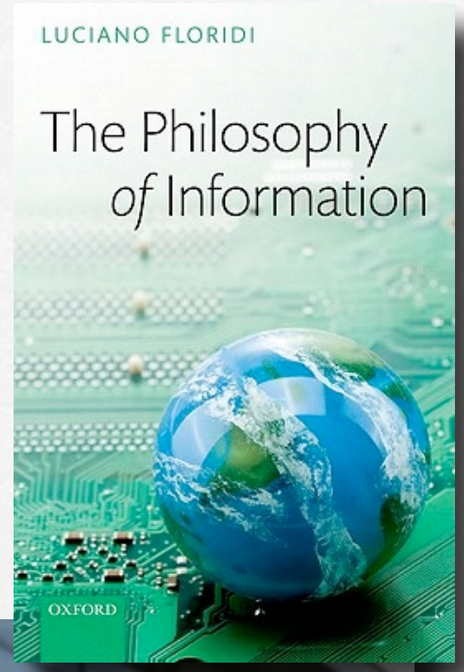
WHAT IS interesting about ...





... the Mandelbrot Set ?

LEVELS of ABSTRACTION



A *level of abstraction (LoA)* is a finite but non-empty set of observables. …
The introduction of LoAs is often an important step prior to mathematical modelling of the phenomenon under consideration … Use of LoAs is effective … where a typed theory would be effective … **[but] analysis …may be conducted at different levels of epistemological levelism.**

TYPES & LEVELS of ABSTRACTION

By the theory of simple types I mean the doctrine which says that the objects of thought ... are divided into types, namely: individuals, properties of individuals, relations between individuals, properties of such relations, etc. ... , and that sentences of the form: " a has the property φ ", " b bears the relation R to c ", etc. are meaningless, if a, b, c, R, φ are not of types fitting together. Mixed types (such as classes containing individuals and classes as elements) and therefore also transfinite types (such as the class of all classes of finite types) are excluded. That the theory of simple types suffices for avoiding also the epistemological paradoxes is shown by a closer analysis of these.

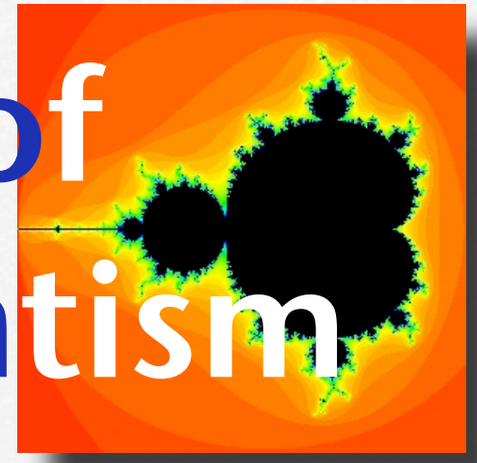
(Kurt Godel: *Russell's mathematical logic*, 1944)

Emergence and Surprise?

Emergence is often invoked in an almost mystical sense regarding the capabilities of behavior-based systems. Emergent behavior implies a holistic capability where the sum is considerably greater than its parts. It is true that what occurs in a behavior-based system is often a surprise to the system's designer, but does the surprise come because of a shortcoming of the analysis of the constituent behavioral building blocks and their coordination, or because of something else?

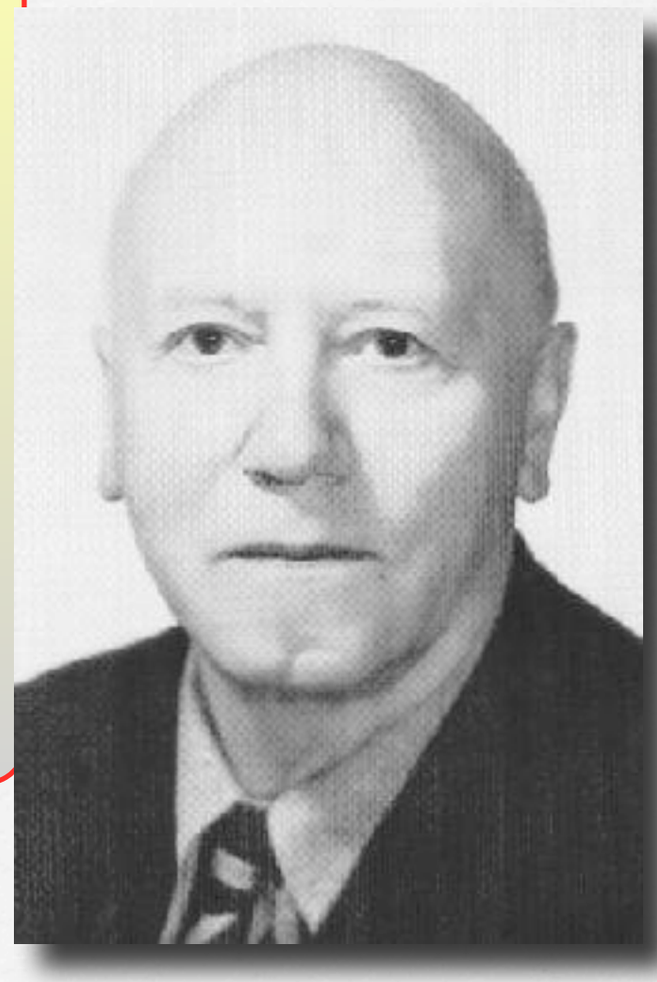
Ronald C. Arkin in "Behaviour-Based Robotics", MIT Press, 1998, p.105

The Rise & Fall of British Emergentism



" ... the mental properties of those events which do have mental properties are completely determined by the material properties which these events also have ... it is certainly not ... a form of Reductive Materialism; it is a form of the theory ... of Emergent Materialism."

- C.D. Broad, The Mind and Its Place In Nature, Kegan-Paul, London, 1923, p.623



C. D. Broad 1887-1971

- See Brian P. McLaughlin: "The Rise and Fall of British Emergentism", in "Emergence or Reduction? - Essays on the Prospects of Nonreductive Physicalism" (A. Beckermann, H. Flohr, J. Kim, eds.), de Gruyter, Berlin, 1992, pp.49-93

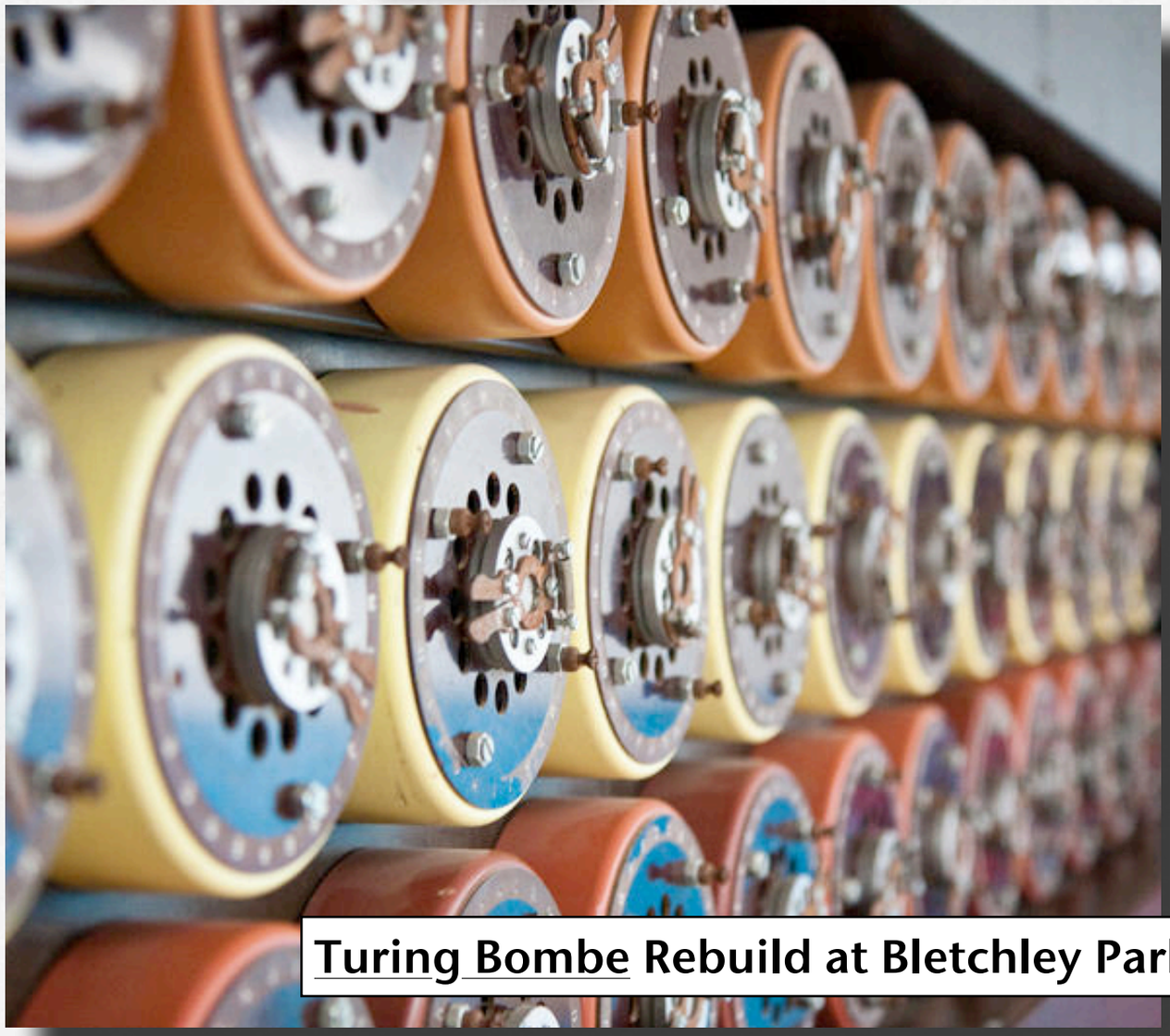
Higher Type

INCOMPUTABLE

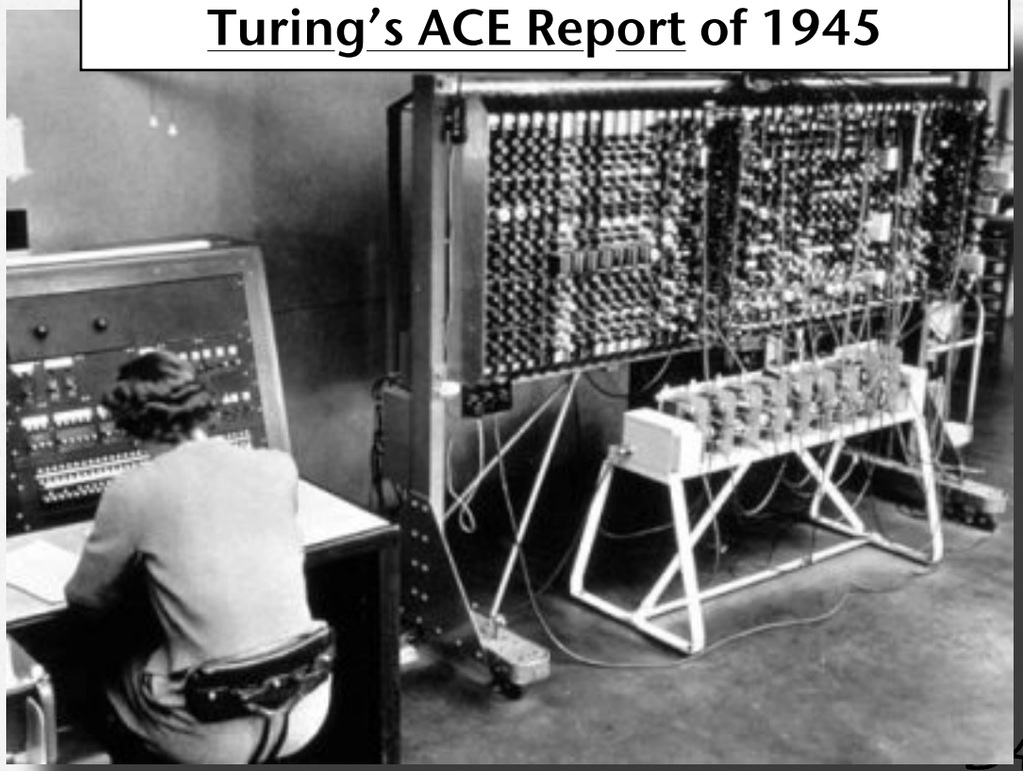
INFORMATION and Emergence

Programs as Data Embodied

Pilot ACE, May 10, 1950 - small version of plan contained in Turing's ACE Report of 1945



Turing Bombe Rebuild at Bletchley Park



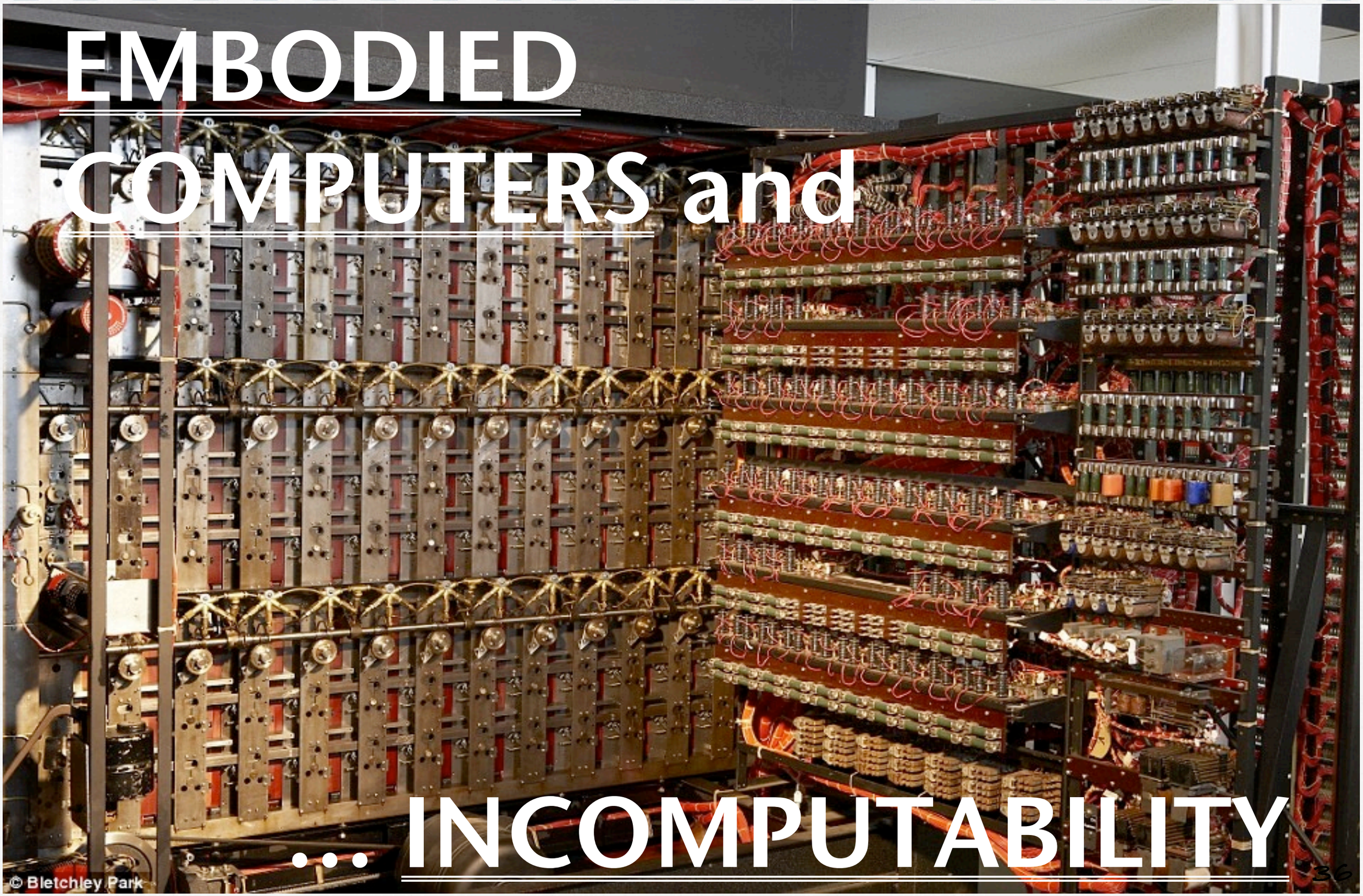
The Discovery of 'Incomputability'

- Turing (1936): Take a Universal Turing machine **U**:

Unsolvability of the Halting Problem for U:
No computer can tell us, for each given input x, whether U will compute - where, remember - we allow an input to include a coded program!

'Church's Theorem':
No computer can tell us, for each given sentence, whether it is logically valid or not.

EMBODIED COMPUTERS and ... INCOMPUTABILITY



© Bletchley Park

Incomputability and randomness

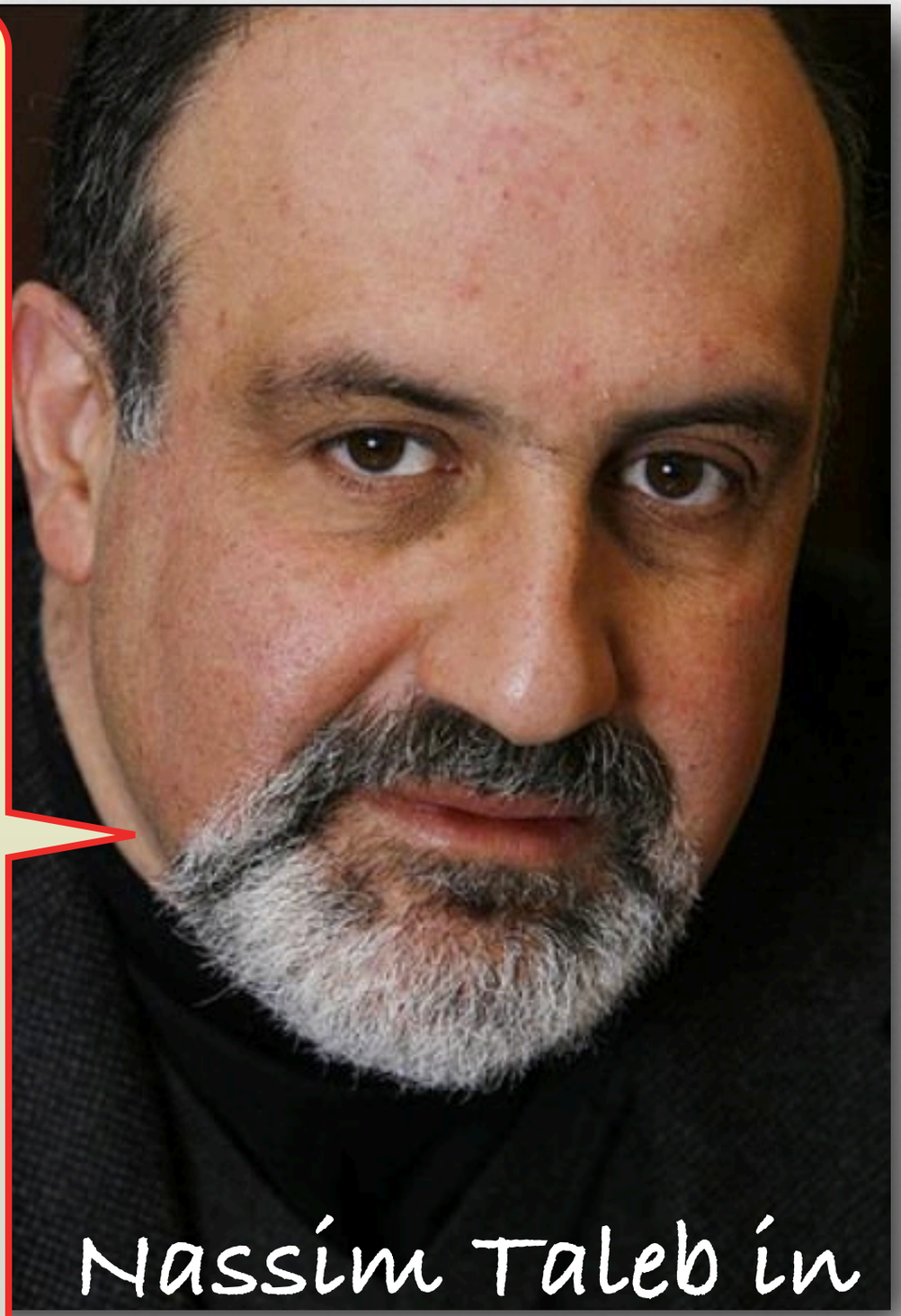
- Quantum randomness is a familiar experimental and theoretical phenomenon

It passes all reasonable statistical properties of randomness

- Cris Calude/Karl Svozil: It is Turing incomputable

Open question: How random is quantum randomness ? Maybe not random

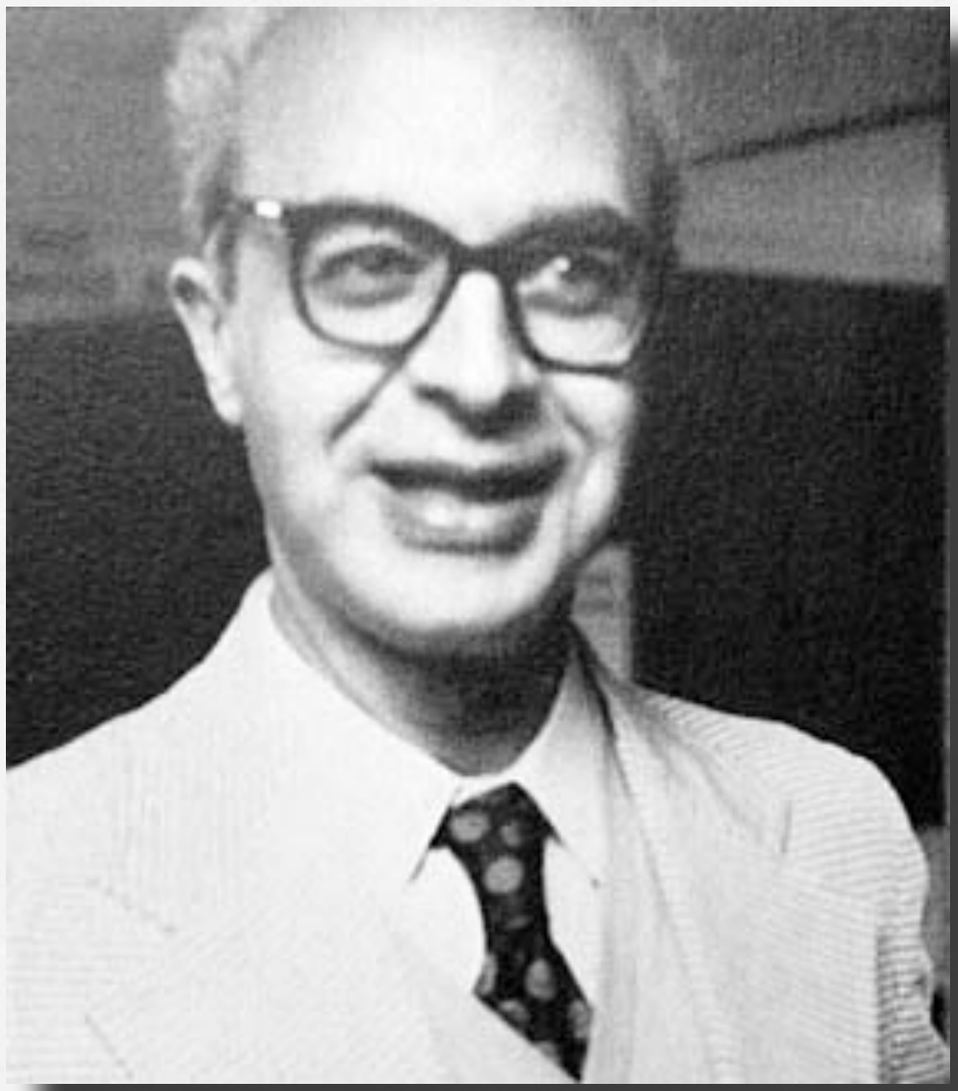
I have spent my entire life studying randomness, practicing randomness, hating randomness. The more that time passes, the worse things seem to me, the more scared I get, the more disgusted I am with Mother Nature ..... The more I think about my subject, the more I see evidence that the world we have in our minds is different from the one playing outside.



Nassim Taleb in "The Black Swan"

Co-operative phenomena

1970 - Georg Kreisel proposes a collision problem related to the 3-body problem, which might result in "an analog computation of a non-recursive function"



Simple rules …
… Complex Interactivity …
…. Emergent Forms ….
… and Definability

Definability in the Real World

Hans Reichenbach
(1891-1953)

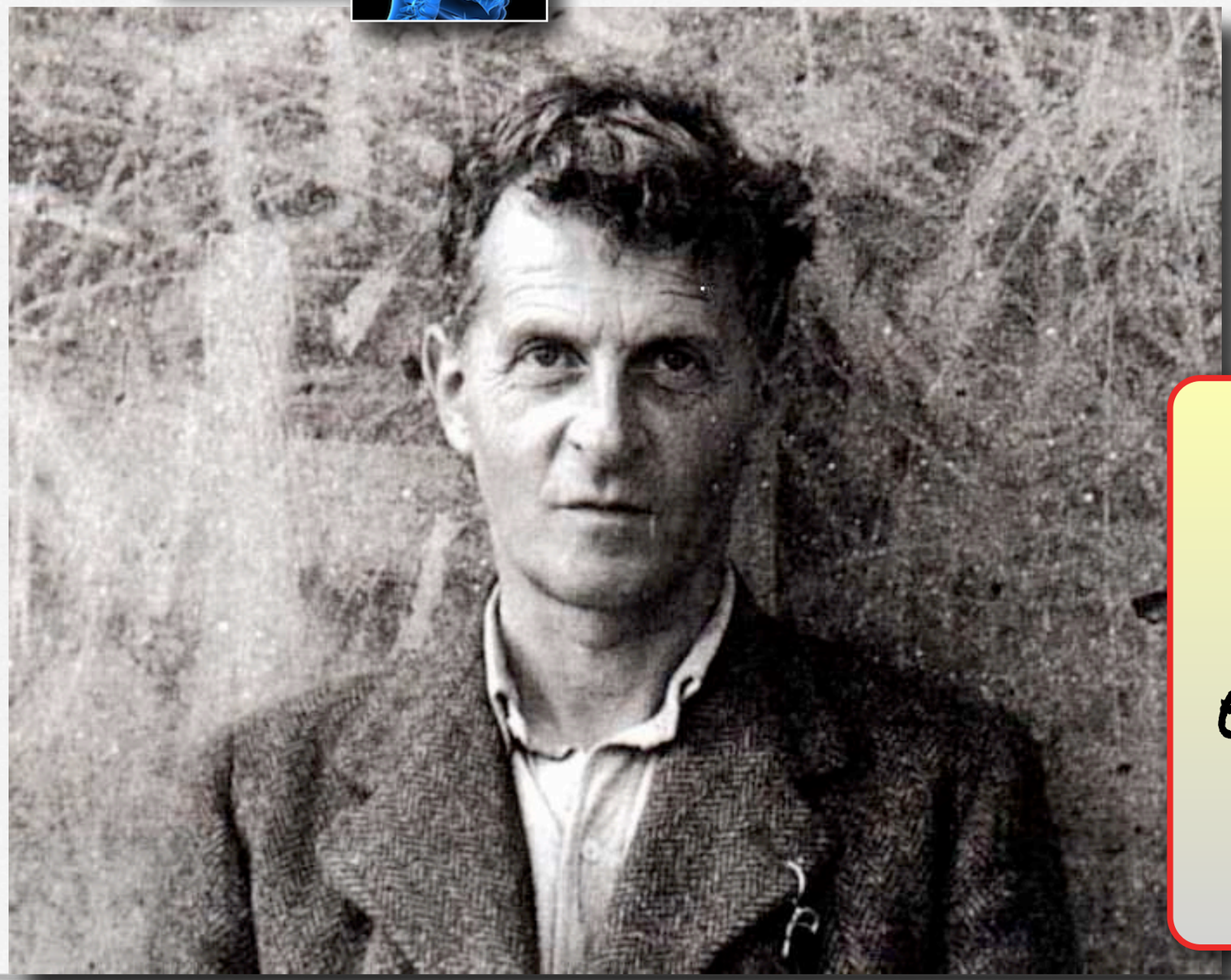
- Intuition: Natural phenomena not only generate descriptions, but arise and derive form from them . . .

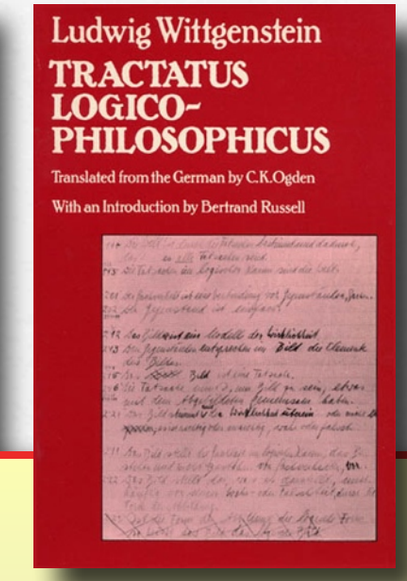
- . . . so – connecting with a useful abstraction – that of mathematical definability – or, more generally, invariance (under appropriate automorphisms) . . .

Alfred Tarski

- . . . giving precision to our experience of emergence as a computationally non-algorithmic determinant of phenomena

Definability in the Real World

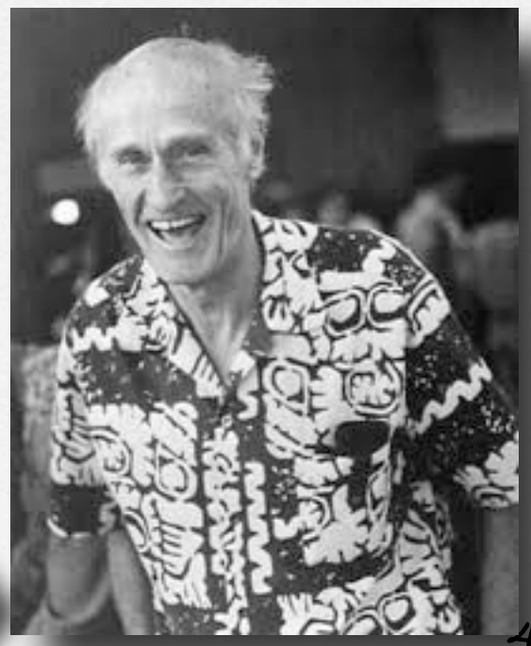




"The world is everything that is the case"

Definability as Higher Order Computation Embodied ...

- Aim to describe global relations in terms of local structure ...
- ... so capturing the emergence of large-scale formations ...
- Mathematically formalised as definability
- Or ... as computability over higher type information



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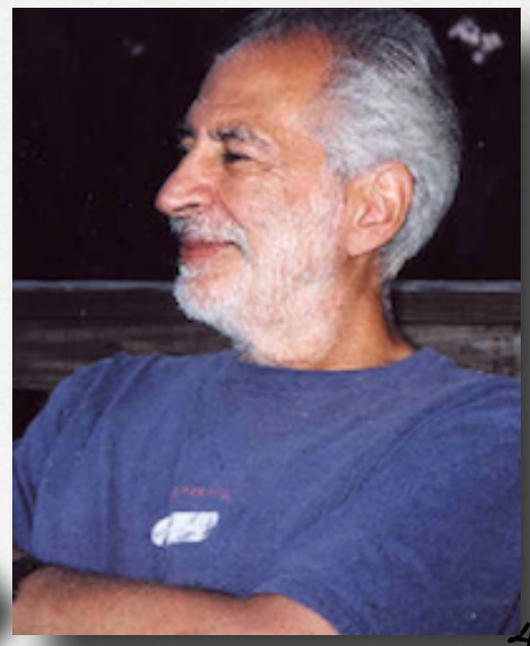
- Mathematically formalised as definability
- Or ... as computability over higher type information



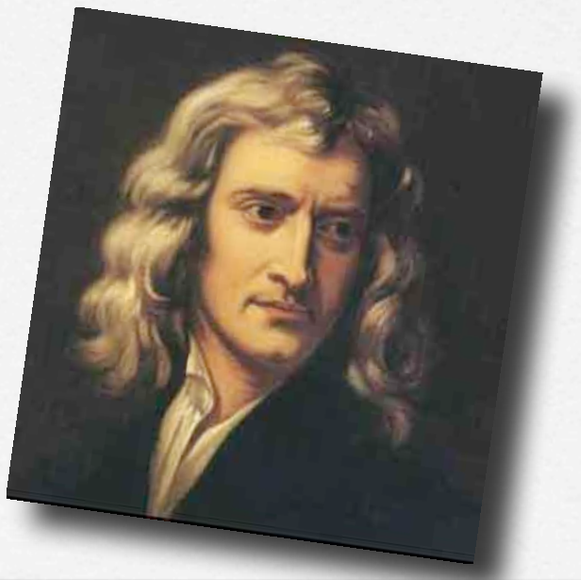
Computational Host for Information

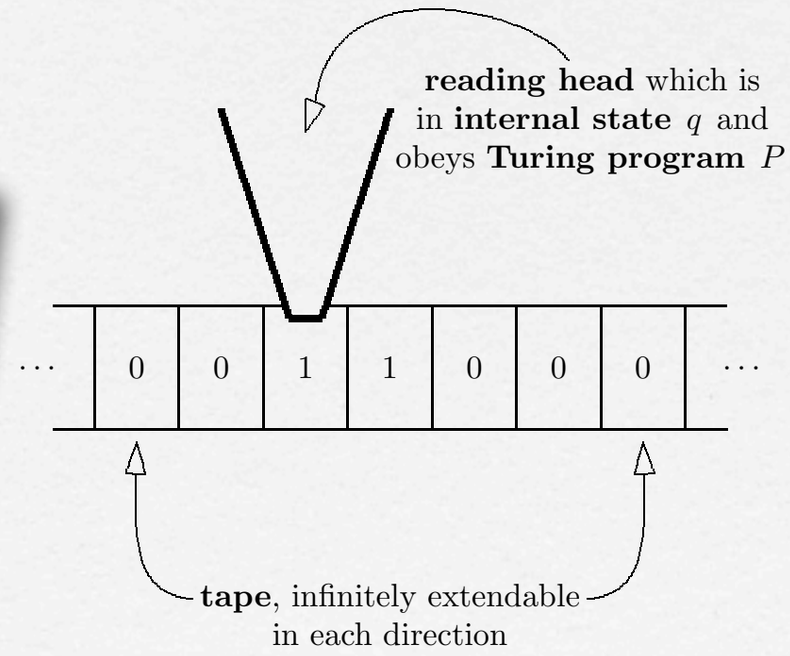
Computation also about Information

- Turing, 1939 - Oracle Turing Machines ...

- Provide a model of how we compute using data given to us from unknown sources

- A model within which Newtonian computability etc comfortably fit ...





The Mathematics of Relative Computability

- A model of computable content of structures, based on p.c. functionals over the reals

- 1944 - Post defines the degrees of unsolvability as a classification of reals in terms of their relative computability

Phyllis, Emil and Gertrude Post

- Giving a landscape with a rich structure

Hartley Rogers' Programme

Fundamental problem: Characterise the Turing definable/invariant relations

- Intuition: These are key to pinning down how higher order relations on the real world can appear to be computed

- So: The richness of Turing structure discovered so far becomes the raw material for a multitude of definable - and hence visibly computable - relations

Bi-interpretability

Bi-interpretability Conjecture(Harrington): The Turing definable relations are exactly those with information content describable in second-order arithmetic



- Where: Conjecture rules out there being non-trivial Turing automorphisms …
- While: Partial results underpin observed certainties …

Physical Computation, Turing Landscape and Emergence ...

- Description of global relations in terms of local structure ...

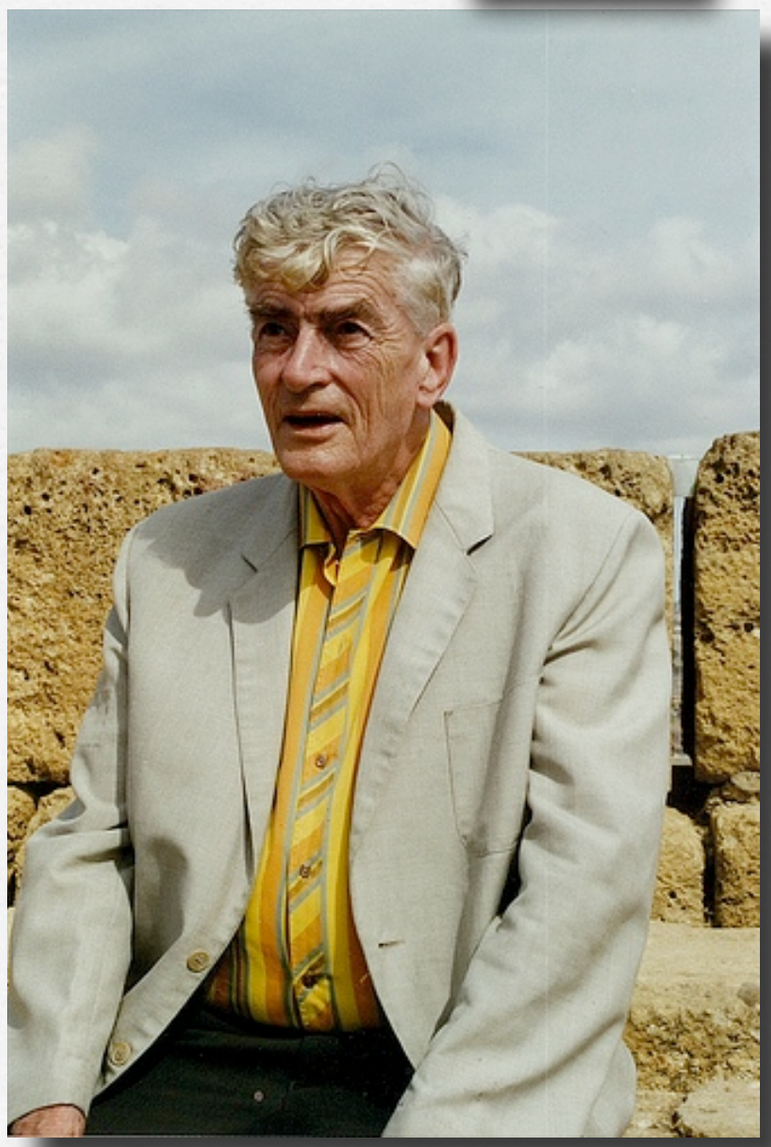
- ... capturing the computation of large-scale formations

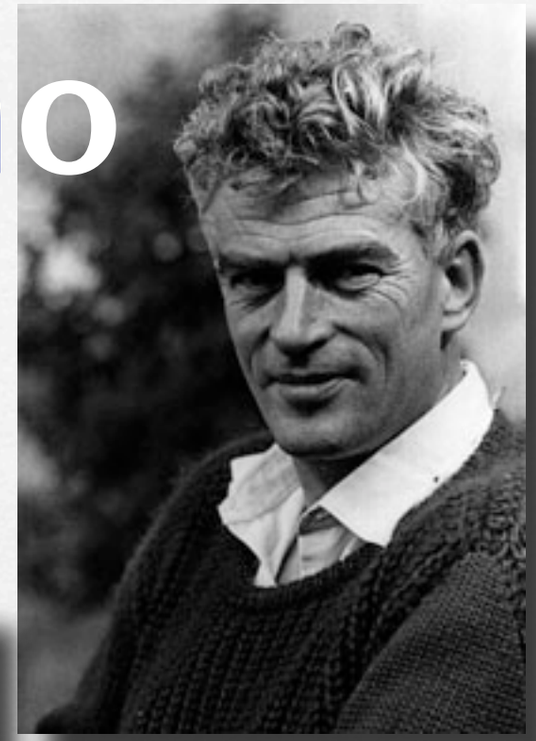
Mathematically - formalised as definability over structure based on oracle computations

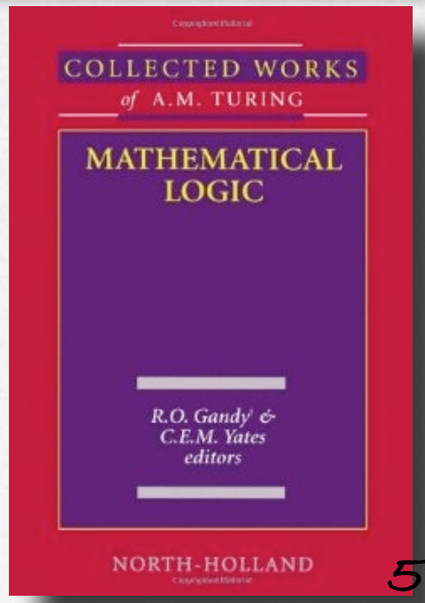
Or - as invariance / higher type computation

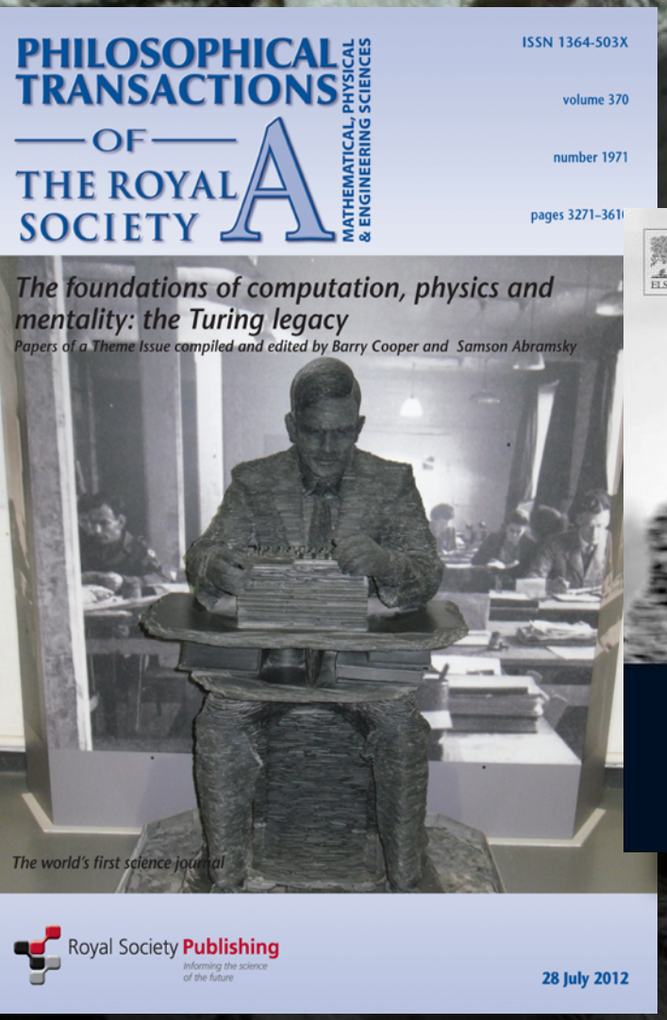
“Some things happen for no reason …”

- Robin Gandy (1919-95)





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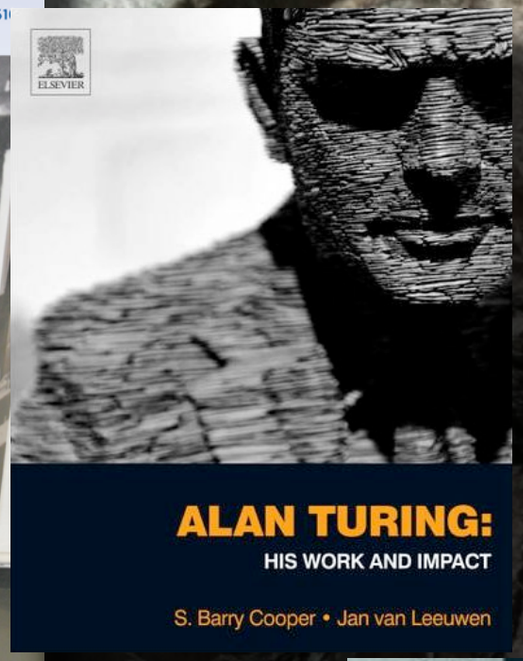
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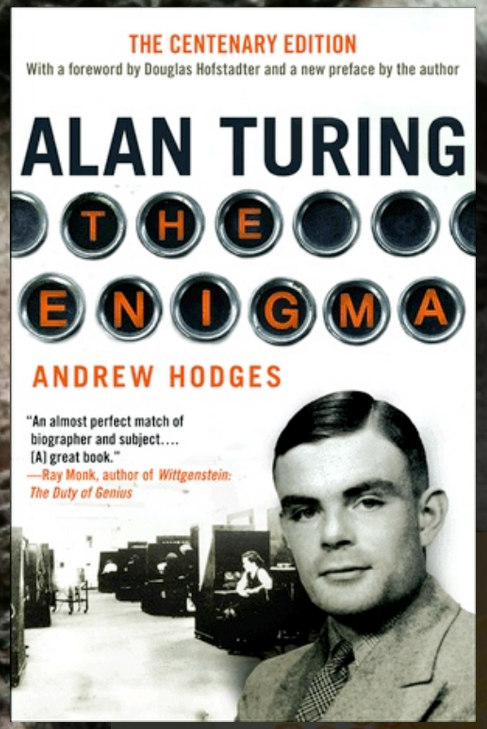
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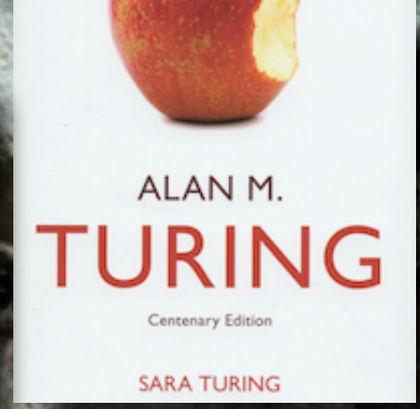
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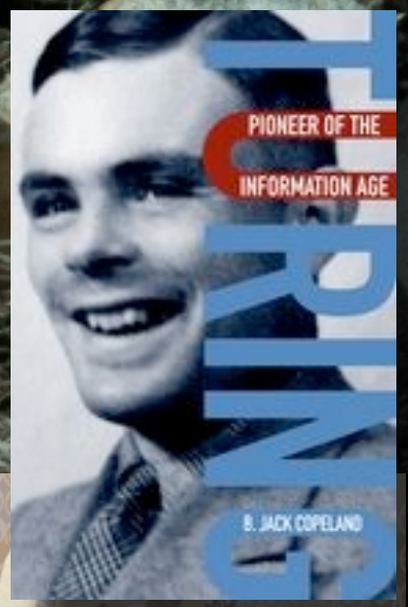
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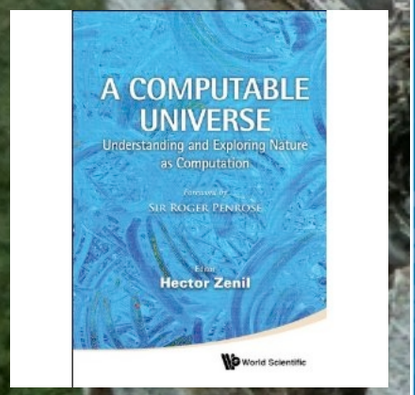
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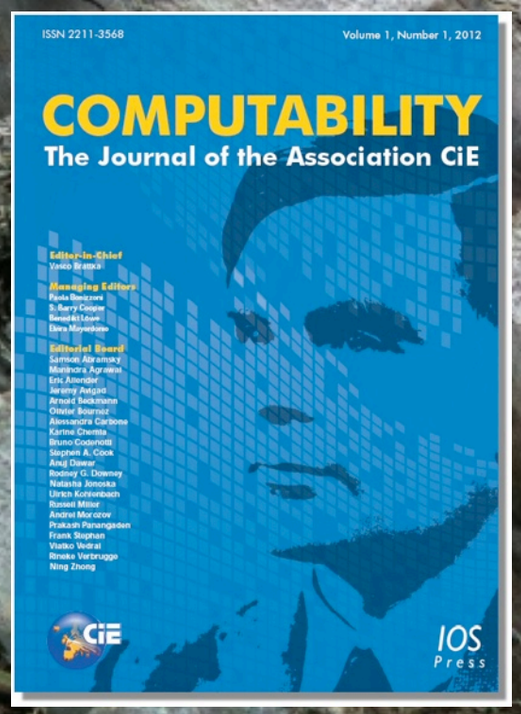
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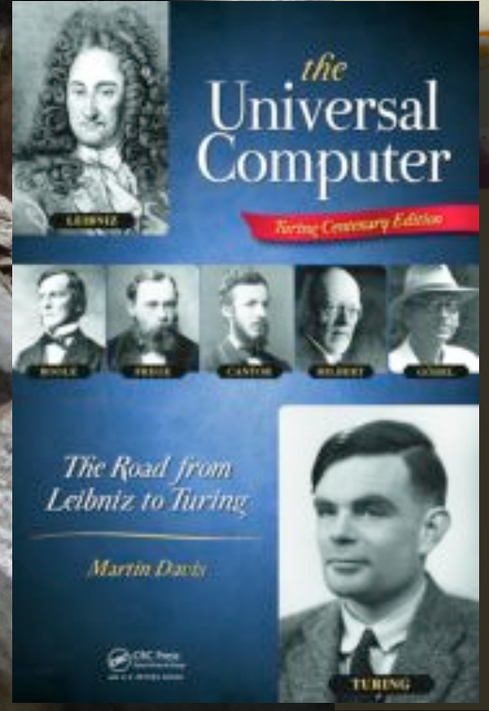
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