# Loopholes: Social Learning and the Evolution of Contract Form*

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**Abstract**

We explore an overlapping-generations model of contractual evolution via "loopholes." Bayesian rational principals, uncertain about what actions are feasible, form beliefs about them through limited observation of previous agents' behavior and offer contracts that only deter the harmful behaviors they deem sufficiently likely. Agents discover whether cheating is feasible, and do so if not deterred by the contract (i.e., if there is a loophole).

In a loophole equilibrium, principals who observe cheating close loopholes when they offer contracts. But loophole-free contracts deter all cheating, thereby conveying little information to other principals about feasible actions, who may then believe that cheating is unlikely and choose loopholey contracts. The result is cycling of contract types that alternately deter and encourage undesired behavior, yielding heterogeneity across principals. There are also bureaucratic equilibria in which contracts deter behavior that is actually infeasible.

Depending on whether principals sample concurrently or historically, population dynamics may display aggregate cycling or convergence to a stationary, nondegenerate distribution of contracts.

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# 1 Introduction

The State of California passes a law requiring motorcyclists to "wear helmets" when riding on public roads. The riders soon comply by strapping the helmets to their arms.

A manufacturing firm equips its typewriters with counters that register the number of times the keys are operated, and pays its stenographers by the keystroke. It is soon noticed that one typist is earning much more than any of the others: she takes lunch at her desk, using one hand for eating and the other for punching the most convenient key on the typewriter as fast as she can.

Division managers at a large food products company receive bonuses only when earnings increase from the prior year. They respond by manipulating the timing of shipments, falsifying dates on sales invoices, and accruing expenses in the wrong period.

All of these are examples of *loopholes*: a principal offers an incentive scheme to an agent, who then "complies" with the letter of the scheme in ways that harm the principal, but cannot be deterred by punishments since they are not violations of the contract. Given the information that this kind of cheating is possible, the principal would redesign the contract.

Indeed, the helmet law was rewritten to specify (and more important, enforce) that helmets be worn on heads.[1] The stenographers' piece rate scheme was withdrawn by their company, Lincoln Electric, and replaced by fixed wages and close supervision (Berg and Fast, 1975). And the food products company, H.J. Heinz, implemented a costly set of accounting and supervisory procedures and modified the company's reporting structure to deal with the problems with its division managers (Goodpaster and Post, 1981).

Examples of this kind, in which firms attempt to boost profits by intro-

[1]For instance the California Vehicle Code now specifies that "... 'wear a safety helmet' or 'wearing a safety helmet' means having a safety helmet...on the person's head that is fastened with the helmet straps and that is of a size that fits the wearing person's head securely without excessive lateral or vertical movement." (Division 12, Chapter 5, Article 7, Section 27803.

ducing incentive schemes that appear to backfire, pepper the management literature.[2] Economists often favor them as pedagogical tools for illustrating the pitfalls in the design of incentive systems, or simply to prove that people really do respond to incentives. There is no shortage of economic analysis showing why rational agents would respond the way they have (Holmström-Milgrom, 1991; Baker, 1992), and plenty of empirical evidence illustrating just how real people manipulate the schemes that govern their workplaces. Piece rate schemes such as that for the stenographers have well known quality problems;[3] timing of sales and other performance quotas such as those at Heinz have been noted in a variety of settings and widely studied (Chevalier and Ellison, 1997; Oyer, 1998; Courty and Maschke, 2004). And motorcyclists around the world continue to comply with their local ordinances by strapping helmets to their arms. In other words, the same mistakes seem to be made many times over.

Of course, firms and other principals respond. Once the harmful action is discovered, the firm will attempt to deter it in future by revising its rules or contracts.[4] Sometimes though, employees find new ways to harm the principal, leading to new contracts. And so on.

These phenomena do raise the question why so many profit maximizing firms would provide perverse incentive schemes in the first place. Indeed, they sit somewhat uneasily with the standard contracting setting, in which the principal designs a contract to induce her agent to behave in desirable ways. There, in equilibrium, nothing “unexpected” happens: the agent chooses the action prescribed by the contract, and every verifiable outcome of the relationship is an anticipated one, in the support of the distribution of outcomes corresponding to the prescribed action. Looking across similar economic environments, contracts chosen are similar, and unless some feature of the environment changes, there is no reason for the form of the contract to change over time. From this perspective, the undesired and apparently unexpected behavior by agents that sometimes happens in the real world, together with subsequent revision of the incentive schemes that govern them,

[2] The classic reference on this is Kerr (1975), "On the Folly of Rewarding A while Hoping for B."

[3] See for instance Stiglitz (1975), though the problems had been documented in the sociology literature and much earlier, e.g. Whyte et al. (1955) and were apparent to Frederick Taylor and his followers.

[4] Thus we distinguish the loophole case from the one in which occasional cheating is anticipated and tolerated simply because it is too costly to deter.

seems to be an anomaly.

In this paper we take the view that this pattern can be understood as a natural and necessary consequence of the fact that real principals – rational as they may be – must offer incentive schemes with only incomplete information about the environment they are operating in, and that learning about this environment is a social process. Learning – in particular by *future principals* – takes place after the scheme is in place: if it is imperfect, agents will "cheat" (do A instead of B), after which new, more costly, schemes will be invoked that deter A.

One might think such a process would converge to a state of affairs in which principals know pretty much everything about the environment. But a key feature of learning about agency environments, which often differs from learning about "natural" environments (e.g. technology), is that *the perfect incentive scheme inhibits learning*: since it deters all undesired behavior, an observer will never see that the undesired behavior is feasible, or the damage it can do. So he will rationally reduce his belief that the behavior is feasible or at least has adverse consequences. As a result, in order to reduce the costs of incentive schemes that deter behavior which in his view is unlikely to happen anyway, he may offer a scheme that turns out to have loopholes. Unfortunately for him, his agents will cheat, but now others will learn from his mistake and close the loophole. The process begins anew.

Thus loopholes, though damaging to principals who offer them, generate a *positive learning externality*, once which in the simplest settings may lead to cycling through a limiting number of possible incentive schemes. More generally, they may be viewed as an engine of the "constancy of change" in organizations.

The framework we use to analyze loopholes is a perfectly standard principal-agent setting except for two assumptions. First, principals do not in general have full knowledge of the actual state of the world (here, the set of actions that an agent may take). Second, since information about the environment is costly to acquire, they are limited in the amount they can observe about other people's actions and therefore in how much they learn. In fact these are the basic assumptions made in the *social learning* literature: loopholes are a likely outcome in organizational design precisely when principals acquire their environmental knowledge through social learning.

The basic setting is as follows. An agent can potentially take different types of actions, only one of which benefits the principal; though he is aware of all the logically possible ways in which he may be cheated, the principal is initially uncertain as to which actions are possible. We consider an overlapping generations model in which at any date there is a large number of principals and agents.

Before offering a contract to an agent, the principal wanders the earth, observing a random selection of other principal-agent relationships. We consider a number of assumptions about what is observable during these encounters, but the simplest is to imagine that the agents can be observed either to be cheating or not cheating.

If cheating is observed, then it is obviously known to be possible and the principal will offer a contract that deters it. If it is not observed, then the principal is not sure whether that is because it is impossible or because he happened to observe only agents who were governed by contracts that deter cheating. Indeed, he will update his beliefs in favor of the scenario that cheating is impossible, conditional on this observation. He then designs a contract optimally given those beliefs. This setup captures what appears to be an important feature of the managerial "styles," namely that they are largely conditioned by early career experiences (Schoar, 2007).

Other than the limitations on what is observed and how many earlier relationships principals may observe, all assumptions are standard: principals and agents are Bayesian rational, there is a common prior about the true state and strategies are assumed to be known in equilibrium.[5]

We obtain the following findings:

- A fully informed contract cannot emerge in equilibrium: that is, sometimes principals don't choose the optimal contract given the actual set of feasible actions.

- There are two basic types of equilibria: (1) In *bureaucratic* equilibria, principals offer contracts that deter behaviors that cannot actually happen; (2) In *loopholey* equilibria, the offered contracts have "loopholes," and agents will sometimes "cheat," i.e., take actions not recommended by the contract.

- In loopholey equilibria, when cheating is feasible, contracts offered by a lineage of principals change over time without converging — since the number of possible optimal contracts is finite, this means that there

[5]For the simplest model, for principals to act they need only know the strategies and not the priors. So the common prior assumption can be relaxed.

is always "cycling" (usually with stochastic frequency) at the lineage level. There may also be heterogeneity in the contract offers across otherwise identical lineages.

- In these equilibria, as the sample size grows without bound, a vanishing fraction of the population offer loopholey contracts, while the rest offer tight ones, so that the outcome approximates the "standard" case in which principals are fully informed about the environment.

- Cycling may also occur at the population level – *all* principals may *simultaneously* offer loopholey contracts, followed by all offering tight ones, followed by loopholes, etc. The nature of these cycles depends on the temporal structure of observation: if principals look only concurrently at other relationships, rather than observing the outcomes of historical relationships, the aggregate cycles are stable; if they look historically, they tend to converge to a steady state in which a constant fraction of the population offers loopholey contracts.

- In an extension of the basic model, principals may pick new contracts that are "more complicated" than earlier ones in that they explicitly prohibit more types of actions; sufficiently complex contracts may be replaced by simple ones (either loopholey ones or ones that deter all undesired behavior by paying agents large rents, depending on parameters; in the latter case these simple high rent contracts will eventually be replaced by simple loopholey contracts).

- In another extension, principals can choose whether to look concurrently or historically. If the world is known to change slowly enough, historical sampling is optimal, and aggregate cycles vanish, although lineage-level ones persist. If the world changes more quickly, concurrent sampling is optimal, and the aggregate limit cycles result; these typically move faster than the underlying environment.

## 1.1 Literature

This paper is related to several theoretical literatures in economics. Most close in spirit is a series of recent papers studying aspects of contracting under unforeseen contingencies and/or cognitive constraints (e.g. Tirole 2009, Bolton and Faure-Grimaud 2005). Part of the distinction here is how we model "cognitive limitations": we take it as representative of the human condition that the signal structure we are able to process is coarse relative to the world we are trying to understand whereas Tirole (2009) and Bolton and Faure-Grimaud (2005) interpret the unwillingness of agents to get full information about the state of the world as bounded rationality. More significant is the difference in focus: we are interested in how contracts evolve as a result of observational limitations rather than on the static properties of contracts that are chosen by cognitively limited agents. Ellison and Holden (2009) seeks to understand how rules are developed, but it uses a different framework, one in which principals are perfectly informed but have difficulty communicating to agents who always obediently follow instructions. It does admit increasing complexity or refinement of rules but the tendency is always toward greater efficiency over time, with eventual convergence to a stable set of rules. There is also a literature on strategic incompleteness (Spier 1992, Bernheim and Whinston 1998, Chung and Fortnow 2007), but there principals deliberately write incomplete contracts with perfect anticipation of the behavior that will result and again there is no evolution of contract forms.

There is also a very large "social learning" literature in which individuals observe one aspect or another of other individuals' behavior and use those observations to update their beliefs about the optimal course of action. Examples include papers on social multi-armed bandits (Bolton and Harris 1999) or game theoretic or economic models in which signals are of limited dimension relative to other individuals' information (Ellison and Fudenberg 1993, Piketty 1995). The difficulty of making correct inferences from coarse signals is also at the heart of the information cascades and herding literature (Banerjee 1992, Bikchandani, Hirshleifer and Welch 1992); Scharfstein and Stein (1990) uses a somewhat different logic to derive herding behavior in a setting in which agency rather than pure inference problems are crucial. None of these papers has the feature that the "right" decision (in our case, the optimal contract that deters the entire set of harmful behaviors that exist) hides information from others, and thus while they will often have convergence to the wrong action, they do not display the constancy of change (e.g. cycling) that our model displays.

We should also mention the literature on learning in games and in particular the idea of self-confirming equilibrium (Fudenberg and Levine, 1998) in which the observation made by subjects does not allow them to pin down a unique interpretation of others' strategies in a dynamic learning process (in particular, about what strategies would be off the equilibrium path).

Our model differs from that literature in that principals' beliefs derive from Bayesian updating and a common prior assumption, and are not just assumed to be subjectively consistent with the observation.[6]

Finally, our model bears some connection with papers on imperfect recall and/or coarse information processing (Piccione and Rubinstein 1997; Mullainathan, 2000; Wilson, 2002; Jehiel, 2005; Baliga and Ely 2009) though formally our agents don't forget anything they observe, they simply have no incentive to remind anyone else. Moreover, that literature doesn't look at implications for contracting or organizational evolution.

# 2 Model

## 2.1 Technology and Preferences

The economy lasts forever, with time measured in discrete periods. In each period, there is a continuum with unit measure of principals and agents who enter into one-period production relationships. All individuals live for two periods, youth and adulthood. Principals only offer contracts in adulthood; in their youth they serve an "apprenticeship," in which they watch and learn. Agents are idle in youth and work in adulthood.

At any time $t$, all (risk-neutral) principals own identical projects. In the simplest case, there are two logically possible dimensions of action that an agent might take. Following Kerr, we refer to them as $B$ and $A$ (this can be generalized to $B$ plus $A_1, ..., A_{n-1}$; we will refer to $n$-dimensional worlds and states $i$ corresponding to the $2^{n-1}$ sets of $A$-dimensions that exist).

Now, either all projects are in fact one-dimensional or they are two-dimensional. Principals don't know what state they are in. They do know that the world either in one state or the other for all time, and they have an (exogenous) prior $\mu^0$, the probability of the "good state," in which only the $B$ action is available. This belief will be updated based on observations of a sample of previous principal-agent relationships.[7] A project yields $R > 0$ to

[6] Moreover, in our setting, even on the equilibrium path, some principals may propose contracts that trigger harmful behavior, and this is because we assume there is not a public record of what happens in past interactions. At the same time the analysis reveals the value of such a record: of course, it is likely to be costly to maintain.

[7] Formally we treat this prior as common among all young principals and exogenous. However, the reader may want to keep in mind the followiong derivation, which will be pertinent in our discussion in Section 4.0.6. Suppose that over time, the economy may be

the principal with probability $e_B$, 0 with probability $1 - e_B$, where $e_B$, which is not verifiable, is the effort exerted in the $B$ dimension.

All agents are identical: they are risk neutral in income, have limited liability, and an outside option equal to zero. Agents preferences are given by ($w$ is income)

$$w + \alpha e_A - \frac{1}{2}(e_B + e_A)^2,$$

where $\alpha \geq 0$ is an agent's private benefit of engaging in action $A$. Of course in the state of the world in which $A$-actions exist don't exist, we necessarily have $e_A \equiv 0$, while in the "bad" state in which they do, $e_A \in [0, 1]$.

We assume that agents learn the state after contracting, when the production period begins. The parameters satisfy

$$1 > R > \alpha > R/2.$$

This assumption ensures that it is both efficient and costly to deter the $A$ action. A first-best allocation in which effort is verifiable would have $e_B = R$ and $e_A = 0$.

## 2.2 State-Optimal Second-best Contracts

In general, principals will be uncertain about the dimensionality of the world they are operating in. One might imagine then that the optimal contract would be some sort of "compromise" that would perform well on average, where the average is computed with respect to the principal's belief about the world he is in.

In our model, however, the optimal contract with uncertainty about the state of the world is always equal to one of the certainty optima. This serves

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in either state. However transition times between states are very long, so for all intents and purposes a principal can be quite certain that he spends his whole life in one state. Specifically, if it is in state 0, the economy transitions out of it into state 1 with probability $\epsilon_0$ and from 1 to 0 with probability $\epsilon_1$. The economy therefore spends $\mu^0 = \frac{\epsilon_1}{\epsilon_0 + \epsilon_1}$ of the time in state 0 with average duration of a spell equal to $1/\epsilon_0$. We think of the $\epsilon_i$ as very small so that principals (and we) may neglect them in their updating calculations. The generalization to any finite number of states is straightforward.

The common prior assumption can be substantially relaxed without changing many of our results; indeed in some cases they are "easier" to obtain. We leave discussion of that case to a future draft.

to highlight the effects we are interested in. In more general settings, contracts and efforts taken by agents might adjust incrementally to principals' beliefs, but it would still be true that principals with strong beliefs in the good state would find agents exerting higher than expected levels of $A$ effort, and principals with strong beliefs in the bad state would offer contracts that lead to low levels of $A$ effort; insofar as observing $A$ effort is more likely when there is more of it, our results will stand.

We compute the state optimal contracts for our model here.

In the 1-dimensional world (state 0), the optimal second best (effort not verifiable) contract in the world pays $w_0$ if $R$, 0 otherwise, where $w_0$ solves

$$\begin{aligned} & \max_w e_B(R-w) \\ \text{s.t. } e_B &= \arg\max_w e_B w - \frac{1}{2}(e_B)^2; \end{aligned}$$

since the agent sets $e_B = w$, maximizing $w\,(R-w)$ yields

$$w_0 = R/2$$

In a 2-dimensional world (state 1), there are two ways to encourage agents to choose the productive effort. The first is simply to pay a sufficiently high incentive wage $w_1$ if $R$ and 0 otherwise, where $w_1$ solves

$$\begin{aligned} & \max_w e_B(R-w) \\ \text{s.t. } e_B &= \arg\max_w e_B w + e_A\alpha - \frac{1}{2}(e_B+e_A)^2. \end{aligned}$$

Since the agent sets $e_B = 0$, $e_A = \alpha$ if $w < \alpha$ and $e_B = w$, $e_A = 0$ if $w \geq \alpha$, the principal gets 0 if $w < \alpha$; $w\,(R-w)$ is decreasing for $w \geq \alpha$ (since $\alpha > R/2$), so

$$w_1 = \alpha > w_0$$

Hence deterring the $A$ effort is costly, and it may be preferable to use a second method of deterrence, namely to add a clause to the contract that prohibits the $A$ action. Such a clause will be costly to enforce, because it requires monitoring to make it verifiable (in some cases the cost may assume the form of foregone production benefits – employees socializing in the coffee room may occasionally communicate information that might enhance productivity and benefit the principal – in short the two actions may not be perfect production substitutes). We model this by supposing that the principal incurs a cost $c$ for each enforceable clause he adds to the contract. Then a clause contract consisting of the incentive wage $w_0$ and a clause would pay the principal $\frac{R^2}{4} - c$.[8]

There is thus an optimal contract $C_0$ for state 0 in which cheating is not possible and a different optimal contract $C_1$ for state 1 in which cheating is possible. Denote by $\pi_i$ the optimal profit in state $i$; thus $\pi_0 = R^2/4$ and $\pi_1 = \max\{\frac{R^2}{4} - c, \alpha(R-\alpha)\}$.

## 2.3 Contracting with Principal Uncertainty about Environment

Let $\mu_0$ be the Principal's (posterior) belief that the world is 1-dimensional; he gets $\mu_0\pi_0$ by offering the $C_0$ contract and $\pi_1$ with $C_1$. It is easy to see that he does not gain by hedging with some sort of compromise contract: any optimal contract will either induce the agent to be productive (choose $B$ effort) in both states or it will induce him to be productive only in state 0 (it cannot be productive in state 1 without also being productive in state 0). If the contract is productive in both states, it is dominated by $C_1$. If it is productive only in state 0, it is dominated by $C_0$ : offering less than $w_0$ yields him less than $R^2/4$ in state 0 and zero in state 1, since it is not incentive compatible there; and offering more than $w_1$ will always yield him less than offering $w_1$, which in turn is (weakly) dominated by contract $C_1$ Finally, offering $w \in (w_0, w_1)$ is weakly dominated by offering $w_0$. Note the argument does not depend on whether $C_1$ is the wage or clause contract.[9]

We thus have a simple characterization of how the contract offered will depend on the Principal's belief about the dimensionality of the agent's action set:

**Lemma 1** *The Principal offers $C_0$ if $\mu_0 > \mu_0^* = \pi_1/\pi_0$ and $C_1$ otherwise.*

[8] For simplicity, we rule out random monitoring, wherein the principal incurs the cost $pc$ for detecting $A$ effort with probability $p$ while paying a wage of $(1-p)\,\alpha$ – little substantive would change if we allowed for it.

[9] In fact, if one thinks of the state as the agent's preference that he learns after contracting, then one can use Myerson's (1982) framework to show that these remain the optimal contracts even if agents's were allowed to report their types once they learn them. Separating agent types is diffcult because both "good" ($\alpha = 0$) and "bad" ($\alpha > R/2$) agents like high wages and (weakly) prefer no clauses. Details are omitted from this draft.

Note that $\mu_0^*$ is small if $c$ (or $\alpha$, in case the high wage contract is optimal in state 1) is "large" – close to $R^2/4$ ($\alpha$ is close to $R$). In other words, if deterrence is costly, the principal does not need a strong belief in state 0 in order to offer the contract suited for it. By the same token, when $c$ (or $\alpha$) is small, the principal might as well deter, even if he is fairly certain that it is not necessary.

We should emphasize that our basic results do not depend on the discreteness of the principal's optimal action set. Whatever contract that would be offered would either deter action $A$ or it would not, and it is the action rather than the contract that we are taking to be observed by young principals.

# 3 Learning

Of course, the immediate question is where does $\mu_0$ come from? Beliefs are endogenous in our model: we suppose a principal learns (imperfectly, in general) about the state of the world based on observations only of what other *agents* do. In particular, principals do not base their beliefs on "stories" told by other principals (though we could allow for some relaxation of this assumption without changing things very much). The main point is that learning will be based on a limited (compared to the complexity of the world) set of observations. This is the extent of "bounded rationality" or coarse information processing in our model.

The sequence of activity during a principal's life time is as follows.

- **Principals have a prior** $\mu^0$ that the world is one dimensional, i.e. that there is no way to cheat other than to withhold $B$-effort.This prior is exogenous, perhaps, as we mentioned, a long historical average.

- **Each apprentice (young principal) gets** $k$ **draws about past interactions** that (almost certainly) took place in the same state of the world that he is in.

- **While an apprentice, principal observes which actions are taken by agents governed by the earlier** $k$ **relationships she sees** (as well as success/failure of the projects, i.e. she observes only whether $e_B > 0$ and $e_A > 0$, but not their magnitudes.

- **Principal forms a posterior** based on observations, knowing what fraction of the population are offering each type of contract in each state of the world (the equilibrium assumption).
- **Principal offers contract to his agent, who then takes an action,** possibly while another apprentice watches.

For now we assume that the apprentice principals observe very little. They don't observe the size of wage paid or the contract itself, or the magnitude of $e_B$ (which would reveal the wage). In fact it would make no difference if they observe the magnitude of $e_A$; even observing $e_B$ would not matter if one extends the model slightly – see the section on observing contracts).

Our calculations assume that all relationships the principal sees are in the same state as her own. The results we get will not depend on this provided transition probabilities between states are small.

We are otherwise making conventional assumptions about rationality and equilibrium play: principals update beliefs using Bayes' rule, and know the strategies other principals and agents are playing. In particular they know the map from states to distributions of contract offers (but not, of course, the actual distribution prevailing at the time).

The equilibria we study are all symmetric and stationary: all agents use the same strategies, and those strategies do not vary over time.

#### 3.0.1 Stationary Equilibria

We consider first stationary equilibria of our model, that is situations in which the distribution of contracts that are offered within a state of the world is constant over the time that state persists (this distribution is simply the fraction $x$ offering $C_0$; the rest offer $C_1$). We shall refer to principal beliefs in terms of the probability of the one-dimensional world, which is the one they would prefer; (given the optimal contracts, agents have the opposite preference).

Assume that each principal views $k$ relationships, where $k$ is exogenous. Since we are focused on stationary equilibria, it does not matter whether the observations the principal gets are from one period (say, his youth) or from relationships that occurred deeper in the past.

**Proposition 2** ***(Low Priors):*** *Assume that $\mu^0 < \mu_0^*$. For any $k$, the following is a stationary equilibrium: all principals offer $w_1$; the sample always delivers that only task $B$ was performed.*

That this is an equilibrium should be clear: with everyone offering $w_1$ in both states of the world, no agent will ever be observed cheating in either state, no matter what the sample size. Thus, what is observed cannot reveal any information about the state, and the posterior equals the prior; since this is less than $\mu_0^*$,it is optimal to offer $w_1$. Notice that in this stationary equilibrium, when the world is one dimensional, contracts are *bureaucratic* in the sense of providing the principals with more protection than they need, and this is costly. Of course no principal *knows* her contract is bureaucratic.

This is not the only stationary equilibrium compatible with a prior that is less than $\mu_0^*$. There are others in which the $w_0$ contract is offered, at least some of the time, even in state 1.To see this, suppose $k = 1$. We have:

**Proposition 3** ***(High Priors):*** *Assume that $\mu^0 > \frac{\mu_0^*}{2-\mu_0^*}$ and each apprentice observes one prior relationship ($k = 1$). The following is a steady state equilibrium: Upon observing no effort on task A, a principal offers $C_0$; upon observing effort on task A, a principal offers $C_1$; in the 0-state, only $C_0$ is offered; in the 1-state, half of the contracts are $C_0$ and half are $C_1$.*

If the entire population use the strategy described in the proposition, then in state 0, no one ever sees $A$, and immediately everyone offers $C_1$.In state 1, if half offer $C_0$ in state 1 and half don't, a randomly chosen relationship will have been governed by a $C_0$ contract with probability $\frac{1}{2}$ and those agents will exert $A$ effort. The observer will offer $C_1$ when it is his turn. The other half of the young principals will observe no $A$, and will offer $C_0$; thus once again half the contracts are $C_0$ To verify that this strategy is optimal, note that those who don't see $A$ but know that in state 1 half the population offer $C_1$ forms the posterior $\mu_0$, where

$$\begin{aligned} \mu_0 &= \frac{\text{Pr(Observe no A}|0)\text{Pr}(0)}{\text{Pr(Observe no A}|0)\text{Pr}(0)+\text{Pr(Observe no A}|1)\text{Pr}(1)} \\ &= \frac{1 \cdot \mu^0}{1 \cdot \mu^0 + \frac{1}{2} \cdot (1 - \mu^0)} > \mu_0^* \Longleftrightarrow \mu^0 > \frac{\mu_0^*}{2 - \mu_0^*}, \end{aligned}$$

so that offering $C_0$ is optimal.

Since $\frac{\mu_0^*}{2-\mu_0^*} < \mu_0^*$ if $\mu_0^* < 1$, these two propositions imply that there are (at least) two stationary equilibria when $\mu_0^* > \mu^0 > \frac{\mu_0^*}{2-\mu_0^*}$. Comparing the two, in both cases state 0 leads to homogeneity of contract form and stationarity at the lineage level (no one ever engages in different behavior than his predecessor), though in the bureaucratic case (Proposition 2) everyone is "overpaying" – the contract is not the one optimally suited for the actual state. In state 1, though, the first stationary equilibrium is homogenous and stationary, while the second one displays heterogeneity of the offered contracts. Moreover, there is "lineage evolution": if a principal observes what happened in his "father's" firm, he will do the opposite, his son will do as his father did, etc. If principals draw randomly from the whole population, the evolution occurs stochastically at the lineage level.[10]

The stationary equilibrium in Proposition 3 is a *loophole equilibrium*: there are states of the world in which some of the contracts being offered are suboptimal for that state and do not deter cheating behavior by the agents. Loophole equilibria involve (ex-post) suboptimal contracting and heterogeneity of contracts across otherwise identical relationships. In slightly more complex environments to be examined below, they will also involve heterogenous *responses* of currently identical firms facing the same problems (i.e., agent misbehaviors).

#### 3.0.2 Welfare

Loopholes, while inconvenient for the principal who offers them, are not all bad. They do provide a *positive externality*: apprentice principals are able to learn from other principals' mistakes. It is worth contrasting the loophole equilibrium, which in state 1 is suboptimal, with the loophole-free equilibrium in Proposition 2. In that case, everyone always writes a tight contract, but because of that there is no possibility of observing that cheating is infeasible, and so it is suboptimal in state 0. In fact, when the loophole equilibrium exists, it is better on average for principals than the stationary, loophole-free equilibrium:

**Proposition 4** *Assume that $\mu_0^* > \mu^0 > \frac{\mu_0^*}{2-\mu_0^*}$. The stationary loophole equilibrium always delivers a higher expected payoff (averaging over states according to the common prior) to the principals than the stationary loophole-free equilibrium*

The calculation is done averaging over states and contracts in each equilibrium. The idea is that it may be (socially) better to make mistakes sometimes (even half the time, as happens in state 1 of the loophole equilibrium)

[10]Note that apart from this detail, the stationary equilibria do not depend on whether the sample is supposed to include the father's firm.

and learn than to persistently do the wrong thing (state 0 of the bureaucratic equilibrium). In fact, the condition for this to be the case is precisely the one that allows the loophole equilibrium to exist: without loopholes the payoff is always $\pi_1$; with loopholes, the expected payoff is $\mu^0\pi_0 + \frac{1}{2}(1-\mu^0)\pi_1$, which exceeds $\pi_1$ whenever $\mu^0 > \frac{\mu_0^*}{2-\mu_0^*}$.

Of course, depending on what alternatives one imagines a social planner would have at his disposal, it is not straightforward to assess whether the loophole equilibrium is optimal: ideally one would have a very small fraction of the population "experiment" by offering the contract that is not ex-ante optimal, and convey the results to the rest. But the costs of this information transmission are possibly quite high (after all, this is implicit in the assumption that $k$ is finite), not to mention the costs imposed on the principals who conduct the experiment. Indeed, with the same instruments, the planner could improve upon the bureaucratic equilibrium as well: forcing a principal to offer a $C_0$ contract would reveal the state. The point here is simply that loopholes allow agents to convey useful information, while agent behavior under tightly written contracts tells the rest of us very little about the world.

It is worth noting that the principals are better off in the loophole equilibrium than in the bureaucratic equilibrium regardless of whether the optimal contract for state 1 is the clause contract or the high wage. Agents, however, will feel differently about the two equilibria depending on which contract is optimal in state 1. If the high wage is optimal, agents prefer getting that all the time (bureaucratic case) to getting it only in state 1. But if the clause contract is optimal, they prefer the loophole equilibrium, because they receive a low wage no matter the state in the bureaucratic equilibrium, while benefiting from occasional cheating in the loophole equilibrium (the wage is the same in both equilibria). Thus, the loophole equilibria may actually Pareto dominate.

### 3.0.3 Larger samples

Principals are unlikely to cast their lot entirely with one mentor and will instead get their information from several sources. In terms of our model, the sample size could be larger than 1. Indeed a natural question to ask is whether the outcome converges to the standard case (in which the principal always offers the contract optimal for the state) as the sample gets large.

We have already seen that this need not be true if the prior is low (Proposition 2), since for any $k$ there is an equilibrium in which only $C_1$ is offered every period in both states. This is similar to the standard incomplete-learning results familiar in the multi-armed bandit literature.]

However, it is also true that for any prior, there is a sufficiently large sample size for which there is a stationary loophole equilibrium (i.e. one in which a positive measure of principals are offering $C_0$ in state 1). Stationarity will require that the set of unfortunates who offer loophole-laden contracts in state 1 is very small, because with large sample sizes, they are still very likely to be observed by future principals, for whom the state is therefore revealed. This means that nearly everyone will be offering $C_1$ in state 1, while everyone offers $C_0$ in state 0. Thus the equilibrium approximates the standard full-information case.

Let $\bar{x}_k$ be the unique solution in $[0, 1]$ to

$$x = (1 - x)^k$$

We note that $\bar{x}_k$ decreases with $k$ and tends to 0 as $k$ tends to infinity. If $\bar{x}_k$ is the proportion of contracts in the 1-state leading to positive effort on task $A$. (i.e., $C_0$ contracts) the probability that a $k$ sample includes no observation of effort $A$ is $(1 - \bar{x}_k)^k$ or $\bar{x}_k$ and the posterior after observing no $A$ effort in the $k$-sample that the state is $B$ would thus be $\frac{\mu^0}{\mu^0+\bar{x}_k(1-\mu^0)}$. For someone to be willing to offer a such a contract, we need this posterior to exceed $\mu_0^*$. But for $k$ large enough (therefore $\bar{x}_k$ small enough) we can make this expression as close to unity as we like, even if $\mu^0$ is small.

To summarize, we have :

**Proposition 5** *For all $\mu^0 > 0$ there exists a $\overline{k}$ such that $k \geq \overline{k}$ implies $\frac{\mu^0}{\mu^0+\bar{x}_k(1-\mu^0)} > \mu_0^*$. For such $k$ sample size the following is a steady state equilibrium: Upon observing no effort on task $A$, a principal chooses $C_0$ ; upon observing effort on task $A$, a principal offers $C_1$. As $k$ tends to infinity, this equilibrium approaches the first-best situation in which the state is perfectly observed by principals.*

This proposition underscores once again the positive nature of the loophole externality: an arbitrarily small fraction of the population can perform the social experiment (offering $C_0$) on behalf of everyone else, provided only that everyone else is sufficiently likely to observe them. It also underscores that any notion of optimality of equilibrium will depend sensitively on what the sampling structure (size of $k$) is. In particular, if very large values of $k$ are costly to sustain, it may be (constrained) optimal for significant portions of the population to offer loophole-laden contracts. We shall discuss this further below in the section on population dynamics.

#### 3.0.4 Observing Contracts

So far we have been assuming that the information available to young principals is limited to observation of the dimension of agents' effort choices. This raises the question of what happens if they can observe more. Of course, if we suppose that individuals are capable of processing more information than we have allowed so far, we must also recognize that the world is also far more complex than what we have discussed. In keeping with the precept that what people can observe and process must surely be small compared to the complexity of the world they are trying to understand, we allow principals to observe more, but we also give them a more complicated inference to make.

To see why observing contracts reveals more information, suppose that principals observe previous contracts as well as effort types. Possible observations are then $(C_0, \tilde{}A)$ and $(C_1, \tilde{}A)$ in state 0 and $(C_0, A)$ and $(C_1, \tilde{}A)$ in state 1.

If $A$ is part of any observation, the state is revealed. It cannot be part of a (symmetric) equilibrium for principals to offer $C_0$ if $(C_0, \tilde{}A)$ or $(C_1, \tilde{}A)$ is observed and $C_1$ otherwise: for the only way an earlier principal would offer $C_1$ according to this equilibrium is in response to having seen $(C_0, A)$. But observing $C_1$ would then tell the current apprentice that the state is 1 and he would offer $C_1$ and not $C_0$, a contradiction. In fact, the only equilibrium strategy involving possibly different choices of contracts will be to offer $C_0$ if $(C_0, \tilde{}A)$ is observed and $C_1$ otherwise. But then, everyone will offer the optimal contract for the state. If the prior is sufficiently low ($\mu^0 < \mu_0^*$), there is also the bureaucratic equilibrium of Proposition 2, since under that scenario no information is transmitted by the observation. Whatever the equilibrium, in each state there is a single contract being offered, which would seem to undermine the main insights about the persistence of heterogeneous contracts in state 1.

The above assumes though that the greater complexity of observation is unmatched by greater complexity of the world that players are trying to understand. Suppose instead that $C_1$ contracts might be offered for several reasons besides the possibility of cheating. For instance a clause prohibiting congregating in the coffee room might be there because there is asbestos in the ceiling as much as because it encourages nonproductive chit-chatting among workers. Contracts list provisions without explaining their purpose. So an apprentice will not be able to infer the payoff relevant state from observing the contract.

To see this formally, consider the case in which $C_1$ is the high wage contract and suppose that $\nu$ of the principals are exceptional, with profitability $\hat{R} = 2\alpha$, and the rest are regular (profitability $R$). The exceptional principals optimally offer $w_1$ regardless of the state, so nothing can be learned from observing them or the contracts they offer. As long as the apprentice isn't sure whether the principal he observes is exceptional or not, he may still offer a $C_0$ contract even in state 1, since observing $(C_1, \tilde{\ } A)$ doesn't tell him for sure that he is in state 1 – he may simply have an exceptional principal. Formally, we have:

**Proposition 6**: Assume that $\frac{\nu\mu^0}{\nu\mu^0 + \frac{1-\mu^0}{2-\nu}} > \mu_0^*$. The following is a steady state with a $k = 1$ sample: Upon observing $(w_1, \tilde{\ } A)$ or $(w_0, \tilde{\ } A)$, a regular principal offers $w_0$; upon observing $(w_0, A)$ she offers $w_1$; exceptional principals always offer $w_1$

If $C_1$ is the clause contract, we simply interpret the exceptional principals as ones for whom the cost of enforcement is negligible or for whom cheating is effectively always possible, as in the asbestos example.

### 3.0.5 Summary

So far we have studied stationary equilibria. The main finding is that stationary loophole equilibria often exist in which lineages of principals switch (stochastically) from offering the optimal contract for state 1 to the "wrong" contract, and then back again. Looking across the population, there will be both types of contracts offered at each period. (In such equilibria, in state 0 everyone offers the optimal contract for that state.) This captures important features of the phenomena we are interested in: heterogeneity, misguided incentive schemes, and responses by patching with schemes that work, only for those schemes to contribute to their own undoing.

The fact that firms that offer tight contracts effectively release little information has interesting implications of the persistence of success and failure within organizations and their relation to managerial turnover. In the bad state, individual lineages might be successful for a while – if their principals happen to observe loopholey firms in their apprenticeships and therefore offer the optimal contract for the state (of course realized profits also have to be high). If we assume that every apprentice observes his "father" (consistent with our assumptions, but not necessary), then every lineage that offers a loopholey contract will correct this in the ensuing period. But if observations are entirely of other principals, the possibility of persistent loopholes within lineages arises. Interpreting lineages as organizations à la Crémer (1986), with the overlapping generations as turnover of managers, this suggests that organizations that recruit managers from outside, particularly from successful rivals, may be in for disappointment: those managers may be "naive" in the sense that they do not believe sufficiently in the possibility of cheating to offer tight contracts. From the organizational point of view, it may be better to recruit from within, or to even seek outside managers from failures. (Obviously this is meant more as empirical implication than policy advice, since there are other reasons why one might not want to recruit exclusively from failed firms.)

# 4 Population Dynamics

This still leaves open the question of whether these steady state equilibria are representative of the overall behavior of an economy in which loopholes are possible, something we would want to know both for descriptive and welfare analysis. In this subsection , we ask whether the steady state is stable, or does the aggregate behavior follow some other temporal pattern?

It turns out that the answer to these questions depend on what assumptions we make not only about sample size but more importantly on the *lag structure* of observations. We shall consider two distinct lag structures. In the first, which we call "Ford," each principal takes all $k > 1$ observations from the period in which he is young. In the second, "Santayana," case, the principals take one observation from each of the $k$ periods prior to the one in which they offer a contract. We have a complete characterization of Ford, and a partial one for Santayana ($k = 2$) , though numerical simulations (and intuition) suggest that the results don't change for higher $k$ when the number of states is 2.

Let $x_t^s$ = fraction of the population offering $C_0$ at $t$ in state $s$. We assume as before that $\mu^0$ is constant over time and common for the population. We maintain the standard equilibrium assumptions, though it is worth emphasizing that away from a steady state this means in particular that individuals are aware of $t$ and $x_t^s$.[11] We begin with a

**Lemma 6** *Consider the dynamical system*

$$x_{t+1} = F(x_t) \equiv (1 - x_t)^k, k > 1$$

*There is a unique steady state* $\bar{x}_k < 1/2$, *which is unstable, and a limit cycle with basin of attraction* $[0, 1] \setminus \{\bar{x}_k\}$.

The uniqueness of the steady state, along with the fact that is less than $1/2$, has already been established. The presence of a period-2 cycle is easy: if $x_t = 1$, then $x_{t+1} = 0$, $x_{t+2} = 1$, etc. That this cycle is an attractor is proven in the Appendix.

**Ford** Apprentices born at $t$ draw all $k$ of their observations from principals active at $t$. Consider the following strategy. Those who observe $A$ in their samples offer $C_1$ (they know the state is 1). Those who never observe $A$ offer contract $C_0$. This strategy is optimal whenever $\mu^0/[\mu^0 + (1 - \mu^0)(1 - x_t^1)^k] \geq \mu_0^*$, i.e., when $x_t^1$ exceeds some minimum value $\underline{x}_k$ (which is 0 if $\mu^0 \geq \mu_0^*$ and positive if $\mu^0 < \mu_0^*$). In this case, we have $x_{t+1}^0 = 1$ (where it remains forever after), while $x_{t+1}^1 = (1 - x_t^1)^k$. If instead $x_t^1 < \underline{x}$, then it is optimal to offer $C_1$ in period $t + 1$ regardless of whether $A$ is observed.

We thus have two sorts of long run behavior, depending on the relation between $\mu^0$ and $\mu_0^*$.

**In the case** $\mu^0 < \mu_0^*$**,** then in state 0 only $C_1$ contracts are offered ($x_t^0 = 0$ for all $t$). In state 1, if $x_t^1 < \underline{x}$ for some $t$, then $x_{t+1}^1 = 0$ from which it follows that $x_{t'}^1 = 0$ for all $t' > t$. And if the economy starts with $x_t^1 \geq \underline{x}$, then from Lemma 6 the sequence $x_{t+i}^1$, $i = 1, 2, 3, ...$must eventually (in finite time) fall below $\underline{x}$ as it approaches the 0-1 limit cycle; thus eventually the economy enters the bureaucratic equilibrium described above. (The only exception is if it starts out at the steady state loophole equilibrium – which exists if and only if $\bar{x}_k \geq \underline{x}$ – in which case it remains there, but that is unstable, as we have seen.)

**In the case** $\mu^0 \geq \mu_0^*$, then $\underline{x} = 0$. In state 0, everyone offers $C_0$. In state 1, anyone who does not observe $A$ will offer $C_0$, and we have $x_{t+1}^1 = (1 - x_t^1)^k$; as

[11] In fact, for the analysis we conduct, it will become apparent that individuals need not know this much in order to form their strategies.

long as $x_t^1 \neq \bar{x}_k$, the economy converges to the limit cycle. This is a loophole equilibrium of sorts, though rather than having a constant fraction of the population offering loopholey contracts every period (as in the steady state, which exists but is unstable), *the whole population is eventually synchronized,* alternately offering $C_0$ then $C_1$.

The reason this state-1 cycle is stable is that if $x$ is initially small, it is unlikely that anyone will observe the $C_0$ firms and see cheating; thus most principals will be convinced to offer $C_0$ contracts; but the next generation is now *likely* to see cheating, making the ensuing fraction of $C_0$ shops small, and so on. The fact that a successful incentive scheme by definition masks undesirable behavior makes it all the more likely that the possibility of that behavior will be "forgotten," not just by individual lineages, but by the population as a whole, contributing to its widespread re-emergence in the future.

To summarize, we have the following

**Proposition 7** *If $k > 1$, then under the "Ford" (concurrent-sampling) assumption:*
*(1) The loophole steady state, if it exists, is unstable*
*(2) If $\mu^0 < \mu_0^*$ and the economy does not start out at the loophole steady state, it reaches the bureaucratic steady state in finite time*
*(3) If $\mu^0 \geq \mu_0^*$ and the economy does not start out at the loophole steady state, then it converges to a period-2 loophole cycle, in which the entire population offers $C_0$ in state 0 and alternates between offering $C_0$ and offering $C_1$ in state 1.*

**Santayana** Suppose that instead of taking their observations simply from concurrent experience, the apprentices take a more historical view, drawing one observation from each of the last $k-1$ periods as well as their own. It is easy to see that the 2-period cycle in state 1 cannot persist as an equilibrium as long as $k > 1$. For if everyone offers $C_0$ in one period and $C_1$ in the next, every apprentice is sure to get one observation from a $C_0$ shop and therefore learn of the feasibility of cheating, so that it can never be a best response to offer a $C_0$ contract.

In a steady state on the other hand, the population from which one samples is by definition the same no matter what period it is in, so that historical sampling and concurrent sampling will always yield the same information. Thus the steady state loophole equilibrium exists under the same conditions as were established in Proposition 5 (indeed that result is proved without reference to the temporal sampling structure). We now establish a stronger result, namely that the loophole steady state is stable starting from any point in the interior of the unit square. Consider first the case $\mu^0 \geq \mu_0^*$.

Formally, the probability that an apprentice observes no $A$ in state 1 is $\Pi_{i=1}^{k}(1-x_{t-k+i}^{1})$; his posterior after such an observation is $\frac{\mu^0}{\mu^0+(1-\mu^0)\Pi_{i=1}^{k}(1-x_{t-k+i}^{1})} \geq \mu^0 \geq \mu_0^*$. So $\Pi_{i=1}^{k}(1-x_{t-k+i}^{1})$ of the population offer $C_0$ at $t+1$, while the rest offer $C_1$. In state 0, of course, no one observes $A$ and so everyone offers $C_0$.

Note first that in state 1, there is a cycle of period $k+1$ : the economy spends $k$ periods with no $C_0$ contracts offered; in the ensuing period, everyone will have observed no $A$, conclude they are likely in state 0 and offer $C_0$, so that $x=1$. This of course reveals the state is 1 to everyone in the next $k$ periods, so that $x=0$ then, and the cycle continues.

This cycle serves to illustrate once again the way information is revealed in this economy. After a sufficient time in which the correct contracts are being offered, the weight of evidence, which precisely because of the correct design of incentive schemes conceals information about feasible actions, serves to convince the population that the expense of protecting against those actions is not worthwhile, and thus loopholes are offered.

The basin of attraction for this cycle to turns out to be very small, however.[12] Rather, the long run behavior of the Santayana economy is better described by the steady state equilibrium, with individual but not aggregate cycling, that was discussed in the previous section.

The dynamics are given by $x_{t+1} = S\left(x_t, x_{t-1}, ..., x_{t-k+1}\right) \equiv \Pi_{i=1}^{k}(1-x_{t-k+i}^{1})$. Clearly, $\bar{x}_k$ is (the unique) steady state of $S(\cdot)$. Denoting $x_t - \bar{x}_k$ by $z_t$, the linearization of $S(\cdot)$ about $\bar{x}_k$ is

$$z_{t+1} = -(1-\bar{x}_k)^{k-1}\sum_{i=0}^{k-1} z_{t-i}.$$

For $k=2$, the eigenvalues of the matrix representation of this equation are readily computed to have norm $1-\bar{x}_2 < 1$, so the steady state is locally stable. This is already a departure from the Ford case, where $\bar{x}_2$ is unstable. In fact we can show much more:

[12]Though larger than the orbit itself, the basin is a strict subset of the boundary of the unit square.

**Proposition 8** *When principals are all Santayanas, any sequence of population frequencies $x_t$ of contract $C_0$ with initial conditions $(x_0, x_1, ..., x_{k-1})$ in $(0,1)^k$ converges to the steady state $\bar{x}_k$.*

The proof is somewhat lengthy and is deferred to the Appendix.

Why are cycles unstable under historical sampling? In contrast to the Ford case in which sampling is concurrent, a Santayana's learning is not particularly sensitive to the time in which he is born. A Ford who is born in state 1 when many $C_0$ contracts are being offered will be very likely to learn the state is 1, since there will be many instances of cheating agents. In response, many $C_1$ contracts will be offered next period. That in turn makes observing $A$ very unlikely for the next generation, leading to a large number of $C_0$ offers, and so on. Thus concurrent sampling will tend toward aggregate volatility.

Such volatility does not survive when everyone is a Santayana. Compare one who is born when few $C_0$ contracts to one who is born subsequently when many are offered. They both look back at a history that is broadly similar: the latter day apprentice still samples from the (previous) low-$x$ period, as does his predecessor, for whom it is contemporaneous. Before that, history looks nearly the same to both (it is one period shorter for the latter day apprentice). Thus despite the difference in $x$ at the times of their births, they will be likely to behave similarly, offering $C_0$ contracts with similar probabilities. Thus differences in $x$ across periods do not persist. Historical sampling has a dampening effect on aggregate fluctuations in organizational form.

Lastly, in the case $\mu^0 < \mu_0^*$, the bureaucratic equilibrium exists as before. But if a loophole steady state exists with $\bar{x}_k > \underline{x}$, the bureaucratic equilibrium will have a smaller basin of attraction than in the Ford case. For since the steady state is stable starting from $(0,1)^k$ under historical sampling, there is an open set of initial conditions from which the trajectories will never fall below $\underline{x}$ and will instead converge to $\bar{x}_k$.

**Welfare** Comparing the expected payoff of Ford and Santayana principals over the long period, we see that when $\mu^0 \geq \mu_0^*$, the Santayanas are better off if $k \geq 2$ : since they converge to $\bar{x}_k$ in state 1, they offer the loophole contract only $\bar{x}_k < 1/2$ of the time, whereas the Fords in their 2-cycle do so half the time (both offer $C_0$ in state 0). In this sense, the volatility that results from ignoring history is welfare reducing.[13] Moreover, not only does historical sampling lead to higher welfare of loophole equilibria than concurrent sampling, it also is less likely to result in the (lower welfare) bureaucratic equilibrium when $\mu^0 < \mu_0^*$.

#### 4.0.6 Extensions

**Increasing the number of states** As the dimension of the world increases, the state space rapidly grows large. In fact, since with $n$ possible cheating dimensions, any subset (including the empty one) of those $n$ dimensions might be feasible, the state space has $2^n$ elements. Rather than attempt an exhaustive analysis, we will merely explore some of the possibilities.

It will be particularly illuminating to consider a special case in which the state space is a restricted to contain the following $n + 1$ elements: $\varnothing$ (i.e., cheating is not feasible), {1}, {1,2}, {1,2,3}, .., {1,2,3,...,n}. The agents' marginal utility for $A_i$ is $\alpha_i$ and their cost function is $\frac{1}{2}(e_B + \sum_i^{n-1} e_{A_i})^2$. Each of the $\alpha_i$ is in $(R/2, R)$, so that it is both efficient and costly to deter each type of cheating. We also assume that $\alpha_i$ is decreasing in $i$. Thus, agents will prefer to cheat first in dimension 1, then in dimension 2, etc. All of this is assumed common knowledge; only the actual state is unknown to the principals.

Principals may deter $A_i$ (and $A_{j,}$ $j > i$) by paying a success wage $w \geq \alpha_i$; the minimum wage that will deter all cheating is then $\alpha_1$. There would be no need for further analysis if this was only means of controlling cheating, since states 1 through $n$ could be treated together and treated as our previous state 1. Things become more interesting with clause contracts. In general there will be an $m < n$ for which $\frac{R^2}{4} - nc > \alpha_j(R - \alpha_j) - (j-1)c$, for all $j < m$ but $\frac{R^2}{4} - nc \leq \alpha_m(R - \alpha_{_m}) - (m-1)c$: at some point it's better to pay the agent a larger rent than to try to deter all conceivable misbehaviors with rules. This keeps the effective dimension of the problem down to $m$. Here we take $m = 2$.

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[13]The Santayana cycle yields even higher welfare: in it, the incorrect contracts are offered only $1/(k + 1)$ of the time, which is less than $x_k$.

To see this, note that for the map $F(\cdot)$ defined in Lemma 6, $F^2(x) = [1-(1-x)^k]^k$ has derivative $k^2[1-(1-x)^k]^{k-1}(1-x)^{k-1}$; at $\bar{x}_k$, this is just $[k(1-\bar{x}_k)^{k-1}]^2$. As mentioned in the proof of that Lemma, $F^2$ has three fixed points, $0, \bar{x}_k$, and 1; its derivative is 0 at 0 and 1 and therefore $[k(1-\bar{x}_k)^{k-1}]^2 > 1$ or $k(1-\bar{x}_k)^{k-1} > 1$. Since $(1-\bar{x}_k)^{k-1} = \bar{x}_k/(1-\bar{x}_k)$, it follows that $\bar{x}_k > \frac{1}{1+k}$.

There will now be three state optimal contracts, $C_0$ for the state in which no cheating is possible, $C_1$ in which only the first type of cheating is feasible, and finally $C_2$ when both types can happen, with corresponding profits $\pi_0 > \pi_1 > \pi_2$. Denote the weight the posterior beliefs place on states 0 and 1 by $\mu_0$ and $\mu_1$; the belief space is now cut into three regions in which posteriors must sit in order for each of the contracts to be offered.

As before, there will be several types of equilibria depending on parameters. Here we focus on a particular loophole equilibrium in which all three contracts are offered in state 2. In state 0 apprentices never observe $A_1$ or $A_{2.}$ In state 1, they may observe $A_1$ (if they draw a $C_0$ shop). Finally, in state 2 they may observe neither $A_1$ nor $A_2$, $A_1$ but no $A_2$, or $A_2$. Letting $x_i$ denote the frequency of $C_0$ state $i$, and $y_i$ the frequency of $C_1$ in state $i$, we have the following

**Proposition 9** *There exist priors $(\mu^0, \mu^1)$ such that the following is a steady state: strategies are*
*Offer $C_0$ if never observed $A_1$ or $A_2$*
*Offer $C_1$ if observed $A_1$ but never $A_2$*
*Offer $C_2$ if ever observed $A_2$,*
*and the outcome is: In state 0, only $C_0$ is offered; in state* 1, *$C_0$ and $C_1$ offered by different principals; in state 2, all three contracts are offered by some part of the population.*

The proof is straightforward, and simply requires calculating steady-state values of $(x_i, y_i)$ resulting from the strategies and the corresponding posteriors corresponding to the values of $(x_i, y_i)$ to ensure optimality of the strategies. Details are omitted from this draft.

Within a lineage, there will continue to be (stochastic) "cycling" from one period to the next. Thus there will continue to be heterogeneity across organizations in states 1 and 2. Moreover, in state 2, if one assumes that each apprentice samples from his predecessor organization, there will be a progression of contract forms within each organization. For instance if $k = 1$, then $C_0$ will be followed by $C_1$ followed by $C_2$: the contracts get increasingly "complex" (at least if they are all clause contracts); but after that they return to the "simple" $C_0$ form.[14]

[14] It is also possible that the progression will be through a series of increasingly complex clause contracts, followed by simple high wage contract, and finally back to a simple loophole contract.

More interestingly, consider what happens when apprentices also sample from other organizations (so that $k > 1$). Then an apprentice who grew up in a $C_0$ shop but who stumbles into only $C_2$ shops on his travels will close the loophole (in which $A_1$ was the agent's response) by offering $C_1$, thereby revealing another loophole. But an apprentice who happened to enter a $C_1$ will know about $A_2$, and will offer a $C_2$. Thus not only is there heterogeneity across fundamentally similar organizations, but *there will be heterogeneous responses by similar organizations facing similar problems.* This is certainly consistent with evidence that heterogeneity in "managerial fixed effects" plays a significant role in the differential performance of organizations (Bertrand and Schoar, 2003): in this case, the differences in the early experiences of principals lead to differences in beliefs and therefore in the decisions they make.

If principals don't observe their predecessors, but only take over organizations from outside, then there is the possibility that the same organization will have persistent loopholes, since each period the new principals may not believe sufficiently in state 2 based on what they have observed. By contrast, those who come from within will always offer (possibly unsuccessful) solutions.

**Allowing for Agents to take a while to discover how to cheat** The assumption that agents always discover the way to cheat may seem somewhat stylized, so it is worth asking what happens if they do so only sometimes. We offer a brief description of what happens here.

Suppose in state 1 the agent discovers $A$ only with probability $p$; discovery is independent across agents and periods. To avoid the trivial case in which the principal always offers $C_0$,tolerating occasional cheating, assume $\mu_0^* > 1-p$. With $x$ as the fraction of $C_0$ contracts offered in state 1, a sampled firm will now reveal $A$ with probability $1-px$ instead of $1-x$.The analysis is similar to before except that the Bayesian update for Pr(0|Observe no A) is just $\frac{\mu^0}{\mu^0+(1-\mu^0)(1-px)^k}$ and the new critical prior $\mu_p^*$ is just $\frac{\mu^*-1+p}{p} < \mu^*$. This changes little in terms of the possibility of loophole equilibria, though principals now need to be less optimistic in order to offer loopeholey contracts – they won't be victimized as often. A loophole steady state still exists.

However it does change the dynamics somewhat. First of all, a principal may offer a loopholey contract and be lucky enough to avoid having his agent cheat. An ensuing principal is therefore less likely to observe information that reveals the state is 1. In other words, a contract appears to work for a while, but then begins to break down. So the rate at which contracts change is, not surprisingly, lower. Since $(1 - px)^k$ is decreasing in $p$, the (unique) steady state value of $x$ is larger, the smaller is $p$.

At the aggregate level, things depend somewhat delicately on $p$. If $p$ is close to 1, then under the Ford assumption, the limit cycle still exists, but will have a smaller amplitude. As $p$ decreases, eventually the limit cycle disappears, and the steady state becomes globally stable. The reason is similar to the one giving stability of the steady state under Santayana that we had before: when agents are less likely to cheat, changes in the number of $C_0$ contracts has less impact on what apprentices see, so their behavior is less responsive to those changes. Under the Santayana assumption, the steady state is stable as before.

So aggregate cycling appears to be most likely when agents are significantly better informed than principals ($p$ close to 1) and when principals ignore history. Otherwise, the aggregate behavior is better described by our steady state, in which lineages but not the population as a whole change behavior over time.

**Endogenous Sampling Structures** So far we have assumed that $k$ and the temporal structure of samples are exogenous. But of course, principals might *choose* both the size of the sample and whether or not to ignore history.

On the sample size front, obviously it is costly to acquire one more observation. If everyone had a very large $k$, then in the loophole steady state, one more observation is not worth much, since the chance of seeing $A$ is very small. But then no one should choose a large $k$. Likewise, if everyone had a low $k$, the informational value of another observation would be quite large. This suggests there will be some equilibrium value of $k$ that is not excessively large. Our conclusions should go through with little modification.

Things are more interesting when one considers endogenizing the choice of temporal structure. In fact, in the model we have considered so far with $k \geq 2$, and the states perfectly correlated over time, if everyone else was a Ford, then since the economy would be cycling in aggregate, it would be informationally more valuable from the individual point of view to look historically. In other words, in an extension of the game in which individuals first choose whether to be a Ford or Santayana, it cannot be an equilibrium for everyone to be a Ford. Similarly, if there is nearly zero correlation across periods, it cannot be an equilibrium for everyone to be a Santayana. In the general case, there will be a mix of Fords and Santayanas, with an accompanying limit cycle if the correlation is moderately low (so that Fords predominate). In this case we get the result that a moderate level of fundamental volatility engenders high volatility in the population distribution of incentive schemes.

# 5 Conclusion

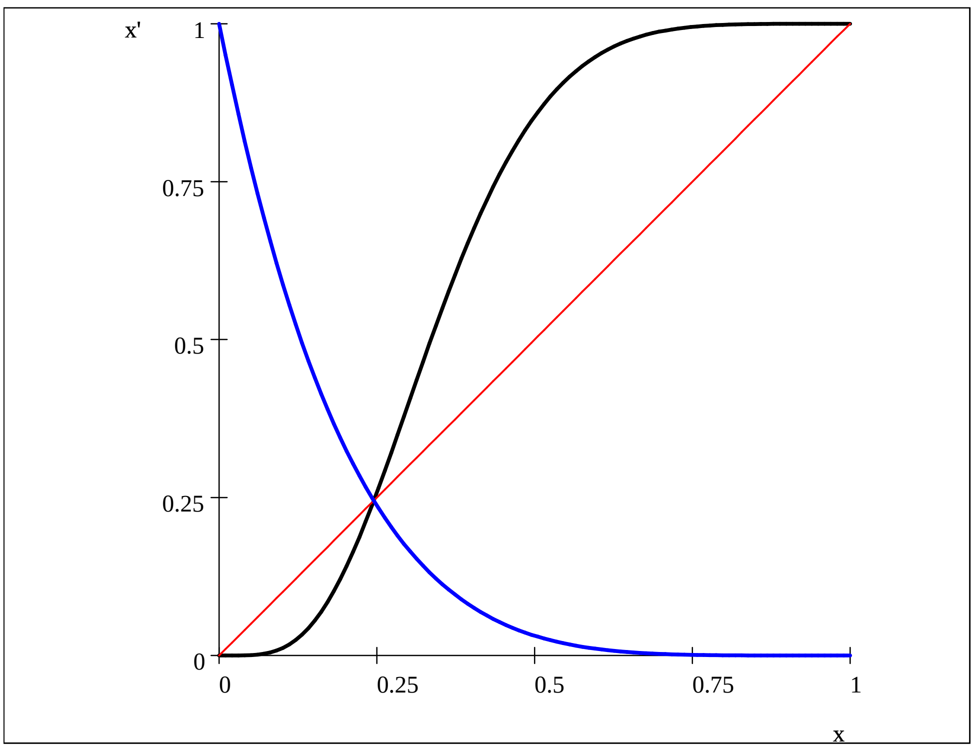
This paper has explored the possibility that incomplete knowledge of the contracting environment that is remediated through social learning may serve as an engine of contractual and organizational change. The key observation is that a "tight" contract hides information about the environment, inhibiting social learning and opening the door for future "loopholey" contracts. Very large sample sizes apart, the learning process cannot converge to everyone getting things right. On the contrary, getting things right sows the seeds of its own destruction, and the model generates the possibility of "cycling" and organizational heterogeneity.

A number of further extensions are possible. We might want to consider more general contracting environments – risk aversion, multiple agents, or the possibility that agents' outside options bind. The last case raises the interesting possibility that agents' opportunities affect the amount of learning: in booms, when they are high, principals are forced to pay high wages to attract agents, misbehavior is deterred, regardless of principal's beliefs, and there is no learning. Thus the amount of cheating, learning, and organizational cycling and/or heterogeneity will tend to ebb and flow with the business cycle.

We also want to consider larger state spaces. Some empirical forms of "organization" (e.g. regulatory settings – regulate or not, executive compensation–stock options or not) do seem to conform to our low period cycles. In other cases, (human resource management strategies) what is more striking is the rich set of schemes that have been tried over time, and how those tend to evolve without obvious repetition (piece rates being a possible exception). Large state spaces allow for the latter possibility (period length longer than the age of the universe) and more important for understanding what factors determine whether cycles or short or long.

# 6 Appendix

**Proof of Lemma** 6. Let $F(x_t) = (1-x_t)^k$. Then $x_{t+1} = F(x_t)$ and $x_{t+2} = F^2(x_t) \equiv F(F(x_t)) = [1-(1-x_t)^k]^k$. These two functions are plotted in the figure. It is straightforward to verify that $F(0) = 1 = 1 - F(1)$, $F$ is decreasing, and has a unique fixed point $\bar{x}_k$. Meanwhile $F^2$ is increasing with fixed points $0, \bar{x}_k$, and 1. Simple calculations show there is a unique $\hat{x}_k$ such that $F^2$ is strictly convex on $(0, \hat{x}_k)$ and strictly concave on $(\hat{x}_k, 0)$. Thus $F^2$ has no other fixed points in the unit interval, and $F^2(x) < x$ in $(0, \bar{x}_k)$ and $F^2(x) > x$ in $(\bar{x}_k, 1)$, as depicted in the figure.




Observe that there is a cycle of period 2: if $x_t = 1$, then $x_{t+1} = 0$, $x_{t+2} = 1$, etc. (In fact, the inflection point $\hat{x}_k = 1 - \left(\frac{1}{k+1}\right)^{1/k}$ exceeds $\bar{x}_k$, which follows from the argument in footnote 13.)

We show that all trajectories starting away from the point $\bar{x}_k$ converge to the 2-cycle $(0, 1, 0, 1, ...)$. Let $x_0 \in (0, \bar{x}_k)$. Then $x_1 > \bar{x}_k$ (since $F(x) > \bar{x}_k$ on $(0, \bar{x}_k)$) . But the "even iterates" $x_2$, $x_4$,..given by $F^2(x_0)$, $F^4(x_0)$,...converge monotonically to 0. Meanwhile the odd iterates $x_1, x_3, x_5, ... = F(x_0), F^2(F(x_0)), F^4(F(x_0))$,... converge monotonically to 1. Thus, the sequence $\{x_t\}$ converges to the 2-cycle. A similar argument holds for $x_0 \in (\bar{x}_k, 1)$; thus any solution originating in $(0,1)\backslash\{\bar{x}_k\}$ converges to the 2-cycle.

**Santayana Dynamics and Proof of Proposition 8**

We study the equation

$$x_{t+1} = \prod_{i=1}^{k} (1 - x_{t+1-i}).$$

To save some notation, it will be convenient to study the dynamics of $y = 1 - x$; our equation becomes

$$y_{t+1} = 1 - \prod_{i=1}^{k} y_{t+1-i}. \tag{1}$$

Note that $y = 1 - y^k$ has a unique solution $\bar{y}$ in $[0,1]$ that is the (unique) steady state of (1), and that $\bar{y}$ is increasing in $k$; we suppress this dependence to save notation. As pointed out in the text, there are periodic solutions to this equation with $y_t = 0, y_{t+i} = 1, i = 1, ..., k$. However for solutions originating within the interior of the unit cube, all trajectories converge to $\bar{y}$, as we will now show.

Denote by $\bar{C}$ the unit cube $[0,1]^k$ and $C$ its interior. Write $\overline{Co}(X)$ for the convex hull of an arbitrary $X \subset \bar{C}$ and $Co(X)$ for its interior. Let initial conditions $\mathbf{y}_0 = (y_{-k+1}, y_{-k+2}, ..., y_0)$ be given; a solution $\mathbf{y}(\mathbf{y_0}) =\{y_{-k+1}, ..., y_0, y_1, y_2, ...\}$ with $\mathbf{y}_0 \in Y \subseteq \bar{C}$ is said to *originate* in $Y$. Call a finite sequence of consecutive $y$ values $\{y_t, y_{t+1}, ..., y_{t+n}\}$ that are part of a solution an *$n$-string*. An $n$-string $\{y_t, y_{t+1}, ..., y_{t+n}\}$ in which all elements weakly exceed (are less than or equal to ) $\bar{y}$, with at least one inequality strict, and for which $y_{t-1}, y_{t+n+1} < \bar{y}$ ($y_{t-1}, y_{t+n+1} > \bar{y}$) is called a positive (negative) *semicycle*. We provide a series of lemmas that help to characterize any solution originating in $C$.

**Lemma 10** $\mathbf{y}_0 \in C$ *implies* $y_t \in C$ *for all* $t \geq 1$.

**Lemma 11** *No semicycle exceeds* $k$ *in length.*

**Proof.** Note that $y > \bar{y}$ if and only if $1 - y^k < \bar{y}$. Thus for a $k$-string $\{y_{t+1}, ..., y_{t+k}\}$ that is part of a positive semicycle with $y_{t+i} > \bar{y}$ for at least one $i \in \{1, ..., k\}$, we have $y_{t+k+1} = 1 - \prod_{i=1}^{k} y_{t+i} < 1 - \bar{y}^k = \bar{y}$; the negative semicycle case is similar. ■

**Lemma 12** *Let* $\mathbf{y}_t^{k+1} = \{y_t, y_{t+1}, ..., y_{t+k}\}$ *be a* $k+1$*-string. Then* $\bar{y} \in Co(\mathbf{y}_t^{k+1}) \subset (0,1)$.

**Proof.** If $\bar{y} \in \mathbf{y}_t$, there is nothing to prove. In the other case, from the previous lemma, $\mathbf{y}_t$ cannot contain elements that all exceed $\bar{y}$ or are all less than $\bar{y}$, else there would be a semicycle of length greater than $k$. So there are $y_{t+i}, y_{t+j} \in \mathbf{y}_t$ with $y_{t+i} < \bar{y} < y_{t+j}$, $i, j \in \{0, ..., k\}$. Thus $\bar{y} \in (y_{t+i}, y_{t+j}) \subseteq Co(\mathbf{y}_t) \subset (0,1)$, where the second inclusion follows from lemma 10. ■

**Lemma 13** *Let* $\{y_t, y_{t+1}, ..., y_{t+k}\}$ *be a* $k+1$*-string, and let* $M$ *and* $m$ *be its maximal and minimal elements. Then*
*(a)* $y_{t+k+1} \in [m, M]$
*(b)*$\min\{y_t, y_{t+k}\} \leq y_{t+k+1} \leq \max\{y_t, y_{t+k}\}$.

**Proof.** We first show that $y_{k+1} \leq M$. Assume instead that $y_{t+k+1} > y_{t+i}$, $i = 0, 1, ..., k$. Since $y_{t+k} = 1 - y_t y_{t+1}..y_{t+k-1}$ and $y_{t+k+1} = 1 - y_{t+k} y_{t+1}...y_{t+k-1}$, the case $i = k$ is equivalent to $y_{t+k} < y_t$ or

$$1 - \pi_t < y_t \tag{2}$$

where $\pi_t = y_t y_{t+1}..y_{t+k-1}$.

For the case $i = 0$, note that $y_{t+k+1} > y_t$ can be written $1 - \frac{(1-\pi_t)\pi_t}{y_t} > y_t$, or

$$(1 - \pi_t)\pi_t < y_t(1 - y_t) \tag{3a}$$

We now show that 2-3a are inconsistent. First, we cannot have $\pi_t = 1/2$, since then $y_t(1 - y_t) \leq (1 - \pi_t)\pi_t$, violating (3a). If $\pi_t < 1/2$, then (3a) implies $1 - \pi_t > 1 - y_t$ and $1 - \pi_t > y_t$, and the latter violates (2). And if $\pi_t > 1/2$, then (3a) implies implies that $\pi_t > y_t$, which contradicts lemma 10.

Similarly, we can show that $y_{t+k+1} \geq m$. If instead $y_{t+k+1} < y_{t+i}$, $i = 0, 1, ..., k$, then we must have from $y_{t+k} > y_{t+k+1}$

$$y_t < 1 - \pi_t \tag{4}$$

and from $y_t > y_{t+k+1}$

$$(1-y_t)y_t < \pi_t(1-\pi_t) \tag{5}$$

By an argument similar to the one used to show $y_{t+k+1} \leq M$, (4) and (5) are inconsistent and we conclude $y_{t+k+1} \geq m$. This establishes (a). ■

Since only the inequalities involving $y_t$ and $y_{t+k}$ have been used, the argument also establishes (b).

**Lemma 14** *If $y_{t+k+1} = y_t$ then $y_{t+k} = y_t$; if $y_{t+k+1} = y_{t+k}$ then $y_{t+k+1} = y_t$*

**Proof.** $y_{t+k+1} = y_t$ is equivalent to $y_t(1-y_t) = \pi_t(1-\pi_t)$ using the previous notation. From lemma 10 we have $\pi_t < y_t$. Thus, we must have $\pi_t = 1 - y_t$. Since $y_{t+k} = 1 - \pi_t$, we establish the first claim. The second hypothesis is equivalent to $1 - \frac{\pi_t(1-\pi_t)}{y_t} = y_{t+k} = 1 - \pi_t$, whence $\frac{1-\pi_t}{y_t} = 1$,therefore $y_t = y_{t+k} = y_{t+k+1}$, as desired. ■

Assume now that our solution $\mathbf{y}$ originates in $C$. Define $H_t = \overline{Co}(\mathbf{y}_t^{k+1})$ for every $k+1$-string $\mathbf{y}_t^{k+1}$ that is part of $\mathbf{y}$. By lemma 13, $H_{t+1} \subseteq H_t$. Denote $H = \bigcap_{t=1}^{\infty} H_t$, which is closed, convex, and, by lemma 12, contains $\bar{y}$. Assume that $H = [l, L]$, where $l \leq \bar{y} \leq L$. Since the $H_t$ are nonincreasing in the set inclusion order, we must have $l > 0$ and $L < 1$. We will now show that $L = \bar{y}$; combined with a similar argument that shows $l = \bar{y}$, we will thereby establish

**Proposition** 8. *When principals are all Santayanas, any path of population frequencies of $C_1$ that originates in $C$ converges to the steady state $\bar{y}$.*

The proof reasons on the accumulation points of the sequence of $k+1$ strings of $\mathbf{y}$, considered as points in $(0,1)^{k+1}$ . Denote a typical element of the sequence by $\sigma^t = (y_t, y_{t+1}, ..., y_{t+k})$, where by definition $y_{t+k} = \hat{S}(y_t, y_{t+1}, ..., y_{t+k-1}) = 1 - y_t y_{t+1} ... y_{t+k-1}$. All accumulation points of the sequence $\{\sigma^t\}$ (which exist since $\{\sigma^t\}$ is a subset of the a compact set) are contained in $H^{k+1}$. Since every $\sigma^t$ has a component that is a maximal element $M_t$ of the corresponding $k+1$-string, and the maxima $M_t \to L$, $\bar{\sigma}$ has at least one component equal to $L$. Since $H = [l, L]$, we must have $\sigma_i \leq L$, $i = 0, 1, ..., k$.

We now show

**Lemma 15** *For any accumulation point $\bar{\sigma} = (\bar{\sigma}_0, \bar{\sigma}_1, ..., \bar{\sigma}_k)$ of $\{\sigma^t\}$, we have $\bar{\sigma}_k = L$.*

**Proof.** Suppose not. Let $\{\sigma^{t_n}\} = \{(y_{t_n}, y_{t_n+1}..., y_{t_n+k})\}$ be a subsequence converging to $\bar{\sigma}$. Begin by assuming $\bar{\sigma}$ has exactly one component $\bar{\sigma}_j = L$, $j < k$. By continuity of $\hat{S}(\cdot)$, $y_{t_n+k+1} = \hat{S}(y_{t_n+1}..., y_{t_n+k})$ converges to a limit $\bar{\sigma}_{k+1} = \hat{S}(\bar{\sigma}_1, ..., \bar{\sigma}_k)$ and thus $\bar{\sigma}' = (\bar{\sigma}_1, ..., \bar{\sigma}_k, \bar{\sigma}_{k+1})$ is also an accumulation point of $\{\sigma^t\}$, the limit of the shifted subsequence $\{\sigma^{t_n+1}\} = \{(y_{t_n+1}..., y_{t_n+k+1})\}$. Since $\min\{y_{t_n}, y_{t_n+k}\} \leq y_{t_n+k+1} \leq \max\{y_{t_n}, y_{t_n+k}\}$ by lemma 13(b), we have $\min\{\bar{\sigma}_0, \bar{\sigma}_k\} \leq \bar{\sigma}_{k+1} \leq \max\{\bar{\sigma}_0, \bar{\sigma}_k\}$. Since $\bar{\sigma}_k < L$, we cannot have $\bar{\sigma}_{k+1} = L$ unless $\bar{\sigma}_0 = L$; but then by lemma 14, we have $\bar{\sigma}_k = L$, a contradiction; thus $\bar{\sigma}_{k+1} < L$. Since $\bar{\sigma}$ has at most one component equal to $L$, so does $\bar{\sigma}'$.

Continuing in this way through the shifted subsequences $\{\sigma^{t_n+i}\}$, we arrive at $\{\sigma^{t_n+j}\}$ whose limit is $(\bar{\sigma}_j, \bar{\sigma}_{j+1}, ..., \bar{\sigma}_{j+k})$, with $\bar{\sigma}_{j+i} < L$, $i = 1, ..., k$. Then $\bar{\sigma}_{j+k+1} < L$, else by lemma 14 $\bar{\sigma}_{j+k} = L$, a contradiction. But then $(\bar{\sigma}_{j+1}, \bar{\sigma}_{j+1}, ..., \bar{\sigma}_{j+k+1})$ is an accumulation point of $\{\sigma^t\}$ with all components less than $L$, which contradicts that $H = [l, L]$. We conclude that we cannot have an accumulation point with $\bar{\sigma}_k < L$ that has only one component equal to $L$.

Suppose now that if $r$ components of $\bar{\sigma}$ are equal to $L$ with $\bar{\sigma}_k < L$, then there is an accumulation point $\bar{\sigma}'$ with $r - 1$ components equal to $L$ with $\bar{\sigma}'_k < L$, where $2 \leq r \leq k$. Since this implies by induction that there is an accumulation point $\bar{\sigma}''$ with one component equal to $L$ and $\bar{\sigma}''_k < L$, we have a contradiction, and we conclude that $\bar{\sigma}_k = L$.

For the inductive step, starting from $\bar{\sigma}^0 = (\bar{\sigma}_0, ..., \bar{\sigma}_k)$, construct accumulation points $\bar{\sigma}^j = (\bar{\sigma}_j, \bar{\sigma}_{j+1}, ..., \bar{\sigma}_{j+k})$, $j = 1, ..., k$ as above. Let $\underline{j}$ be the lowest $j$ st. $\bar{\sigma}_j = L$. For $j < \underline{j}$, $\bar{\sigma}^j$ has $r$ components equal to $L$ and the $k^{th}$ component less than $L$. And $\bar{\sigma}_{\underline{j}+k+1} < L$, else by lemma 13(b) $\bar{\sigma}_{\underline{j}+k} = L$, so that $\bar{\sigma}^{\underline{j}}$ has $r + 1$ components equal to $r$, a contradiction. But then $\bar{\sigma}^{\underline{j}+1} = \left(\bar{\sigma}_{\underline{j}}, \bar{\sigma}_{\underline{j}+1}, ..., \bar{\sigma}_{\underline{j}+k}\right)$ is an accumulation point with $r - 1$ components equal to $L$ and it's $k^{th}$ component less than $L$, as claimed. This completes the proof of the lemma. ■

Now we show that in fact there is an accumulation point $\bar{\sigma}^* = (L, L, ..., L)$. Starting with $\bar{\sigma}^0$, which by lemma 15 is of the form $(\bar{\sigma}_0, \bar{\sigma}_1..., L)$, construct the accumulation point $\bar{\sigma}^1 = (\bar{\sigma}_1..., L, \bar{\sigma}_{k+1})$; also by lemma 15 we must have $\bar{\sigma}_{k+1} = L$. After repeating this for $k$ steps, we obtain an accumulation point $\bar{\sigma}^* = (L, L, ..., L)$ as desired. Since $\bar{\sigma}^*_k = \hat{S}(L, L, ..., L) = 1 - L^k = L$, we conclude that $L = \bar{y}$.

A similar argument can be applied to $l$ to conclude that $l = \bar{y}$. Then

$H = \{\bar{y}\}$, and the proof that $y_t \to \bar{y}$ is complete.

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