Startling simplicity

$$(6)\quad \sum \psi_i \delta u_i \;=\; \delta L -$$

$$-\frac{d}{dx}\left\{\sum\left(\binom{1}{1}\frac{\partial L}{\partial u_i^{(1)}}\delta u_i+\binom{2}{1}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i^{(1)}+\cdots+\binom{\varkappa}{1}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-1)}\right)\right\}$$
$$+\frac{d^2}{dx^2}\left\{\sum\left(\binom{2}{2}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i+\binom{3}{2}\frac{\partial L}{\partial u_i^{(3)}}\delta u_i^{(1)}+\cdots+\binom{\varkappa}{2}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-2)}\right)\right\}$$
$$\cdots+(-1)^{\varkappa}\frac{d^{\varkappa}}{dx^{\varkappa}}\left\{\sum\binom{\varkappa}{\varkappa}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i\right\}.$$

The last three lines are a derivative $\frac{d}{dx}$.

The last three lines are a derivative $\frac{d}{dx}$.

On its face, an equation Euler or Lagrange could have written.

The last three lines are a derivative $\frac{d}{dx}$.

On its face, an equation Euler or Lagrange could have written.

Direct consequence of additivity, chain rule, and integration by parts.

The last three lines are a derivative $\frac{d}{dx}$.

On its face, an equation Euler or Lagrange could have written.

Direct consequence of additivity, chain rule, and integration by parts.

In several independent variables those lines become a sum:

$$\mathrm{Div}(A) \;=\; \frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}.$$

The last three lines are a derivative $\frac{d}{dx}$.

On its face, an equation Euler or Lagrange could have written.

Direct consequence of additivity, chain rule, and integration by parts.

In several independent variables those lines become a sum:

$$\mathrm{Div}(A) \;=\; \frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}.$$

Noether's A_j are not functions. They are differential operators.

Noether *thought* in formulas (always).

circa. 1930

Noether *thought* in formulas (always).

1918: not *symmetry*, but *transforming variables*: $x_1, \ldots, x_n \;\to\; y_1, \ldots, y_n$.

circa. 1930

Noether *thought* in formulas (always).

1918: not *symmetry*, but *transforming variables*: $x_1, \ldots, x_n \;\rightarrow\; y_1, \ldots, y_n$.

1918: defines a *divergence* as a kind of “expression” –

circa. 1930

Noether *thought* in formulas (always).

1918: not *symmetry*, but *transforming variables*: $x_1, \ldots, x_n \rightarrow y_1, \ldots, y_n$.

1918: defines a *divergence* as a kind of "expression" – for good reasons we will see.

circa. 1930

Noether *thought* in formulas (always).

1918: not *symmetry*, but *transforming variables*: $x_1, \ldots, x_n \rightarrow y_1, \ldots, y_n$.

1918: defines a *divergence* as a kind of “expression” – for good reasons we will see.

circa. 1930

She (like Klein) put “conservation laws” in quote marks.

Noether *thought* in formulas (always).

1918: not *symmetry*, but *transforming variables*: $x_1, \ldots, x_n \;\rightarrow\; y_1, \ldots, y_n$.

1918: defines a *divergence* as a kind of "expression" – for good reasons we will see.

circa. 1930

She (like Klein) put "conservation laws" in quote marks.

Thus Noether's originality, generality, simplicity.

Background.

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation.
Hamilton's whole method of "ignorable coordinates."

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough:

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation.
Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality.

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

> *A combination of the methods of the formal calculus of variations with those of Lie's group theory. (Noether 1918)*

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation.
Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

> *A combination of the methods of the formal calculus of variations with those of Lie's group theory. (Noether 1918)*

Merely put the two together:

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

> *A combination of the methods of the formal calculus of variations with those of Lie's group theory. (Noether 1918)*

Merely put the two together: the result lies on the page before you.

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

> *A combination of the methods of the formal calculus of variations with those of Lie's group theory. (Noether 1918)*

Merely put the two together: the result lies on the page before you.

(Vast generality $\rightarrow$ unforeseen applications.

Background.

Euler and Lagrange knew roughly: Symmetry $\leftrightarrow$ conservation. Hamilton's whole method of "ignorable coordinates."

Noether cites complicated derivations of SR and Newtonian mechanics from symmetries; extremely specific on-going GR.

Her breakthrough: Utter generality. It is all simply:

> *A combination of the methods of the formal calculus of variations with those of Lie's group theory. (Noether 1918)*

Merely put the two together: the result lies on the page before you.

(Vast generality $\rightarrow$ unforeseen applications. Not Noether's concern.)

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

It is not.

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

It is not.

Noether knew Lie never saw this.

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

It is not.

Noether knew Lie never saw this.

No one before her saw it

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

It is not.

Noether knew Lie never saw this.

No one before her saw it – though several had applied Lie's ideas to calculus of variations and conservation laws.

A combination of the methods of the formal calculus of variations with those of Lie's group theory.

Has been interpreted as modesty.

It is not.

Noether knew Lie never saw this.

No one before her saw it – though several had applied Lie's ideas to calculus of variations and conservation laws.

"Formal calculus of variations" did not exist before Noether.

Noether's proofs were (and remain) startling in their simplicity.

— Nathan Jacobson

Noether's proofs were (and remain) startling in their simplicity.

— Nathan Jacobson

- Her life.

Noether's proofs were (and remain) startling in their simplicity.

— Nathan Jacobson

- Her life.
  - 1913 Erlangen research program.

Noether's proofs were (and remain) startling in their simplicity.

— Nathan Jacobson

- Her life.
  - 1913 Erlangen research program.
- Her mature program for mathematics.

Noether's proofs were (and remain) startling in their simplicity.

— *Nathan Jacobson*

- Her life.
  - 1913 Erlangen research program.
- Her mature program for mathematics.
  - Why did she not pursue the Conservation Theorems?

Noether's proofs were (and remain) startling in their simplicity.

— Nathan Jacobson

- Her life.
  - 1913 Erlangen research program.
- Her mature program for mathematics.
  - Why did she not pursue the Conservation Theorems?
- Noether calculates variance and divergence.

Noether's proofs were (and remain) startling in their simplicity.
— *Nathan Jacobson*

- Her life.
  - 1913 Erlangen research program.
- Her mature program for mathematics.
  - Why did she not pursue the Conservation Theorems?
- Noether calculates variance and divergence.
  - What did "Formal" mean to Noether in 1918?

She already had a notable career in Nineteenth Century Erlangen.

1905

1916

She already had a notable career in Nineteenth Century Erlangen.

1905

1916

The *long Nineteenth Century*, in comfortable and distinctly old-fashioned Erlangen.

Noether:

- Dissertation 1908 with Gordan.

Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.

Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.
- Deutsche Mathematiker-Vereinigung 1909.

Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.
- Deutsche Mathematiker-Vereinigung 1909.
- Supervised Hans Falckenberg dissertation 1911 (with E. Schmidt).

Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.
- Deutsche Mathematiker-Vereinigung 1909.
- Supervised Hans Falckenberg dissertation 1911 (with E. Schmidt).

Reliable, traditional even by Erlangen standards.

Noether:

- Dissertation 1908 with Gordan.
- Circolo Matematico di Palermo 1908.
- Deutsche Mathematiker-Vereinigung 1909.
- Supervised Hans Falckenberg dissertation 1911 (with E. Schmidt).

Reliable, traditional even by Erlangen standards. Nothing that people today recall.

Life in Erlangen

Weyl's eulogy: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."

Life in Erlangen

Weyl's eulogy: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."

1933

This is probably from little brother Fritz.

Life in Erlangen

Weyl's eulogy: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."

1933

This is probably from little brother Fritz.

Likely true,

Life in Erlangen

Weyl's eulogy: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."

This is probably from little brother Fritz.

Likely true, in Erlangen,

1933

Life in Erlangen

Weyl's eulogy: "There was nothing rebellious in her nature; she was willing to accept conditions as they were."

This is probably from little brother Fritz.

Likely true, in Erlangen, before 1915.

1933

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

- He never did justice to developing concepts from the fundamentals (*Grundlagen gehenden Begriffsentwicklungen*).

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

- He never did justice to developing concepts from the fundamentals (*Grundlagen gehenden Begriffsentwicklungen*).
- His lectures entirely avoided fundamental conceptual definitions, even such as *limit*.

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

- He never did justice to developing concepts from the fundamentals (*Grundlagen gehenden Begriffsentwicklungen*).
- His lectures entirely avoided fundamental conceptual definitions, even such as *limit*.
- His lectures rested on lively expression and the power gained from his own studies, rather than on logic and rigor (*Systematik und Strenge*).

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

- He never did justice to developing concepts from the fundamentals (*Grundlagen gehenden Begriffsentwicklungen*).
- His lectures entirely avoided fundamental conceptual definitions, even such as *limit*.
- His lectures rested on lively expression and the power gained from his own studies, rather than on logic and rigor (*Systematik und Strenge*).

(Indeed this was standard in calculus of variations, for Noether 1918, and long after.

Erlangen: Compare Max and Emmy Noether on Paul Gordan:

- He never did justice to developing concepts from the fundamentals (*Grundlagen gehenden Begriffsentwicklungen*).
- His lectures entirely avoided fundamental conceptual definitions, even such as *limit*.
- His lectures rested on lively expression and the power gained from his own studies, rather than on logic and rigor (*Systematik und Strenge*).

(Indeed this was standard in calculus of variations, for Noether 1918, and long after. Notations not standardized, definitions not given.)

Noether's 1913 *Erlanger Programme*.

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's *Fields of Rational Functions* was this, in effect (1913 *Jahresbericht der Deutschen Mathematiker-Vereinigung*).

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's *Fields of Rational Functions* was this, in effect (1913 *Jahresbericht der Deutschen Mathematiker-Vereinigung*).

To bring Gordan's perspective to Dedekind/Weber/Hilbert algebra.

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's *Fields of Rational Functions* was this, in effect (1913 *Jahresbericht der Deutschen Mathematiker-Vereinigung*).

To bring Gordan's perspective to Dedekind/Weber/Hilbert algebra.

Anyone might have found it very ambitious.

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's *Fields of Rational Functions* was this, in effect (1913 *Jahresbericht der Deutschen Mathematiker-Vereinigung*).

To bring Gordan's perspective to Dedekind/Weber/Hilbert algebra.

Anyone might have found it very ambitious.

Fulfilled by a series of four papers to 1926,

Noether's 1913 *Erlanger Programme*.

Programme= a new professor's declaration of research plans.

Noether's *Fields of Rational Functions* was this, in effect (1913 *Jahresbericht der Deutschen Mathematiker-Vereinigung*).

To bring Gordan's perspective to Dedekind/Weber/Hilbert algebra.

Anyone might have found it very ambitious.

Fulfilled by a series of four papers to 1926, then far surpassed.

By 1913 Max and Fritz (and teacher/colleague Ernst Fischer) proudly, happily aware that Emmy is on a level they are not, By 1913 Max and Fritz (and teacher/colleague Ernst Fischer) proudly, happily aware that Emmy is on a level they are not, and few have ever been.

By 1913 Max and Fritz (and teacher/colleague Ernst Fischer) proudly, happily aware that Emmy is on a level they are not, and few have ever been.

Paul Gordan.

Von

MAX NOETHER in Erlangen.

(Mit Unterstützung von Felix Klein in Göttingen und von Emmy Noether in Erlangen.)*)

*) Von Ersterem wurde ich in der Gesamtwürdigung, von Letzterer in der Würdigung der algebraischen Arbeiten wesentlich unterstützt.

The Twentieth Century came on fast.

Erlangen August 8, 1914, one week after Germany declared war.

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed,

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested.

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested. Nearly as many killed as arrested.

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested. Nearly as many killed as arrested.

Noether joins the Independent Social Democrats (USPD).

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested. Nearly as many killed as arrested.

Noether joins the Independent Social Democrats (USPD).

(By 1920s, Noether well known to colleagues as pro-Soviet.)

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested. Nearly as many killed as arrested.

Noether joins the Independent Social Democrats (USPD).

(By 1920s, Noether well known to colleagues as pro-Soviet.)

Weyl calls the USPD "close to the Social Democrats."

1914–1918: 2 million German soldiers die in war, a half million civilians in famine.

German Revolution 1918–1919.

Berlin, early March 1919, 1200 workers and protesting soldiers killed, 1600 arrested. Nearly as many killed as arrested.

Noether joins the Independent Social Democrats (USPD).

(By 1920s, Noether well known to colleagues as pro-Soviet.)

Weyl calls the USPD "close to the Social Democrats." Like calling Students for a Democratic Society "close to the Democratic Party."

Weyl says her "period of relative dependence" extended to 1919:

Weyl says her "period of relative dependence" extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan's formal standpoint to the Hilbert method of approach was accomplished.

Weyl says her "period of relative dependence" extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan's formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source,

Weyl says her "period of relative dependence" extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan's formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source, but wrong about this.

Weyl says her “period of relative dependence” extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan’s formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source, but wrong about this.

From 1913 her own “method”:

Weyl says her “period of relative dependence” extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan’s formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source, but wrong about this.

From 1913 her own “method”: Gordan plus Hilbert,

Weyl says her “period of relative dependence” extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan’s formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source, but wrong about this.

From 1913 her own “method”: Gordan plus Hilbert, plus Dedekind and Lie,

Weyl says her "period of relative dependence" extended to 1919:

Her dependence on Gordan did not last long. . . . [By 1913] the transition from Gordan's formal standpoint to the Hilbert method of approach was accomplished.

Weyl is a indispensable source, but wrong about this.

From 1913 her own "method": Gordan plus Hilbert, plus Dedekind and Lie, in ways no one, including those four, saw before her!

Weyl beautifully describes her “fundamental characteristic:”

> *the ability to find the formulation which reveals the essential logical nature of the question, stripped of any incidental peculiarities which complicate matters and obscure the fundamental point.*

Weyl beautifully describes her “fundamental characteristic:”

> *the ability to find the formulation which reveals the essential logical nature of the question, stripped of any incidental peculiarities which complicate matters and obscure the fundamental point.*

But he is quite wrong to write of:

> *Emmy Noether, the ardent foe of computation and algorithms in mathematics.*

Weyl beautifully describes her “fundamental characteristic:”

> *the ability to find the formulation which reveals the essential logical nature of the question, stripped of any incidental peculiarities which complicate matters and obscure the fundamental point.*

But he is quite wrong to write of:

> *Emmy Noether, the ardent foe of computation and algorithms in mathematics.*

Noether disparaged idle, wasted calculation,

Weyl beautifully describes her “fundamental characteristic:”

> *the ability to find the formulation which reveals the essential logical nature of the question, stripped of any incidental peculiarities which complicate matters and obscure the fundamental point.*

But he is quite wrong to write of:

> *Emmy Noether, the ardent foe of computation and algorithms in mathematics.*

Noether disparaged idle, wasted calculation, as did Gordan,

Weyl beautifully describes her “fundamental characteristic:”

> *the ability to find the formulation which reveals the essential logical nature of the question, stripped of any incidental peculiarities which complicate matters and obscure the fundamental point.*

But he is quite wrong to write of:

> *Emmy Noether, the ardent foe of computation and algorithms in mathematics.*

Noether disparaged idle, wasted calculation, as did Gordan, as does everyone who wants actual calculated answers to problems.

Philosophic point:

Philosophic point: *constructive* mathematics “finding solutions in principle,” Philosophic point: *constructive* mathematics “finding solutions in principle,” turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Philosophic point: *constructive* mathematics "finding solutions in principle," turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Kronecker really only cared about solutions in principle.

Philosophic point: *constructive* mathematics "finding solutions in principle," turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Kronecker really only cared about solutions in principle.

Much of his work remained purely "principled" until 1950s when Noether's student/ colleague Olga Taussky-Todd used computers to apply it.

Philosophic point: *constructive* mathematics "finding solutions in principle," turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Kronecker really only cared about solutions in principle.

Much of his work remained purely "principled" until 1950s when Noether's student/ colleague Olga Taussky-Todd used computers to apply it.

Gordan, Dedekind, Noether (inter alios) wanted solutions in fact.

Philosophic point: *constructive* mathematics "finding solutions in principle," turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Kronecker really only cared about solutions in principle.

Much of his work remained purely "principled" until 1950s when Noether's student/ colleague Olga Taussky-Todd used computers to apply it.

Gordan, Dedekind, Noether (inter alios) wanted solutions in fact.

Noether was constructive when that helped,

Philosophic point: *constructive* mathematics "finding solutions in principle," turns out surprisingly irrelevant to *applied* mathematics which needs solutions in fact.

Kronecker really only cared about solutions in principle.

Much of his work remained purely "principled" until 1950s when Noether's student/ colleague Olga Taussky-Todd used computers to apply it.

Gordan, Dedekind, Noether (inter alios) wanted solutions in fact.

Noether was constructive when that helped, and non-constructive when that helped.

In 1915, Noether notes she has outdone one of Hilbert's results by Gordan's perspective:

> *The following is an entirely elementary finiteness proof ... for the invariants of a finite group, which at once supplies an actual statement of a complete system of invariants while the usual proof using the Hilbert basis theorem is only an existence proof. *(See for example Weber, Lehrbuch der Algebra §57.)*

In 1915, Noether notes she has outdone one of Hilbert's results by Gordan's perspective:

> *The following is an entirely elementary finiteness proof ... for the invariants of a finite group, which at once supplies an actual statement of a complete system of invariants while the usual proof using the Hilbert basis theorem is only an existence proof. *(See for example Weber, Lehrbuch der Algebra §57.)*

Weber's existence proof took 3pp. after Hilbert's Theorem.

In 1915, Noether notes she has outdone one of Hilbert's results by Gordan's perspective:

> *The following is an entirely elementary finiteness proof ... for the invariants of a finite group, which at once supplies an actual statement of a complete system of invariants while the usual proof using the Hilbert basis theorem is only an existence proof. *(See for example Weber, Lehrbuch der Algebra §57.)*

Weber's existence proof took 3pp. after Hilbert's Theorem.

Noether gives two independent explicit calculations, each 1/2 page.

In 1926, Hilbert's Göttingen, at the peak of her commutative algebra, with Gordan's framed picture in her study, Noether supervises her first official doctoral dissertation.

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Hermann was the first to realize: To effectively verify a routine you should find "an upper bound for the number of calculations needed" for the routine (as a function of input size).

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Hermann was the first to realize: To effectively verify a routine you should find "an upper bound for the number of calculations needed" for the routine (as a function of input size).

Did this for routines giving Noether's primary decompositions in the polynomial case.

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Hermann was the first to realize: To effectively verify a routine you should find “an upper bound for the number of calculations needed” for the routine (as a function of input size).

Did this for routines giving Noether’s primary decompositions in the polynomial case.

Noether enthusiastically promoted Hermann’s work.

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Hermann was the first to realize: To effectively verify a routine you should find "an upper bound for the number of calculations needed" for the routine (as a function of input size).

Did this for routines giving Noether's primary decompositions in the polynomial case.

Noether enthusiastically promoted Hermann's work.

Still cited today.

Grete Hermann: *The question of finitely many steps in the theory of polynomial ideals.*

Hermann was the first to realize: To effectively verify a routine you should find "an upper bound for the number of calculations needed" for the routine (as a function of input size).

Did this for routines giving Noether's primary decompositions in the polynomial case.

Noether enthusiastically promoted Hermann's work.

Still cited today. "The foundational paper in computer algebra."

Two related points: Noether's mature/Göttingen program, and why she abandoned her conservation theorems.

With all the fervor of her nature, she was herself ready to forget what had been done in the first years of her mathematical activity, considering these results as standing apart from her true mathematical path—the creation of a general abstract algebra. (Alexandroff, 1981, p. 101)

Alexandroff, Brouwer, Urysohn in Laren, Holland 1922?. Noether also visited and talked about topology.

With all the fervor of her nature, she was herself ready to forget what had been done in the first years of her mathematical activity, considering these results as standing apart from her true mathematical path—the creation of a general abstract algebra. (Alexandroff, 1981, p. 101)

Alexandroff, Brouwer, Urysohn in Laren, Holland 1922?. Noether also visited and talked about topology.

Her "true path" was in no way limited to her specific theorems.

Not everybody trusted that her achievements were what they were later accepted to be. She irritated people by bragging about them.
– Olga Taussky-Todd (1981, p. 84)

Not everybody trusted that her achievements were what they were later accepted to be. She irritated people by bragging about them.
– Olga Taussky-Todd (1981, p. 84)

Noether's student/colleague in Göttingen and Bryn Mawr.

Not everybody trusted that her achievements were what they were later accepted to be. She irritated people by bragging about them.
– Olga Taussky-Todd (1981, p. 84)

Noether's student/colleague in Göttingen and Bryn Mawr.

Everyone *did* trust Noether's theorems!

Not everybody trusted that her achievements were what they were later accepted to be. She irritated people by bragging about them.
– Olga Taussky-Todd (1981, p. 84)

Noether's student/colleague in Göttingen and Bryn Mawr.

Everyone *did* trust Noether's theorems!

Not her vision of algebraizing all mathematics.

Not everybody trusted that her achievements were what they were later accepted to be. She irritated people by bragging about them.
– Olga Taussky-Todd (1981, p. 84)

Noether's student/colleague in Göttingen and Bryn Mawr.

Everyone *did* trust Noether's theorems!

Not her vision of algebraizing all mathematics. Yet it happened.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
    - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
  - Later differential geometry, notably via fiber bundles.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
    - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
    - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
  - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.
  - Later Weil, Serre, Grothendieck.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
  - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.
  - Later Weil, Serre, Grothendieck.
- Arithmetic – beginning with Artin, Herbrand, Brauer.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
  - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.
  - Later Weil, Serre, Grothendieck.
- Arithmetic – beginning with Artin, Herbrand, Brauer.
  - Then another Artin, Chevalley.

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
    - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
    - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.
    - Later Weil, Serre, Grothendieck.
- Arithmetic – beginning with Artin, Herbrand, Brauer.
    - Then another Artin, Chevalley.
- All of these (plus complex analysis) unified in *cohomology*

- Abstract algebra – beginning with van der Waerden *Moderne Algebra*.
  - Later the whole *Elements* of Bourbaki.
- Topology – beginning with Alexandroff and Hopf *Topologie*.
  - Later differential geometry, notably via fiber bundles.
- Algebraic geometry – beginning with van der Waerden.
  - Later Weil, Serre, Grothendieck.
- Arithmetic – beginning with Artin, Herbrand, Brauer.
  - Then another Artin, Chevalley.
- All of these (plus complex analysis) unified in *cohomology* – beginning when her student Saunders Mac Lane "did not understand" factor sets.

Noether did not pursue her Conservation Theorems for long.

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

She had people do that.

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

She had people do that.

Hermann began in computational algebra

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

She had people do that.

Hermann began in computational algebra – then left math for politics and philosophy.

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

She had people do that.

Hermann began in computational algebra – then left math for politics and philosophy.

Bessel-Hagen began in Conservation Theorems.

Noether did not pursue her Conservation Theorems for long.

She did not long pursue any of her ideas (except her 1913 program in finite group representations, over the rest of her life).

She had people do that.

Hermann began in computational algebra – then left math for politics and philosophy.

Bessel-Hagen began in Conservation Theorems.

Then turned to history of math.

Some 60 years later, Peter Olver took up Noether's own invariant theory and view of the Conservation Theorems.

Some 60 years later, Peter Olver took up Noether's own invariant theory and view of the Conservation Theorems.

Updated by fiber bundle methods, and cohomology, both descended from her commutative algebra.

ca. 1918?

$$(6)\quad \sum \psi_i \delta u_i \;=\; \delta L -$$

$$- \frac{d}{dx}\left\{\sum\left(\binom{1}{1}\frac{\partial L}{\partial u_i^{(1)}}\delta u_i + \binom{2}{1}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{1}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-1)}\right)\right\}$$

$$+ \frac{d^2}{dx^2}\left\{\sum\left(\binom{2}{2}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i + \binom{3}{2}\frac{\partial L}{\partial u_i^{(3)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{2}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-2)}\right)\right\}$$

$$\cdots + (-1)^{\varkappa}\frac{d^{\varkappa}}{dx^{\varkappa}}\left\{\sum \binom{\varkappa}{\varkappa}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i\right\}.$$

ca. 1918?

$$(6) \quad \sum \psi_i \delta u_i \;=\; \delta L -$$

$$- \tfrac{d}{dx} \left\{ \sum \left(\binom{1}{1} \frac{\partial L}{\partial u_i^{(1)}} \delta u_i + \binom{2}{1} \frac{\partial L}{\partial u_i^{(2)}} \delta u_i^{(1)} + \cdots + \binom{\varkappa}{1} \frac{\partial L}{\partial u_i^{(\varkappa)}} \delta u_i^{(\varkappa-1)} \right) \right\}$$

$$+ \tfrac{d^2}{dx^2} \left\{ \sum \left(\binom{2}{2} \frac{\partial L}{\partial u_i^{(2)}} \delta u_i + \binom{3}{2} \frac{\partial L}{\partial u_i^{(3)}} \delta u_i^{(1)} + \cdots + \binom{\varkappa}{2} \frac{\partial L}{\partial u_i^{(\varkappa)}} \delta u_i^{(\varkappa-2)} \right) \right\}$$

$$\cdots + (-1)^{\varkappa} \tfrac{d^{\varkappa}}{dx^{\varkappa}} \left\{ \sum \binom{\varkappa}{\varkappa} \frac{\partial L}{\partial u_i^{(\varkappa)}} \delta u_i \right\}.$$

In short $=\; \delta L + \operatorname{Div} A$ where $\delta L = \sum \frac{\partial L}{\partial u_i} \delta u_i$.

ca. 1918?

$$(6)\quad \sum \psi_i \delta u_i = \delta L -$$
$$- \frac{d}{dx}\left\{\sum\left(\binom{1}{1}\frac{\partial L}{\partial u_i^{(1)}}\delta u_i + \binom{2}{1}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{1}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-1)}\right)\right\}$$
$$+ \frac{d^2}{dx^2}\left\{\sum\left(\binom{2}{2}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i + \binom{3}{2}\frac{\partial L}{\partial u_i^{(3)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{2}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-2)}\right)\right\}$$
$$\cdots + (-1)^{\varkappa}\frac{d^{\varkappa}}{dx^{\varkappa}}\left\{\sum \binom{\varkappa}{\varkappa}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i\right\}.$$

In short $=\ \delta L + \mathrm{Div}\, A$ where $\delta L = \sum \frac{\partial L}{\partial u_i}\delta u_i$.

But δu is part of a symmetry $\varDelta x, \varDelta u$ of L iff $\delta L = -\mathrm{Div}(L.\varDelta x)$.

ca. 1918?

$$(6)\quad \sum \psi_i \delta u_i \;=\; \delta L -$$

$$- \frac{d}{dx}\left\{\sum\left(\binom{1}{1}\frac{\partial L}{\partial u_i^{(1)}}\delta u_i + \binom{2}{1}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{1}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-1)}\right)\right\}$$

$$+ \frac{d^2}{dx^2}\left\{\sum\left(\binom{2}{2}\frac{\partial L}{\partial u_i^{(2)}}\delta u_i + \binom{3}{2}\frac{\partial L}{\partial u_i^{(3)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{2}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-2)}\right)\right\}$$

$$\cdots + (-1)^{\varkappa}\frac{d^{\varkappa}}{dx^{\varkappa}}\left\{\sum\binom{\varkappa}{\varkappa}\frac{\partial L}{\partial u_i^{(\varkappa)}}\delta u_i\right\}.$$

In short $=\ \delta L + \mathrm{Div}\, A$ where $\delta L = \Sigma \frac{\partial L}{\partial u_i}\delta u_i$.

But δu is part of a symmetry $\varDelta x, \varDelta u$ of L iff $\delta L = -\mathrm{Div}(L.\varDelta x)$.

So δu is part of a symmetry if and only if

$$\Sigma \psi_i \delta u_i \;=\; \mathrm{Div}(A - L.\varDelta x).$$

We seek *divergences*:

We seek *divergences*: “recently often called ‘conservation laws’.”

We seek *divergences*: “recently often called ‘conservation laws’.”

So Noether explains what she will mean by “divergence,”

We seek *divergences*: “recently often called ‘conservation laws’.”

So Noether explains what she will mean by “divergence,” then organizes the equations to bring them out:

die partielle Integration zeigt, sind diese Randglieder Integrale über Divergenzen, d. h. über Ausdrücke

$$\operatorname{Div} A = \frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n},$$

wobei A linear in δu und seinen Ableitungen ist. Somit kommt:

$$(3) \qquad \sum \psi_i \delta u_i = \delta f + \operatorname{Div} A.$$

Enthält f insbesondere nur erste Ableitungen der u, so ist im Fall des einfachen Integrals die Identität (3) identisch mit der von Heun sogenannten „Lagrangeschen Zentralgleichung“:

$$(4) \qquad \sum \psi_i \delta u_i = \delta f - \frac{d}{dx}\left(\sum \frac{\partial f}{\partial u_i'} \delta u_i\right), \qquad \left(u_i' = \frac{du_i}{dx}\right),$$

während für das n-fache Integral (3) übergeht in:

$$(5) \quad \sum \psi_i \delta u_i = \delta f - \frac{\partial}{\partial x_1}\left(\sum \frac{\partial f}{\partial \frac{\partial u_i}{\partial x_1}} \delta u_i\right) - \cdots - \frac{\partial}{\partial x_n}\left(\sum \frac{\partial f}{\partial \frac{\partial u_i}{\partial x_n}} \delta u_i\right).$$

Für das einfache Integral und $\varkappa$ Ableitungen der u ist (3) gegeben durch:

$$(6) \qquad \sum \psi_i \delta u_i = \delta f -$$

$$- \frac{d}{dx}\left\{\sum\left(\binom{1}{1}\frac{\partial f}{\partial u_i^{(1)}}\delta u_i + \binom{2}{1}\frac{\partial f}{\partial u_i^{(2)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{1}\frac{\partial f}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-1)}\right)\right\} +$$

$$+ \frac{d^2}{dx^2}\left\{\sum\left(\binom{2}{2}\frac{\partial f}{\partial u_i^{(2)}}\delta u_i + \binom{3}{2}\frac{\partial f}{\partial u_i^{(3)}}\delta u_i^{(1)} + \cdots + \binom{\varkappa}{2}\frac{\partial f}{\partial u_i^{(\varkappa)}}\delta u_i^{(\varkappa-2)}\right)\right\} + \cdots$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$+ (-1)^\varkappa \frac{d^\varkappa}{dx^\varkappa}\left\{\sum \binom{\varkappa}{\varkappa}\frac{\partial f}{\partial u_i^{(\varkappa)}}\delta u_i\right\}$$

und eine entsprechende Identität gilt beim n-fachen Integral: A

Divergences are, and for Noether can only be:

Expressions $\frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}$ *where A is linear in* δu *and its derivatives. From this follows:* $\Sigma\psi_i\delta u_i \;=\; \delta L + \mathrm{Div}\, A.$

Divergences are, and for Noether can only be:

Expressions $\frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}$ *where* A *is linear in* δu *and its derivatives. From this follows:* $\Sigma \psi_i \delta u_i \; = \; \delta L + \mathrm{Div}\, A$.

Partials only at *independent* variables,

Divergences are, and for Noether can only be:

> *Expressions* $\frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}$ *where* A *is linear in* δu *and its derivatives. From this follows:* $\Sigma \psi_i \delta u_i \;=\; \delta L + \mathrm{Div}\, A.$

Partials only at *independent* variables, while the A_i depend on much more.

Divergences are, and for Noether can only be:

> *Expressions* $\frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}$ *where A is linear in* δu *and its derivatives. From this follows:* $\Sigma \psi_i \delta u_i \;=\; \delta L + \mathrm{Div}\, A$.

Partials only at *independent* variables, while the A_i depend on much more. (Choose values of the dependent variables u, and a symmetry $\varDelta x, \varDelta u$: this gives a vector field on the independent variables x and its ordinary divergence is Noether's divergence for those choices.)

Divergences are, and for Noether can only be:

> *Expressions* $\frac{\partial A_1}{\partial x_1} + \cdots + \frac{\partial A_n}{\partial x_n}$ *where A is linear in* δu *and its derivatives. From this follows:* $\Sigma \psi_i \delta u_i \ = \ \delta L + \mathrm{Div}\, A.$

Partials only at *independent* variables, while the A_i depend on much more. (Choose values of the dependent variables u, and a symmetry $\varDelta x, \varDelta u$: this gives a vector field on the independent variables x and its ordinary divergence is Noether's divergence for those choices.)

Clever, crucial use of "linear in δu."

Other authors of the time made divergences geometric in terms of vector fields (*vektorfelden*),

Other authors of the time made divergences geometric in terms of vector fields (*vektorfelden*), summing one partial for each variable.

Other authors of the time made divergences geometric in terms of vector fields (*vektorfelden*), summing one partial for each variable.

In Noether's calculations the variables in A include "δu and their derivatives" – which no one at the time could define.

Other authors of the time made divergences geometric in terms of vector fields (*vektorfelden*), summing one partial for each variable.

In Noether's calculations the variables in A include "δu and their derivatives" – which no one at the time could define.

Her favorite source, Kneser, gives as a typical "formal fundamental property (*formale Grundeigenschaft*) of the sign δ:"

$$\delta y = \left(\frac{\partial y}{\partial t}\right)_{t=0} dt \quad \text{where } \delta y,\ \partial y,\ \partial t,\ dt \text{ are all "small."}$$

Other authors of the time made divergences geometric in terms of vector fields (*vektorfelden*), summing one partial for each variable.

In Noether's calculations the variables in A include "δu and their derivatives" – which no one at the time could define.

Her favorite source, Kneser, gives as a typical "formal fundamental property (*formale Grundeigenschaft*) of the sign δ:"

$$\delta y = \left(\frac{\partial y}{\partial t}\right)_{t=0} dt \quad \text{where } \delta y,\ \partial y,\ \partial t,\ dt \text{ are all "small."}$$

"Formal fundamental" = a basic calculating rule, not a consequence of any analytic or geometric definition.

What was Noether's Formal Calculus of Variations?

What was Noether's Formal Calculus of Variations?

I am not sure.

What was Noether's Formal Calculus of Variations?

I am not sure.

One key was to eschew solutions of differential equations in favor of calculating with formal linear combinations of variations δu_i.

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma \psi_i \delta u_i = \delta L - \frac{d}{dx}\left(\Sigma \frac{\partial L}{\partial u_i'} \delta u_i\right), \quad \left(u_i' = \frac{du}{dx}\right).$$

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma\psi_i \delta u_i = \delta L - \frac{d}{dx}\left(\Sigma \frac{\partial L}{\partial u_i'} \delta u_i\right), \quad \left(u_i' = \frac{du}{dx}\right).$$

Uses the familiar Lagrangian expressions:

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma \psi_i \delta u_i \;=\; \delta L \;-\; \frac{d}{dx}\left(\Sigma \frac{\partial L}{\partial u_i'} \delta u_i\right), \quad \left(u_i' = \frac{du}{dx}\right).$$

Uses the familiar Lagrangian expressions:

$$\psi_i \;=\; \frac{\partial L}{\partial u_i} \;-\; \frac{d}{dx}\frac{\partial L}{\partial u_i'}.$$

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma\psi_i\delta u_i \;=\; \delta L \;-\; \frac{d}{dx}\left(\Sigma\frac{\partial L}{\partial u_i'}\delta u_i\right), \quad \left(u_i' = \frac{du}{dx}\right).$$

Uses the familiar Lagrangian expressions:

$$\psi_i \;=\; \frac{\partial L}{\partial u_i} \;-\; \frac{d}{dx}\frac{\partial L}{\partial u_i'}.$$

But instead of cancelling each δu_i and solving each equation $\psi_i = 0$; algebraist Noether calculates formally with $\Sigma\psi_i\delta u_i = 0$.

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma \psi_i \delta u_i \;=\; \delta L \;-\; \frac{d}{dx}\left(\Sigma \frac{\partial L}{\partial u'_i} \delta u_i\right), \quad \left(u'_i = \frac{du}{dx}\right).$$

Uses the familiar Lagrangian expressions:

$$\psi_i \;=\; \frac{\partial L}{\partial u_i} \;-\; \frac{d}{dx}\frac{\partial L}{\partial u'_i}.$$

But instead of cancelling each δu_i and solving each equation $\psi_i = 0$; algebraist Noether calculates formally with $\Sigma \psi_i \delta u_i = 0$.

The calculations are like the classical

Noether 1918 writes a linear combination of variations δu_i of the dependent variables $u_1, \ldots, u_\mu$:

$$(4) \qquad \Sigma\psi_i\delta u_i \; = \; \delta L \; - \; \frac{d}{dx}\left(\Sigma\frac{\partial L}{\partial u_i'}\delta u_i\right), \quad \left(u_i' = \frac{du}{dx}\right).$$

Uses the familiar Lagrangian expressions:

$$\psi_i \; = \; \frac{\partial L}{\partial u_i} \; - \; \frac{d}{dx}\frac{\partial L}{\partial u_i'}.$$

But instead of cancelling each δu_i and solving each equation $\psi_i = 0$; algebraist Noether calculates formally with $\Sigma\psi_i\delta u_i = 0$.

The calculations are like the classical – *except* Noether does not use the "fundamental theorem of calculus of variations" to cancel the δu_i!

A powerful idea to this day,

A powerful idea to this day, not unique to Noether but central to her heritage.

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

1. help find actual solutions to equations; or

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

1. help find actual solutions to equations; or
2. provide some information when solutions cannot be found; or

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

1. help find actual solutions to equations; or
2. provide some information when solutions cannot be found; or
3. provide information you actually want, when you do not require a specific solution anyway.

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

1. help find actual solutions to equations; or
2. provide some information when solutions cannot be found; or
3. provide information you actually want, when you do not require a specific solution anyway.

E.g. existence of stable equilibria for a family of ODEs where you are not especially interested in any one equation

A powerful idea to this day, not unique to Noether but central to her heritage.

Calculations with formal linear combinations can be a powerful tool to

1. help find actual solutions to equations; or
2. provide some information when solutions cannot be found; or
3. provide information you actually want, when you do not require a specific solution anyway.

E.g. existence of stable equilibria for a family of ODEs where you are not especially interested in any one equation – let alone in locating its precise stable equilibria.

This is one half of *cohomology*.

This is one half of *cohomology*. The other half was even more crucial to Noether:

This is one half of *cohomology*. The other half was even more crucial to Noether: quotienting out (or modding out) suitable cases.

This is one half of *cohomology*. The other half was even more crucial to Noether: quotienting out (or modding out) suitable cases.

From number theory, to connections on fiber bundles (gauge theories), to complex analysis (conformal field theories), to the *variational complex* (Vinogradov, Olver, others).

This is one half of *cohomology*. The other half was even more crucial to Noether: quotienting out (or modding out) suitable cases.

From number theory, to connections on fiber bundles (gauge theories), to complex analysis (conformal field theories), to the *variational complex* (Vinogradov, Olver, others).

Olver uses this to clarify Noether's Theorems I and II.

This is one half of *cohomology*. The other half was even more crucial to Noether: quotienting out (or modding out) suitable cases.

From number theory, to connections on fiber bundles (gauge theories), to complex analysis (conformal field theories), to the *variational complex* (Vinogradov, Olver, others).

Olver uses this to clarify Noether's Theorems I and II.

Noether 1918 ignores some (somehow trivial) symmetries.

This is one half of *cohomology*. The other half was even more crucial to Noether: quotienting out (or modding out) suitable cases.

From number theory, to connections on fiber bundles (gauge theories), to complex analysis (conformal field theories), to the *variational complex* (Vinogradov, Olver, others).

Olver uses this to clarify Noether's Theorems I and II.

Noether 1918 ignores some (somehow trivial) symmetries.

Olver forms equivalence classes of symmetries/conservation laws by modding out the trivial in a precise sense.

A decisive mathematician, with lifelong ardent support from family and all the world's best placed mathematicians – set out to change the course of mathematics and succeeded beyond her or anyone's dreams – yet never held a secure job, nor even a salary.

Not modest,

Not modest, and not wrong,

Not modest, and not wrong, writing to Helmut Hasse, December, 1931:

Not modest, and not wrong, writing to Helmut Hasse, December, 1931:

My methods are working- and conceptual-methods and so they penetrate everywhere anonymously.

Alexandroff, P. (1981). *In memory of Emmy Noether*, pages 99–114. In Brewer and Smith (1981). This 1935 eulogy at the Moscow Mathematical Society is also in N. Jacobson ed. *Emmy Noether Collected Papers*, Springer Verlag, 1983, 1–11; and Dick 1970, 153–80.

Brewer, J. and Smith, M., editors (1981). *Emmy Noether: A Tribute to Her Life and Work*. Marcel Dekker, New York.

Taussky-Todd, O. (1981). My personal recollections of Emmy Noether. In Brewer and Smith (1981), pages 79–92.

I. *If the integral I is invariant under* $\mathfrak{G}_\rho$*, then* ρ *linearly independent combinations among the Lagrangian expressions become divergences — and conversely, that implies I is invariant under some* $\mathfrak{G}_\rho$*.*

No solutions to the Euler-Lagrange equations.

I. *If the integral I is invariant under* $\mathfrak{G}_\rho$*, then* ρ *linearly independent combinations among the Lagrangian expressions become divergences — and conversely, that implies I is invariant under some* $\mathfrak{G}_\rho$*.*

No solutions to the Euler-Lagrange equations.

Does not call $\mathfrak{G}_\rho$ a group!

I. *If the integral I is invariant under* $\mathfrak{G}_\rho$*, then* ρ *linearly independent combinations among the Lagrangian expressions become divergences — and conversely, that implies I is invariant under some* $\mathfrak{G}_\rho$*.*

No solutions to the Euler-Lagrange equations.

Does not call $\mathfrak{G}_\rho$ a group! (Often just a local group.)

The eulogy of Gordan

- “He compiled volumes of formulas, very well ordered but providing a minimum of text.
- His mathematical friends undertook to prepare the text for press. . . .
- They could not always produce a fully correct conception.”

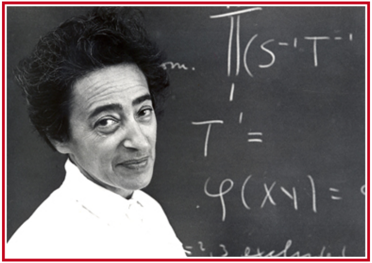
“Only a few of his publications, and especially the earliest, express Gordan’s specific style: bare, brief, direct, uninterrupted theorems one after the other.”

Olga Taussky worked with Noether in Göttingen and Bryn Mawr.

Olga Taussky worked with Noether in Göttingen and Bryn Mawr.

Emmy was not uninterested in the problems women face. She was concerned already in Göttingen. I think it was through her, but am not completely certain about it, I learned about the IFUW,. . . of which the AAUW is a branch. In 1932 she attended one of their meetings when they invited her, or maybe she only mentioned the invitation to me. In any case I do recall that she said that one ought to attend such functions.

She said women should not try to work as hard as men. She remarked that she, on the whole, only helped young men to obtain positions so they could marry and start families. She somehow imagined all women were supported.



Hel Braun's student-eye view.

Number theory at Frankfurt University 1933. Student of Carl Ludwig Siegel. Habilitated Göttingen 1940.

Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.

Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.
- University mathematics became, so to say, more ‘logical.’

Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.
- University mathematics became, so to say, more ‘logical.’
- One learns methods and everything is put into a theory.

Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.
- University mathematics became, so to say, more ‘logical.’
- One learns methods and everything is put into a theory.
- Talent is no longer so extremely important.”

Saw the spread of Göttingen methods:

- “This largely goes back to the algebraists.
- University mathematics became, so to say, more ‘logical.’
- One learns methods and everything is put into a theory.
- Talent is no longer so extremely important.”

“Perhaps I exaggerate but this is the impression I have when I compare the lectures of that time to later ones.”

Or again:

Or again:

- “Still in my student days university mathematics rested strongly on mathematical talent.

Or again:

- “Still in my student days university mathematics rested strongly on mathematical talent.
- Logic and notation were not so well established.

Or again:

- “Still in my student days university mathematics rested strongly on mathematical talent.
- Logic and notation were not so well established.

“The days are gone when one affectionately described one’s professor with ‘He said A, wrote B, meant C, and D is correct’...”

Max Noether

Paul Gordan

Max Noether

Paul Gordan

Does Emmy Noether use limits in her “formale Variationsrechnung”?

Max Noether

Paul Gordan

Does Emmy Noether use limits in her “formale Variationsrechnung”?

Not obvious.

Max Noether

Paul Gordan

Does Emmy Noether use limits in her “formale Variationsrechnung”?

Not obvious.

Her reference, Kneser 1900, gives dx, $\varDelta x$, ∂x, δx distinct roles,

Max Noether

Paul Gordan

Does Emmy Noether use limits in her "formale Variationsrechnung"?

Not obvious.

Her reference, Kneser 1900, gives dx, $\varDelta x$, ∂x, δx distinct roles,

Defined only as "small."

Max Noether

Paul Gordan

Does Emmy Noether use limits in her “formale Variationsrechnung”?

Not obvious.

Her reference, Kneser 1900, gives $dx, \Delta x, \partial x, \delta x$ distinct roles,

Defined only as “small.” Kneser 1900 was a noted advance in rigor.