Did Malament Prove the Non-Conventionality of Simultaneity in the Special Theory of Relativity?*

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David Malament's (1977) well-known result, which is often taken to show the uniqueness of the Poincaré-Einstein convention for defining simultaneity, involves an unwarranted physical assumption: that any simultaneity relation must remain invariant under temporal reflections. Once that assumption is removed, his other criteria for defining simultaneity are also satisfied by membership in the same backward (forward) null cone of the family of such cones with vertices on an inertial path. What is then unique about the Poincaré-Einstein convention is that it is independent of the choice of inertial path in a given inertial frame, confirming a remark in Einstein 1905. Similarly, what is unique about the backward (forward) null cone definition is that it is independent of the state of motion of an observer at a point on the inertial path.

1. Introduction. Reichenbach (e.g., 1957) and Grünbaum (e.g., 1963, 1973), following the lead of Einstein (e.g., 1905), maintained that, in the special theory of relativity, distant simultaneity is conventional in the sense that it can only be established by some stipulation and that

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no matter of physical fact can depend on that stipulation.[1] Let a light signal from a point *a* in some inertial frame be sent at time t_1 (as measured by a clock at rest at *a*) to a point *b* in the same frame and reflected back to *a*, reaching it at t_2. Einstein defined the event at *a* that is simultaneous with the time of arrival of the light signal at *b* as the one with $(t_1 + t_2)/2$ as its temporal co-ordinate. Because Poincaré (1900) had suggested it earlier, we shall call this convention for defining simultaneity the Poincaré-Einstein convention. In the literature, the resulting simultaneity relation is often referred to as "standard simultaneity."

Standard simultaneity has many advantages. In particular, it is independent of the choice of the point *a* in a given inertial frame. Nevertheless, it is not unique: there are other definitions of simultaneity that are consistent with all physical laws. In this sense, any definition of simultaneity is based on a convention. To illustrate the stipulative character of the simultaneity relation, Einstein (1905) noted the possibility of using the family of backward null cones of an inertial world line to define simultaneity. As he put it: "We could content ourselves with evaluating the time of events by stationing an observer with a clock at the origin of co-ordinates, who assigns to an event to be evaluated the corresponding position of the hands of the clock when a light signal from that event reaches him" (893). He rejected this possibility in favor of the Poincaré-Einstein convention because the former "has the drawback that it is not independent of the position of the observer with the clock" (893).

The contention that simultaneity in the special theory of relativity is a conventional relation later aroused considerable controversy (see, e.g., Bridgman 1962, Ellis and Bowman 1967, Grünbaum 1969, Janis 1969). In 1977 Malament proved a theorem that was interpreted by some prominent philosophers of science as establishing the nonconventionality of standard simultaneity. More precisely, the theorem is supposed to show that the Poincaré-Einstein convention for defining simultaneity relative to a straight time-like (i.e., inertial) world line *O* in Minkowski spacetime (which can serve to define an inertial frame of reference) "is the only . . . relative simultaneity relation which is definable from κ [the relation of causal connectibility between points of spacetime which, following Malament, we refer to as "events"] and *O*" (Malament 1977, 297). This result is said to follow from the geometric properties of Minkowski spacetime when some "minimal, seemingly innocuous conditions are imposed" (297).

1. For a historical review of both the philosophical and the physical literature on the conventionality of simultaneity, see Anderson, Vetharaniam, and Stedman 1998.

Malament's result has been interpreted as having resolved the philosophical debate about the conventionality of simultaneity. Norton (1992, 194) asserts that the result brought about "one of the most dramatic reversals in debates in the philosophy of space and time." Torretti (1983, 229) states: "Malament proved that simultaneity by standard synchronism in an inertial frame *F* is the *only* non-universal equivalence between different points of *F* that is definable ('in any sense of "definable" no matter how weak') in terms of causal connectability alone, for a given *F*." This argument is one of the reasons why Torretti rejects the conventionality of simultaneity. Friedman (1973, 310) says: "there is a fundamental fact about the standard . . . simultaneity-relation-for-*s* [an inertial trajectory] that both Reichenbach and Grünbaum overlook: namely, in Minkowski space-time the standard relation is explicitly definable from the space-time metric *g* (in fact, from the conformal structure of *g*), whereas the nonstandard . . . relations are not so definable." According to Friedman (1973, 319), Malament has shown that the "standard simultaneity relation is the one and only would-be simultaneity relation that is 'causally' definable in Minkowski space-time"; hence "it cannot be varied without completely abandoning the basic structure of the theory [of special relativity]" (320).

Even a decidedly less enthusiastic commentator, Redhead (1983, 114), observes: "Malament (1977) has proven the remarkable result that standard synchrony is the only nontrivial equivalence relation even implicitly definable from the relation of causal connectibility and the world line of the origin of an inertial frame of reference." To weaken the impact of this result, Redhead proposes either to abandon the requirement that simultaneity be an equivalence relation or to argue that a convention is still involved in a choice between standard simultaneities defined in all possible inertial frames (that is, the relativity of standard simultaneity). The last alternative has also been pointed out by Janis (1983).

In sharp contrast to these philosophers, the response from physicists, when they have had occasion to discuss the result, has been generally skeptical (Janis 1983; Anderson, Vetharaniam, and Stedman 1998). Havas (1987, 444) suggests that: "What Malament has shown, in fact, is that in Minkowski space-time . . . one can always introduce time-orthogonal coordinates . . . , an obvious and well-known result which implies [standard simultaneity]."

Surprisingly, Einstein's alternative convention (discussed above), which we shall call the backward null cone convention, though obviously invoking only an inertial world line and causal relations, has not received any attention in the discussion of Malament's result.[2] We shall

2. Similarly we will also refer to the "forward" null cone convention. We will use "backward (forward)" to refer to one or the other of them.

show just which of Malament's assumptions rules out this alternative and then argue against the physical plausibility of this assumption. In Section 2 we introduce Malament's framework and outline his proof. In Section 3 we analyze this framework and show that it does not foreclose the adoption of some non-standard simultaneity definitions.[3] Our demonstration of this result involves a construction that distinguishes between forward and backward null cones using only causal relations. The assumptions made in Malament's proof include a crucial one that is physically unwarranted: any simultaneity relation must be invariant under temporal reflections. Dropping that assumption, we prove an alternative result that captures what Einstein saw as truly unique about the Poincaré-Einstein convention (our Theorem 1). We then show that the adoption of an alternative "seemingly innocuous" condition leads to a proof of the uniqueness of the backward (forward) null cone simultaneity convention (Theorem 2). Thus the alternatives, as we see it, are a convention that is: either (i) independent of the position of an observer within an inertial reference frame, leading to the uniqueness of the Poincaré-Einstein convention; or (ii) independent of the state of motion of an observer at each point of an inertial world line, leading to the backward (forward) null cone convention. In Section 4 we discuss our results and compare our conclusions with those of some of the other commentators mentioned above.

2. Malament's Result. According to Malament, a version of the causal theory of time asserts that temporal relations are non-conventional if and only if they are uniquely definable in terms of the relation of causal connectibility, κ (293). While he is generally skeptical of the causal theory of time, his analysis only explicitly addresses the question whether standard simultaneity is the sole simultaneity relation definable from κ.[4] The relation κ is interpreted as follows: two points, p and q are causally connectible ($p\kappa q$ is true) if "it is possible for a photon or particle with non-zero rest mass to travel between them" (294), that is, if a signal can pass from p to q or q to p. Malament observes that the three causal relations, "timelike relatedness (τ)," "lightlike relatedness (λ)," and κ are explicitly (first-order) definable in terms of one another.

3. When we speak of the "non-conventionality of simultaneity" in this paper, we use it only as shorthand for the "uniqueness of a simultaneity convention." (Of course, any convention can be made unique by imposing a sufficient number of auxiliary conditions. We exploit this fact in our proof of Theorem 2.) In order to keep our discussion focused, we also do not distinguish between convention, definition, and stipulation.

4. We do not explicitly take any position on the causal theory of time in this paper. We are concerned with Malament's precise result and, like him, only directly address the question of uniqueness.

Therefore, one may work with any of them in proving the relevant results.

Malament first motivates and partly proves the straightforward and well-known result that, in Minkowski spacetime, the Poincaré-Einstein convention is equivalent to a choice of the family of hyperplanes orthogonal to the inertial world line O as hypersurfaces of simultaneity (his Proposition 1; 295–297). In other words, the Poincaré-Einstein stipulation amounts to a choice of the foliation of spacetime by a family of spacelike hypersurfaces orthogonal to O.[5]

Malament requires that any simultaneity relation be implicitly definable from κ and O. He then imposes three other conditions on any such putative simultaneity relation: (i) that it be an equivalence relation; (ii) that at least one event on O be simultaneous with at least one event not on O; and (iii) that not all events in spacetime be simultaneous with each other.[6] If the second or third requirement is not met, a simultaneity relation is "vacuous" (297). While, as noted above, requirement (i) has been questioned, we accept it here (though the very fact that it can be questioned suggests that a conventional element enters into definitions of simultaneity). There is something attractive about this set of conditions: they effectively ensure that there exists a family of hypersurfaces of simultaneity constituting a foliation of spacetime.

Malament then explicates the notion of "causally definable" in terms of "causal automorphisms" (297). A bijective map $f : \mathbf{R}^4 \to \mathbf{R}^4$ (with the Minkowski inner product) is a "causal automorphism" if and only if, for all points p and q in $\mathbf{R}^4$, $p\kappa q \leftrightarrow f(p)\kappa f(q)$ (p. 297); f is an "O causal automorphism" if it also satisfies the requirement that if $p \in O$, $f(p) \in O$ (297). As Malament notes, the class of O causal automorphisms includes "all rotations, translations, . . . scalar expansions [and] . . . all reflections of $\mathbf{R}^4$ with respect to hypersurfaces orthogonal to O which map O onto itself" (298). The inclusion of such temporal reflections, which we shall call "O-temporal reflections," will be discussed critically in the next section. Finally, a relation is "causally definable from κ and O" if and only if it is preserved under *all* O causal automorphisms (297).

In this framework, Malament's proof of the uniqueness of standard simultaneity (which he denotes as "Sim_O") is straightforward (his Prop-

5. Malament notes that this result is at least implicit in Robb 1914.

6. Malament (297–298) lists the requirement of implicit definability as his condition (i). His condition (ii) corresponds to our (i). His condition (iii) corresponds to our (ii). He then proves that a simultaneity relation that satisfies his three conditions is either standard simultaneity or the universal relation. He later implicitly rejects the latter in order to argue for the uniqueness of standard simultaneity. We impose non-universality at this stage as requirement (iii) of non-triviality.

osition 2; 297–299). If a putative simultaneity relation is Sim_O, the simultaneity hypersurfaces consist of the hyperplanes orthogonal to O. If it is not, such simultaneity hypersurfaces form (full) cones ("double cones" in Malament's terminology) with a vertex at some point on O.[7] Given such a family of (full) conical simultaneity hypersurfaces, it is relatively trivial to show that all events in spacetime are simultaneous with each other.

3. Critical Analysis and Alternative Results. As we noted in Section 1, the backward (forward) null cone convention for defining simultaneity is based on the possibility of using the family of backward (forward) null cones with vertices on an inertial world line, O. The full null cone is trivially causally definable: if λ is the relation of null relatedness, then $\Lambda(e)$, the full null cone at an event e, is defined as the set of all events p such that $p\lambda e$. If the distinction between the backward and forward half null cones of $\Lambda(e)$ can be made using causal definability alone, then we have a seeming contradiction with Malament's result.

We now show that λ can be used to distinguish each of the two separate (backward [$\Lambda_-(e)$] or forward [$\Lambda_+(e)$]) half cones of $\Lambda(e)$. (Note that, at this point, one of them is called "backward [$\Lambda_-(e)$]" and the other "forward [$\Lambda_+(e)$]" merely to distinguish them.[8]) The (spacelike) "elsewhere" $E(e)$ of e is defined as the set of all events p such that $\neg p\kappa e$. (Remember κ is definable in terms of λ.) Two points of $\Lambda(e)$ are on the same half cone if and only if they are not causally connectible or there exists a signal connecting them which does not pass through $E(e)$ or e. It follows that there are two such half null cones, with only e in common, that is, $\Lambda_-(e) \cup \Lambda_+(e) = \Lambda(e)$ and $\Lambda_-(e) \cap \Lambda_+(e) = \{e\}$. Note that this distinction, once made on the null cone at any point of spacetime, can be consistently carried by parallel transport to the null cone at any other point of spacetime.

Thus λ can be used to define two additional simultaneity relations, $Back_{O,e}$ and $Forw_{O,e}$, relative to a time-like world line O: two events are simultaneous relative to an event e on O if and only if they lie on $\Lambda_-(e)$ [$\Lambda_+(e)$]. Clearly neither relation is "vacuous" in Malament's (297) sense: e is simultaneous to events not on O and not all events are simultaneous. Each is also trivially an equivalence relation. Most im-

7. Note that these are not necessarily null cones.

8. This distinction is preserved under all causal automorphisms. The ability to make this distinction does not imply a time orientation but only a time orientability of spacetime. Choosing one of them in a definition of simultaneity (see below) does introduce a time orientation. But even this is not a choice of a direction of time: the "arrow of time" could just as well point to the past as to the future.

portantly, each of these relations can be defined from λ alone, since $A_{-}(e)$ $[A_{+}(e)]$ can be so defined. (Note, that since they are definable from λ, they are also definable from κ).

Clearly something is amiss with Malament's theorem. A correct mathematical result seems to be contradicted by patently good counterexamples. The trouble lies in the interpretation of one of the conditions that Malament imposes on simultaneity relations. Malament requires that a relation be "causally definable from κ and O" if and only if it is preserved under *all* O causal automorphisms. As noted in Section 2, these include O-temporal reflections which preclude our counterexamples. Our result shows that $Forw_{O,e}$ and $Back_{O,e}$ are causally definable from κ and O. This suggests that we exclude O-temporal reflections from the group of admissible O causal automorphisms. Is this physically reasonable?

It is perfectly reasonable to demand that any relation between events be preserved under symmetry transformations of Minkowski spacetime that can be physically implemented as active transformations, that is, as transformations that take the related events from one spatio-temporal configuration to another. The group of proper, orthochronous Poincaré transformations (consisting of spatial rotations, boosts, and spatial and temporal translations with respect to a particular inertial reference frame) can clearly be implemented actively. Scale transformations and temporal and spatial reflections, which are also included in Malament's group of O causal automorphisms, require separate analysis.

All proper orthochronous Poincaré transformations are continuously connected with the identity. Scale transformations are also continuously connected with the identity but they do not preserve all invariants of the Poincaré group, in particular, the length of any non-null vector. Therefore, they are not physically implementable as active transformations on any relation depending on non-null separation between events.[9] Therefore, we shall not use invariance under scale transformations in the proofs of Theorems 1 and 2 below.

Temporal and spatial reflections are discontinuous transformations, that is, they are not continuously connected with the identity. The identification of two physical configurations, one of which is generated from the other by a transformation continuously connected to the identity, is unproblematic. However, what is meant by "the identical physical configuration" after a discontinuous transformation *is* problematic, and must be suitably explicated before invariance can even be inves-

9. This means they are not physically implementable as active transformations in any theory that contains massive particles or fields.

tigated. Whether any explication is possible that extends invariance to discontinuous transformations depends on the dynamical theory being considered. An extensive discussion, starting in the late 1950s with the discovery of parity violation in weak interactions, indicated the limited applicability of (active) spatial- and then, later, time-reversal invariance to quantum field theories of the weak interactions.[10] If we confine ourselves to local special-relativistic quantum field theories, the PCT theorem implies that the time-reversed mirror image of a system of particles with given velocities is a system of anti-particles with reversed velocities. Thus the event corresponding to an *electron* at a point in an inertial reference frame will be a *positron* at the spatially reflected point with equal and opposite velocity. Since reflections are not physically implementable as active transformations, it is not physically reasonable to demand that all relations between events be universally preserved under them.

Hence, there is no physical warrant for requiring that Minkowski spacetime be non-oriented and non-temporally oriented. As is well known (see, e.g., Hermann 1966) the spacetime on which a group of symmetry transformations acts can be constructed from the group. Given the full Poincaré group, Minkowski spacetime may be identified with the quotient of the Poincaré group by its Lorentz subgroup. But, restriction to the proper, orthochronous Poincaré group (and its Lorentz subgroup), as physically warranted, yields oriented, temporally oriented Minkowski spacetime.

We are thus led to broaden the class of relations causally definable from κ and O in the following way (and designate the new relation as causally′ definable):

Definition: A relation is causally′ definable from κ and O if and only if it is preserved under the group of all O causal automorphisms continuously connected to the identity.

The O causal automorphisms continuously connected to identity, which we will call O causal′ automorphisms, include the rotations and translations; they exclude all reflections.[11] The following trivial Lemma will be useful later:

Lemma: Standard simultaneity is causally′ definable from κ and O.

10. For a discussion of the problem of time reversal invariance, see Sachs 1987.

11. Malament's proof uses scale transformations, which are included in the group of causal automorphisms in addition to the Poincaré transformations. As noted in the text, the proofs of our theorems are independent of a requirement of invariance under scale transformations. We retain them in the definition of causally′ definable to keep our discussion as close as possible to that of Malament.

The validity of this lemma is a consequence of Malament's Proposition 1.[12] Since our definition of "causally′ definable from κ and O" only requires invariance under a sub-group of Malament's group of transformations, all relations, including standard simultaneity, that are definable in his framework are *ipso facto* definable in ours.

However, what does *not* follow from the new relation is the uniqueness of the Poincaré-Einstein convention. Malament's proof fails because of its dependence on time reflection-invariance, and the way in which it fails shows—in yet another way—why the backward (forward) null cone definition of simultaneity ($Back_{O,e}$ [$Forw_{O,e}$]) is a relation causally′ definable from κ and O. Malament shows (as part of his proof of Proposition 2) that, besides the Sim_O hypersurfaces induced by the Poincaré-Einstein convention, the other potential simultaneity hypersurfaces (by his definition of causal definability from κ and O) are full (not necessarily null) cones (298). He rejects such full cones—as do we the full *null* cone—because they lead to the unpalatable conclusion that all points in Minkowski spacetime are simultaneous with each other. The reason Malament generates only full cones, rather than also half cones, is that he requires a simultaneity relation to be invariant under O-temporal reflections. Once the demand of O-temporal reflection invariance is dropped, one half cone cannot be transformed into the other, and either half null cone meets all our criteria.[13]

Nevertheless, even after dropping invariance under O-temporal reflections, there is something unique about standard simultaneity, as Stein (1991, 153n) has noted. Drawing once again on the insight of Einstein discussed in Section 1, we characterize it in Theorem 1.

Let $O(e)$ be an inertial world line through the event e and $O(e')$ be the parallel inertial world line through an event e'. Let $Sim_{O(e)}$ [$Sim_{O(e')}$] be the standard simultaneity defined for $O(e)$ [$O(e')$].

Theorem 1: Standard simultaneity is the only non-vacuous simultaneity relation causally′ definable from κ and O that depends only on an inertial frame, and not on the particular world line O initially chosen to define that inertial frame. In other words, $Sim_{O(e)} = Sim_{O(e')}$ for all e and e'.

Proof Sketch: The inertial frame is characterized by the fibration of

12. The important part of this proposition, as Malament notes, goes back to Robb 1914.

13. That the introduction of a temporal orientation of spacetime blocks Malament's proof was first noted by Spirtes (1981, 171–186). However, he interprets this result as showing only that non-standard simultaneity relations are "conventional" in his sense. Stein (1991, 153n) has also noted this weakness of Malament's proof.

spacetime consisting of all inertial world lines parallel to O. So we must show that (i) Standard simultaneity is independent of which of these lines we start from; and (ii) that it is the only simultaneity relation causally′ definable from κ and O that has this property.

(i) is trivial. Consider the hypersurface defined by all events standardly simultaneous with an event p on O. As Malament shows (296–297), this hypersurface is orthogonal to every fiber in the fibration consisting of all straight lines parallel to O. Since standard simultaneity is an equivalence relation, it does not matter which line we start from: the same simultaneity relation will hold for all events in spacetime;

(ii) is only slightly less trivial. According to our definition, any simultaneity relation causally′ definable from κ and O must be invariant under any transformation belonging to the group of O causal′ automorphisms. This implies that it must take the family of hypersurfaces of simultaneity onto itself under any such automorphism. Now, these O causal′ automorphisms consist of rotations about O, translations along O, and scale transformations. If the simultaneity relation is also to be independent of the particular world line in the fibration from which we start, then the family of hypersurfaces of simultaneity must also be invariant under translations that rigidly displace the fibers into each other. Any such translation can be decomposed into a translation along the fibers and a translation orthogonal to the fibers. Translations along the fibers are already included in our O causal′ automorphisms. So, the only additional translations that we need to consider are those orthogonal to the fibers. If they are not to affect the simultaneity relation (which amounts to our assumption that the simultaneity relation is independent of the initially-chosen world line O), these translations must take each simultaneity hypersurface onto itself. So, take any point p on O, and consider the orbit of p under the action of the group of orthogonal translations (i.e., the set of points onto which p is translated by the members of this group). It is clearly the hyperplane orthogonal to O that includes p. Thus, by construction, we have shown that this hyperplane which, as we have seen, corresponds to the standard simultaneity relation, is the only such invariant hypersurface.

Now assume causal′ definability for simultaneity relations, but not including invariance under scale transformations, and conditions (i), (ii), and (iii) of Section 2. We have seen what is then unique about the Poincaré-Einstein convention is its independence of the choice of a point in an inertial frame or, we may say, of the choice of an observer at rest in an inertial frame. It is obviously not invariant under boosts from one inertial frame to another at any point.

What is unique about the backward (forward) null cone convention? It is the only simultaneity relation relative to O at an event e that is independent of the state of motion of an observer at a point e on O such that no event is simultaneous with another in its causal future (past). This observation is formalized in the following theorem:

Theorem 2: $Back_{O,e}$ ($Forw_{O,e}$) is the only simultaneity relation causally′ definable from κ and O satisfying the following conditions: (i) given an event e on an inertial world line O, the relation shall be independent of all boosts at e; and (ii) no event is simultaneous with one in its causal future (past). If $O'(e)$ represents another inertial world line through e, this result can be written as $Back_{O,e} = Back_{O',e}$ ($Forw_{O,e} = Forw_{O',e}$) for all $O(e)$ and $O'(e)$.

Proof Sketch: Let p be any event not on O that is simultaneous with e. Consider the vector ep. By condition (i) of the theorem, under boosts at e, the length of this vector must remain invariant. Thus, the locus of p under all such boosts is either the forward or backward null cone or a time-like hyperboloid within the null cone. Now, e does not belong to any such hyperboloid. Therefore, if such a hyperboloid were used to define the simultaneity relation, e would not be simultaneous with itself violating the reflexivity condition of an equivalence relation. Thus, only the two half null cones remain as potential hypersurfaces of simultaneity. Condition (ii) restricts us to one of the two.

Now, we can compare the Poincaré-Einstein and the backward (forward) null cone conventions. Pre-relativistic kinematics leads to two compatible expectations of the simultaneity relation: (i) it should be independent of the position of an observer; and (ii) it should be independent of the state of motion of an observer. In special relativistic kinematics, these two expectations are no longer compatible. Theorem 1 shows that the Poincaré-Einstein convention meets the first expectation; Theorem 2 shows that the backward (forward) null cone meets the second. In that sense, each convention is natural. Within special relativity there is a trade-off between the two conventions. However, the backward (forward) null cone convention has the advantage that it is generalizable to arbitrary time-like world lines in Minkowski spacetime as well as to many spacetimes with curvature.

4. Final Remarks. Returning to the assessments of Malament's result mentioned in Section 1, we generally agree with Havas's assessment, though not with the details of his criticism. There is more to the result than he indicates. Beyond showing that time-orthogonal coordinates can always be defined, Malament has shown that they can be *causally* defined. Of course, as Malament notes, this result was implicit in Robb 1914, and Torretti (1983, 229) points out that it was explicitly proven by Mehlberg (1935, 1937).

Turning to the philosophical literature, Redhead surrendered prematurely when he accepted Malament's proof as showing that *only* the Poincaré-Einstein convention is causally definable. If our assessment of Malament's result is correct, then Friedman's and Torretti's are not. Even aside from the question of uniqueness of a simultaneity definition, we find most puzzling Friedman's contention that standard simultaneity cannot be varied "without abandoning the basic structure" of the special theory of relativity. As noted earlier, Einstein (1905) mentions the possibility of using the backward null cone convention to define simultaneity. We have shown here how that possibility can be incorporated into a causal definition of a non-standard simultaneity relation within the standard structure of the special theory of relativity. Of course, the best way to demonstrate the conventionality of simultaneity is to formulate the basic structure of the special theory of relativity without the use of any simultaneity convention. Elsewhere, we will show how this can be done using radar ranging coordinates (i.e., advanced and retarded times) to coordinatize events with respect to an inertial world line, and Bondi's (1980) K-calculus (i.e., the radial Doppler shift) to perform coordinate transformations between one inertial world line and another.

Turning to Norton, we only hope that our theorems—together with Havas's critical comments—lead to at least a minor reversal in debates in the philosophy of space and time. After describing Spirtes' (1981) result about temporally oriented spacetimes (see fn. 13), Norton (1992, 226) adds: "However, before modifying the construal of causal definability by adding or subtracting from the list [of Malament's criteria], we would need to find very good reasons for doing so." We believe that we have supplied the needed reasons.

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