DYNAMIC SPECULATIVE ATTACKS

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Abstract

The paper presents a model of rational Bayesian agents with speculative attacks in a regime of exchange rate which is pegged within a band. Speculators learn from the observation of the exchange rate within the band whether their mass is sufficiently large for a successful attack. Multiple periods are necessary for the existence of speculative attacks. Various defense policies are analyzed. A trading policy by the central bank may defend the peg if it is unobserved and diminishes the market's information for the coordination of speculators.

Key words: Speculative attack, currency crisis, coordination, informational externalities, social learning.

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Introduction

A regime of fixed exchange rates is for speculators an open invitation to a stag hunt. If they act in coordination, their combined reserves dwarf those of the central bank, thus forcing a devaluation and a distribution of good profits to the hunters. The game has been formalized by Obstfeld (1996) who also provides some empirical justification: under perfect information on the strategies and the payoffs of individuals, there are two equilibria in pure strategies, as in any standard stag hunt game: either speculators cooperate and gain, or they don't in which case there is neither gain nor loss.

Coordination games with multiple equilibria raise the issue of expectations on the choice of the equilibrium and imperfect information. Carlsson and Van Damme (1993a) and (1993b) "resolve" the issue of multiplicity of a one-period coordination game by assuming that agents have different expectations. Each agent makes his decision not knowing the actions of the others, but *educing* what others will do in a mental process which takes place before all agents make their unique decision[1]. The problem of multiple equilibria disappears because a contagion process from the agents with extreme beliefs leads all other agents either to action or to inaction.

This "global game" method has been applied to the model of Obstfeld by Morris and Shin (1998) who assume that agents have different beliefs on a fundamental parameter of the economy. A speculative attack succeeds only if the mass of hunters exceeds some level which is an increasing function of the funda-

[1] See Binmore (1987), Guesnerie (1992) who use the method in the case of strategic substitutability. For an exposition, see Chamley (2002).

mental. Such an assumption is justified by the consideration of a central bank which defends the currency only if the cost of defense (which is increasing in the mass of hunters and decreasing in the fundamental parameter), does not exceed an exogenous ceiling. The model "resolves" the multiplicity of equilibria and generates conclusions on the feasibility of fixed exchange rates. However, it does not embody some important features of currency markets.

(i) An essential property of the one-period model is that all agents have to make a decision once and simultaneously[2]. There is no interaction between the learning from others' actions and strategic decisions (*i.e.*, to speculate now or delay after some observation of others' actions). Actual currency attacks may be short but during an attack agents intensively observe the actions taken by others and react quickly to their perceptions of these actions.

(ii) The global game method requires the distribution to have a tail of agents on each side for whom the dominant strategy is to attack or not to attack the currency. In a multi-period context, one tail may disappear (*e.g.*, the tail of agents with high expectations who exercise their option to attack the currency), and the method is not applicable in the following period unless a new shock "regenerates" the tail (as in Morris and Shin, 1998). If the period is short (as it

[2] Morris and Shin (1998b) consider a sequence of one-period models with no strategic decisions. The state of nature θ_t evolves randomly from period to period. The assumption that θ_t is learned exogenously in period $t+1$ rules out social learning. In any case, the exogenous learning seems a bit slow compared to the time frame of a speculative attack. Corsetti *et al* (2000) consider the strategic behavior of a large player and infinitesimal players. Given the assumption of the model, the outcome is trivial with the large player moving first.

should be in exchange markets), such a new shock is asymptotically equivalent to a discontinuous jump of the parameters and may not be plausible[3].

(iii) A fixed cost of transaction plays a critical role in the model. When that cost is vanishingly small, the difference between the sustainable exchange rate and the fundamental exchange rate is also vanishingly small. A realistic value of this cost as a fraction of the transaction is very small for positions which last just a few days.

(iv) The model assumes that the exchange rate is fixed with absolutely no room for variations. Unless the currency belongs to a monetary zone such as the Euro between January 1, 1999 and January 1, 2002, some fluctuations as in the regime before 1999 is allowed. These fluctuations have two opposite effects. The first is to provide a channel of communication. A speculative attack typically induces a depreciation of the exchange rate within the allowed band before any devaluation. This communication increases the risk of an attack. The second effect reduces this risk by introducing a penalty if the attack fails: after a failed attack, the price of the foreign currency falls thus generating a loss for the holders of the foreign currency. This penalty is incurred without any *ad hoc* transaction cost.

(v) The payoff after the abandonment of the fixed rate is independent of the actions of the players if the attack is successful. All the bids are carried at the "old" exchange rate and the speculators gain from the devaluation. But actual

[3] For some distributions of agents however, there is a unique equilibrium even when one of the tails of dominance is erased by past learning (Chamley, 1999).

players in a dynamic game face the risk of arriving at the window too late. This is obviously the reason why they may want to rush, like in a bank run. The model should incorporate the trade-off between "going early" at a smaller cost (with a favorable exchange rate) and little information, and delaying for more information with the risk of coming too late.

(vi) In most previous models of speculative attacks, the central bank has perfect information on the state of the world. However, a critical factor for the success of a speculative attack is the distribution of characteristics of the speculators about which the central bank may not have superior information. The one-period approach leaves little room for the role of the central bank during a speculative attack.

These issues are addressed here with a new model where agents act strategically in a multi-period context. The equilibrium exchange rate is allowed to fluctuate within a band and a devaluation takes place only if the ceiling of the band is reached. As emphasized previously, any speculator in a currency attack wonders how many other agents believe that the fundamentals are such that an attack will succeed, how others think about others' beliefs, and so on. This feature is modeled here by assuming that there is a mass of speculators with relatively high beliefs that there is a large mass of agents with high beliefs and so on. The mass of these speculators is the uncertain parameter in the economy. For simplicity, there are two states of nature. In one of the two states, the "high" state, the mass of speculators is sufficiently large to induce a devaluation if all speculators buy the foreign currency while that mass is "sub-critical" in the other "low" state.

The emphasis is not on the resolution of multiple equilibria as in the global game approach, but on the opportunities offered by the dynamic setting for speculative attacks when agents have heterogeneous expectations on the mass of potential speculators. In all the cases considered here, there will be an equilibrium with no speculative attack. Under some conditions, there will also be an equilibrium with a speculative attack. The role of policy, if any, will be to abolish this second equilibrium. The strategic aspect is critical: the interesting cases occur when the parameters of the model are such that if there is one-period (and thus no opportunity to learn from others), there is a unique equilibrium with no attack[4].

Payoff externalities have a particular property here. When speculators face a coordination problem (*e.g.* attack a currency), there is some incentive to delay in order to get more information on others[5]. But there is also a premium for the agents who invest earlier in the coordination, (*e.g.* when the asset price is still relatively low in a financial market, before the bank run really starts or at the beginning of a political revolution). If the agent delays too much, he comes too late for the gain of the attack and does not get anything.

The model presented here builds on a standard structure of a financial market

[4] Gale (1995) has shown how a finite number of agents may achieve coordination in multiple periods when each agent can precipitate a subgame with fewer agents in which coordination is dominant. Rodionova and Surti (2000) analyze the case of speculative attacks with a finite number of players and Corsetti *et al.* (1999) assume one large agent who can induce small and competitive agents to act.

[5] The incentive for delay generates strategic substitutability in Chamley and Gale (1994), and strategic complementarity when agents are heterogeneous in Chamley (2000).

with noise traders. The main feature of the model is that speculators observe the exchange rate at the end of each period and place market orders for the next period. Following Hellwig (1982) and Blume, Easley and O'Hara (1994), these orders depend on all the information available at the time they are placed and the rational expectations about the price in the next period. Speculators may delay the timing of their attack. An equilibrium is constructed analytically for any finite number of periods by backward induction.

In all periods, the subgame has an equilibrium in which there is no speculation for all remaining periods. Under some conditions on the beliefs of the agents, there are other equilibria with speculative attacks. The purpose of the analysis is not to construct a model with a unique equilibrium in which there may be a speculative attack but to analyze how a speculative attack may be facilitated by the learning from prices in markets, and how such equilibria with a speculative attack can be prevented by policy. Proposition 1 shows that a higher number of periods extends the set of beliefs for which a speculative attack is an equilibrium. The equilibrium strategy which is analyzed here tends to a stationary solution when the number of periods tends to infinity (Proposition 2). A numerical example with Gaussian noise illustrates some properties of the model.

Policies are analyzed in the last section. A speculative attack can be prevented by widening the band of fluctuations or through the trading by the central bank. If the central bank intervenes by stabilizing the exchange rate (*i.e.,* selling when the exchange rate increases), and this policy is anticipated by rational speculators, a speculative attack is more likely. Speculative attacks may be prevented either by a rationally anticipated intervention which amplifies the fluctuations of the exchange rate, or by a random intervention which cannot be anticipated.

1 The model

There is a finite number of periods, $T+1$, and a continuum of speculators, called agents, of mass $\theta \in \{\theta_0, \theta_1\}$. The value of θ defines the state which is selected by nature before the first period. In each period, an agent can at most one unit of foreign currency, also called *asset*. This constraint embodies a credit constraint. At the beginning of the first period, all agents hold only the domestic currency, also called *currency*. In period $T+1$, all agents undo their position: by assumption, all agents must hold only the currency at the end of period $T+1$; if an agent holds the asset at the beginning of period $T+1$, he sells it in period $T+1$. The game is effectively played during T periods.

Market orders

At the beginning of any period, an agent who holds the currency can place a *buy* order for the asset, and an agent who holds the asset can place a *sell* order. The orders are market orders, *i.e.*, they specify a quantity to be traded (conditional on no devaluation) at whatever the market price in the period. An order depends on the information available at the beginning of the period and on the rational expectations, given that information, on the transaction price[6]. Market orders embody the sequential nature of trades. Another assumption which is used in the micro-structure of financial markets is that of limit orders. When agents can place limit orders, they submit their entire demand schedule contingent on the information revealed by the equilibrium price. Orders are executed only at the equilibrium price and agents would not change their orders after the closing of

[6] The informational properties of a financial market with market orders have been analyzed by Hellwig (1982), Blume, Easly and O'Hara (1994) and Vives (1995).

the market if they had an opportunity to do so. No rationing can occur. Such a setting is not appropriate for a situation which is similar to that of a bank run and where agents would change their trade after the equilibrium price is known.

We will consider only symmetric equilibria. Because agents are risk-neutral, the payoff of placing an order of amount $a < 1$ for the asset will be equivalent to that of an order of 1 placed with probability a. We will assume the second formulation to facilitate the presentation. In this way, an agent either holds one unit of the asset (and is an asset holder), or no amount of the asset (and is a currency holder).

The market for the asset

There is a game in period t if no devaluation (a process described below) has taken place before period t. Let λ_{t-1} be the fraction of agents who hold the asset at the beginning of period t. The mass of agents who hold the asset at the beginning of period t is therefore $\lambda_{t-1}\theta$. By assumption, no agent holds the asset at the beginning of the first period: $\lambda_0 = 0$.

We will consider only symmetric strategies: there is one strategy (possibly random) for the asset holders and one for the currency holders. Let ζ_t be the fraction of agents who place a buy order for the asset in period t. By an abuse of notation, ζ_t will define the strategy of currency holders who place a buy order with probability $\zeta_t/(1 - \lambda_{t-1})$. The strategy of the asset holders will be simpler as we will see later: if some agents buy in an equilibrium, no asset holder sells, and if some asset holders sell, they all sell. Since there will be no ambiguity, an order will be a buy order. Suppose $\zeta_t > 0$. Given the quantity of orders $\zeta_t\theta$, the demand for the asset by the agents in period t (a stock) is $(\lambda_{t-1} + \zeta_t)\theta$. The total demand for the asset in period t is the sum of the agents' endogenous demands, and of an exogenous noise η_t. The introduction of noise traders is standard in financial markets and facilitates trading between agents with asymmetric information (Grossman, 1981). The noise traders in period t undo their position at the beginning of the next period. The distribution of η_t is stationary. The terms η_t are serially independent and the mean of η_t is equal to zero.

The supply of the asset is a linear function of the price p of the asset: $S(p) = (p-1)/a$, where a is a fixed parameter. The value of S represents the net supply when the price departs from the middle of the band of exchange rate fluctuations which can be interpreted as a long-run value as determined by real trade and policy. This long-run price is an equilibrium value when there is no speculative attack, and it is normalized to 1. The supply schedule $S(p)$ can be defined as minus the net demand of risk-averse "market-makers" who place price contingent orders which may take into account the information revealed by the transaction price. Their net demand is of the form $\kappa E[p_{t+1}|p_t - p_t]/Var(p_{t+1}|p_t)$, where p_t and p_{t+1} are the prices of the asset in period t and $t+1$, and $\kappa > 0$ is a parameter. Because market makers have a lower *ex ante* subjective probability on state θ_1 than the speculators, they interpret the market data differently. (The speculators are more optimistic about their high mass). For simplicity, we assume that the market makers have *ex ante* a vanishingly low probability that there is a high mass of speculators and that a devaluation will take place. Hence, contingent on the observation of the equilibrium price, the revised probability of a devaluation is still vanishingly low: $E[p_{t+1}|p_t] = 1$ and the variance of p_{t+1} in the subjective distribution is constant. In this case, their net demand is of the form $(1-p)/a$ with $a = Var(p_{t+1}|p_t)/\kappa$. If the current price p_t has a significant impact on the expected value $E[p_{t+1}|p_t]$, a higher price p_t shifts the demand curve up, and the effect is equivalent to a less elastic demand curve. We will consider below the impact of such a lower elasticity on the properties of the model.

The schedule $S(p)$ may also incorporate the strategy of other agents whose trades depend only on p_t. The central bank is assumed to perform the function of a clearing house by matching the trade orders. The central bank may also use its reserves for trading. In that case, its net supply is incorporated in the schedule $S(p)$. Policies of the central bank will be discussed in Section 5, and no trading by the central bank will be considered until then.

The regime of the exchange rate within a band of fluctuations stipulates that the price of the asset is allowed to fluctuate in a band below a threshold value $1+\gamma$, with $\gamma > 0$. If the equilibrium price (to be defined later) is above $1+\gamma$, a devaluation takes place according to a specification which will be given later. The event of a price below $1-\gamma$ will have a negligible probability and will be ignored.

Assuming no devaluation prior to period t, let p_t be the price determined by the equation

$$p_t = 1 + a\Big((\lambda_{t-1} + \zeta_t)\theta + \eta_t\Big). \tag{1}$$

If $p_t \leq 1+\gamma$, the price which clears the demand and the supply is within the band and is equal to p_t in (refeq:equiprice). There is no devaluation in period t. All buy orders are satisfied. The fraction of agents who hold the asset at the beginning of the next period is $\lambda_t = \lambda_{t-1} + \zeta_t$.

If $p_t > 1+\gamma$, the price at which supply and demand are equal is greater than that allowed by the band of fluctuations. Let $\bar{X}$ be the critical mass of the demand (speculators and noise traders), *i.e.*, the highest value of the demand which can be accomodated by an equilibrium in the band. From equation (1), $\bar{X}$ is defined such that

$$\gamma = a\bar{X}.$$

A devaluation takes place when the demand is higher than the critical mass. All orders cannot by executed. To simplify the process, it is assumed that: first, noise traders of period $t-1$ undo their positions; second, noise traders of period t place their orders. All these orders are executed (even if the total amount exceeds the critical mass)[7]. The amount of the asset which is available for new orders without devaluation is therefore $Max(\bar{X} - \lambda_{t-1}\theta - \eta_t, 0)$. (We will see later that if new orders come in, no asset holder sells).

Suppose that the mass of new orders in period t is strictly positive: $\zeta_t > 0$. (The case $\zeta_t = 0$ will be described below). By assumption, all the agents' new orders are executed with the same probability and with the highest probability. The transaction price is the highest possible in the band, $1+\gamma$, and the probability of execution of a buy order is

$$\pi = \frac{Max(\bar{X} - \lambda_{t-1}\theta - \eta_t, 0)}{\zeta_t \theta}.$$

By construction, $0 \leq \pi \leq 1$. If a devaluation takes place, the price of the foreign asset is set at $1 + A$, where $A > \gamma$ is a fixed parameter[8].

[7] This assumption is made to simplify the process. The probability of a demand greater than the critical mass at that stage is very small.

[8] One could consider the case where the amount of the devaluation is determined by the intensity of the attack, or by an equibrium mechanism. Such an effect would enhance the strategic complementarity but would not alter the properties of the model. The issue is discussed

Information

The true state θ is not observable. At the beginning of the first period, all agents have a subjective probability μ_0 of the high state θ_1. At the end of each period t, if a devaluation takes place, the game ends. If no devaluation takes place, a subgame begins in period $t+1$. Agents observe the price p_t in period t. They use this observation to update in a Bayesian fashion their belief from μ_{t-1} to μ_t. Since the strategies are common knowledge, agents know the fraction of agents who place orders in period t. Hence, the fraction of agents who hold the asset at the beginning of the next period, λ_t, is known. We will show that the subgame which begins in period t depends only on the period t and on $(\lambda_{t-1}, \mu_{t-1})$.

Payoffs

The payoff of an agent is the sum of the discounted values of the trades in all periods, valued in the (domestic) currency. The discount factor δ, $0 < \delta < 1$, embodies a positive difference between the rate of return in the domestic currency and that of the foreign asset, for a fixed exchange rate. Such a positive difference ensures that in the context of the model, the band of the asset price is "sustainable": if there is no speculative attack, speculators prefer to hold the currency (or to sell the foreign asset which they may own), in an equilibrium. If the difference between interest rates were not strictly positive (or $\delta = 1$), the band might not be sustainable[9]. In the next section, we will introduce a mild

briefly at the end of the paper.

[9] For example, if the support of the distribution of noise traders extends beyond γ/a, holders of the foreign asset never sell, and if the number of periods is sufficiently large, all agents with an option to buy the asset exercise it with no delay.

sufficient condition on the discount rate for the sustainability of the exchange rate band.

2 Zero-equilibria

Suppose that in some period t, all asset holders sell and no currency holder buys. This situation occurs at least in the last period. The value of holding an asset at the beginning of such a period t is denoted by $\bar{V}$ and is given by Lemma 1.

Lemma 1 *If in some period (e.g., in period $T+1$), all asset holders sell and no currency holder buys, the value of selling the asset is*

$$\bar{V} = 1 - \beta, \qquad \textit{with} \quad \beta = a \int_{\eta > \bar{X}} (\eta - \bar{X}) dF(\eta).$$

All formal results are proven in the appendix. We will assume that $0 \leq \beta < 1$. The main purpose of the model is to analyze how the endogenous behavior of speculators can trigger a devaluation with no "real shock" to the economy. We assume that the probability of a devaluation generated solely by the noise traders α is small:

$$\alpha = \int_{\eta > \bar{X}} dF(\eta) \geq 0. \tag{2}$$

The next result shows that under some conditions on (α, β), a speculator does not have an incentive to buy or to hold the asset if he is the only one to do so.

Lemma 2 *For any $\delta \in (0,1)$ and any $\beta^* \in (0,1)$, there exists $\alpha^* > 0$ such that if $\alpha < \alpha^*$ and $\beta < \beta^*$, the following property holds: for any $(\lambda_{t-1}, \mu_{t-1})$, if there has been no devaluation before period t, the subgame which begins in period t has an equilibrium in which all asset holders sell in period t and no agent places an order in period t or after. This equilibrium is called the zero-equilibrium. The value of β^* does not need to be small.*

In the last period $T+1$, the zero-equilibrium is the only equilibrium, by assumption. The value of holding the asset at the beginning of the last period is therefore $\bar{V}$, while the value of the option to buy the asset is zero. We now proceed by backward induction to determine any equilibrium in the subgame which begins in period $t \leq T$.

3 Equilibria with speculation

3.1 Value functions

In any period t, three value functions will be important. They will be defined by backward induction.

1. The payoff of an order in period t depends on (i) the fraction of agents who place an order in the same period, ζ_t, (ii) the fraction of agents who hold the asset at the beginning of the period, λ_{t-1}, (iii) the speculator's probability μ_{t-1} of the state θ_1, which is determined by the history of past prices. The payoff is thus a function $u(\zeta_t; \lambda_{t-1}, \mu_{t-1})$ where $(\lambda_{t-1}, \mu_{t-1})$ are "initial conditions" in period t. We will show that in an equilibrium, the strategy ζ_t depends only on $(\lambda_{t-1}, \mu_{t-1})$. The value of an order in equilibrium will be a function of $(\lambda_{t-1}, \mu_{t-1})$ and will be denoted by $U_t(\lambda_{t-1}, \mu_{t-1})$.

2. The value of holding the asset at the beginning of period t and keeping the asset until the end of the period (not selling), in an equilibrium, is denoted by $V_t(\lambda_{t-1}, \mu_{t-1})$. Since an asset holder does not have to buy the asset, $V_t \geq U_t$.

3. The agents who don't hold the asset have the option to buy. The value

of delay (*i.e.*, keeping the option until at least the next period), will be denoted by $w_t(\zeta_t, \lambda_{t-1}, \mu_{t-1})$.

In period $T+1$, all agents sell by assumption. The payoff of a buy order is set at zero by convention and the payoff of selling the asset if it is held at the beginnin of the period is given in Lemma 1. For all $(\lambda_T, \mu_T) \in [0,1) \times (0,1)$,

$$V_{T+1}(\lambda_T, \mu_T) = \bar{V} = 1 - \beta, \qquad \text{and} \quad U_{T+1}(\lambda_T, \mu_T) = 0.$$

We now proceed by backward induction for all periods $t \leq T$.

3.2 Equilibria

The evolution of beliefs

If there is no devaluation in period t, the equilibrium price is $p_t = 1 + ay_t$. Using this equation, the observation of p_t is equivalent to the observation of the total demand

$$y_t = (\lambda_{t-1} + \zeta_t)\theta + \eta_t, \tag{3}$$

which conveys a signal on the state θ. Since agents know the strategies and $\lambda_{t-1} + \zeta_t$, the observation of y_t is equivalent to the observation of the variable

$$z_t = \theta + \frac{\eta_t}{\lambda_{t-1} + \zeta_t}. \tag{4}$$

The variance of the noise term is reduced when more agents place an order. The information conveyed by the market (conditional on no devaluation) increases with the fraction of agents who place orders. Recall that the belief at the beginning of period t (probability that $\theta = \theta_1$) is denoted by μ_{t-1}. Let $f(\eta)$ be the density of η (which is independent t). If there is no devaluation in period t, the belief on θ_t in the next period is determined by the Bayesian updating formula

$$\frac{\mu_t(y_t; \lambda_{t-1} + \zeta_t, \mu_{t-1})}{1 - \mu_t(y_t; \lambda_{t-1} + \zeta_t, \mu_{t-1})} = \frac{\mu_{t-1} f(y_t - (\lambda_{t-1} + \zeta_t)\theta_1)}{(1 - \mu_{t-1}) f(y_t - (\lambda_{t-1} + \zeta_t)\theta_0)}. \tag{5}$$

Payoff of an order

From the description of the trades, a devaluation takes place in period t if $y_t > \bar{X}$, in which case the *ex post* payoff of an order is $(A-\gamma)\dfrac{Max(\bar{X}-\lambda_{t-1}\theta-\eta,0)}{\zeta_t\theta}$.

If no devaluation takes place in period t, the *ex post* payoff is the value of holding the asset in the continuation of the game minus the purchase price, $1+ay_t$, *i.e*, $\underline{u}(\theta,\eta)=\delta\ V_{t+1}(\lambda_{t-1}+\zeta_t,\mu_t)-(1+ay_t)$. In this expression, the continuation value V_{t+1} depends on the fraction of speculators holding the asset $\lambda_t=\lambda_{t-1}+\zeta_t$, and on the belief μ_t at the end of period t which has been expressed in (5).

The payoff of an order is the expected value of all *ex post* payoffs for all possible values of θ and η. Denoting by $F(\theta,\eta;\mu_{t-1})$ the cumulative distribution[10] of (θ,η), this payoff is

$$\begin{aligned} u_t(\zeta_t;\lambda_{t-1},\mu_{t-1}) = & \int_{y_t>\bar{X}}(A-\gamma)\frac{Max(\bar{X}-\lambda_{t-1}\theta-\eta,0)}{\zeta_t\theta}dF(\theta,\eta;\mu_{t-1}) \\ & +\int_{y_t<\bar{X}}\Big(\delta\ V_{t+1}(\lambda_{t-1}+\zeta_t,\mu_t)-(1+ay_t)\Big)dF(\theta,\eta;\mu_{t-1}). \end{aligned} \tag{6}$$

The method of backward induction which is used here can characterize all equilibria of all subgames but such a complete characterization is beyond the scope of the paper. We will assume that if in period t there is no equilibrium strategy with $\zeta_t>0$ (no new order comes in), then the speculative attacks stops completely[11].

[10] θ and η are independent; the distribution of θ depends on the belief μ_{t-1}, and the distribution of η_t is independent of the period.

[11] Since the incentive to hold is weaker than that to buy, there may be a level of belief such that no speculator buys but asset holders don't sell. The price in period t may convey sufficient information to induce a resumption of new orders in period $t+1$ and eventually a successful attack. While such an equilibrium is theoretically possible, we assume that agents don't coordinate on it.

Such an assumption is possible because the zero-equilibrium is an equilibrium in any period (Lemma 2). In the subgame which begins in period t, there may be multiple equilibrium values for $\zeta_t > 0$. (An example will be given below). When there are such multiple equilibrium strategies, we assume that agents coordinate on the highest equilibrium value of ζ_t.

Assumption 1 *In any period t, if there is no strictly positive equilibrium value of ζ_t for buy orders, agents coordinate on the zero-equilibrium and the game ends at the end of the period. If there are multiple equilibrium strategies with strictly positive ζ_t, agents coordinate on the highest such value. This coordination rule is common knowledge.*

Arbitrage and the payoff of delay

By Assumption 1, the game ends in period $t+1$ if the payoff of an order in that period is negative. Hence, in equilibrium, the payoff of delay is equal to that of making a final decision in the following period, either to place a buy order in period $t+1$ or to never place a buy order. This *one-step* property is the same as in Chamley and Gale (1994) and is a consequence of Assumption 1. The payoff of delay is therefore equal to

$$w_t(\zeta_t; \lambda_{t-1}, \mu_{t-1}) = \delta \int_{y_t < \bar{X}} Max\Big(U_{t+1}(\lambda_{t-1} + \zeta_t, \mu_t), 0\Big) dF(\theta, \eta; \mu_{t-1}), \quad (7)$$

Since the mass of asset holders is not greater than one, $\zeta \in [0, 1 - \lambda_{t-1}]$. A necessary condition for ζ to be an equilibrium value in period t is that the payoff of a buy order is at least equal to that of delay:

$$u_t(\zeta; \lambda_{t-1}, \mu_{t-1}) \geq w_t(\zeta; \lambda_{t-1}, \mu_{t-1}). \quad (8)$$

Case a: If there is no $\zeta > 0$ which satisfies (8), then the equilibrium value is $\zeta_t = 0$ and by Assumption 1, the game ends in period t with the zero-equilibrium. All speculators hold only the currency for all remaining periods.

Case b: If there is a value $\zeta > 0$ such that (8) is satisfied, by Assumption 1, the equilibrium value ζ_t is defined as

$$\zeta_t = Max\Big\{\zeta \in (0, 1-\lambda_{t-1}] \;\mid\; u_t(\zeta; \lambda_{t-1}, \mu_{t-1}) \geq w_t(\zeta; \lambda_{t-1}, \mu_{t-1})\Big\}.$$

In general, the strategy in period t is a function $\zeta_t = \phi_t(\lambda_{t-1}, \mu_{t-1})$ with

$$\phi_t(\lambda_{t-1}, \mu_{t-1}) = Max\Bigg\{0, \Big\{\zeta \in (0, 1-\lambda_{t-1}] | u_t(\zeta; \lambda_{t-1}, \mu_{t-1}) \geq w_t(\zeta; \lambda_{t-1}, \mu_{t-1})\Big\}\Bigg\}. \tag{9}$$

When $\zeta_t = \phi_t > 0$, a speculative attack takes place. It is of one of the following two types: (i) If $\zeta_t = 1 - \lambda_{t-1}$, all agents who have an option place an order. Since there cannot be new orders in period $t+1$, the attack either succeeds in period t or it fails with all agents selling the foreign asset in period $t+1$. (ii) If $0 < \zeta_t < 1-\lambda_{t-1}$, by continuity of u_t and w_t, (8) must be an equality, $u_t(\zeta_t; \lambda_{t-1}, \mu_{t-1}) = w_t(\zeta_t; \lambda_{t-1}, \mu_{t-1})$. In such an equilibrium, there is an *arbitrage* between buying in period t at a relatively low price with less information, and delaying until period $t+1$ to get more information while facing the risk of missing the benefit from a devaluation in period t.

The strategy $\phi_t(\lambda_{t-1}, \mu_{t-1})$ determines the payoff of an order, in equilibrium:

$$U_t(\lambda_{t-1}, \mu_{t-1}) = u_t(\phi_t(\lambda_{t-1}, \mu_{t-1}), \lambda_{t-1}, \mu_{t-1}). \tag{10}$$

Value of holding the asset

Suppose that the belief μ is such that some agents place a buy order. These agents buy after the agents who already own the asset and face a higher price.

In this situation, if some agents find it profitable to place a new order, then the agents who already own the asset and have the same information strictly prefer to hold rather than to sell. This property is formalized in Lemma 3.

Lemma 3 *In an equilibrium, if some agents place a buy order in period t (and $\phi_t(\lambda_t, \mu_t) > 0$), then no asset holder sells in period t.*

The value of holding the asset at the beginning of period t depends on whether the speculative attack continues in the period. Using Lemma 3, this value satisfies the following recursive equations.

- If $\phi_t(\lambda_{t-1}, \mu_{t-1}) > 0$,

$$\begin{aligned} V_t(\lambda_{t-1}, \mu_{t-1}) &= \int_{y_t > \bar{X}} (1 + A) dF(\theta, \eta; \mu_{t-1}) \\ &+ \delta \int_{y_t < \bar{X}} V_{t+1}(\lambda_{t-1} + \phi_t, \mu_t) dF(\theta, \eta; \mu_{t-1}). \end{aligned} \tag{11}$$

- If $\phi_t(\lambda_{t-1}, \mu_{t-1}) = 0$, by Assumption 1 the equilibrium is the zero-equilibrium and
$$V_t(\lambda_{t-1}, \mu_{t-1}) = \bar{V}.$$

Backward determination of the equilibrium

In the last period $T + 1$, we have $U_{T+1} \equiv 0$ and $V_{T+1} \equiv \bar{V} = 1 - \beta$. Assume that for $t \leq T$, U_{t+1} and V_{t+1} are given. The payoffs of a buy order in period t is determined by (6) and that of delay by (7). The policy function $\phi_t(\lambda_t, \mu_t)$ is then determined by (9). This function determines U_t by (10) and V_t by (11). The equilibrium is determined for all periods by backward induction.

The previous definition of the policy function generates an equilibrium which is stable in the sense that a small deviation of all currency holders from the equilibrium strategy induces a stabilizing reaction. Indeed, consider first the corner solution and assume that $\zeta_t = 1 - \lambda_t > 0$. Ruling out an event with probability zero, we can assume that $u_t(\zeta_t; \lambda_{t-1}, \mu_{t-1}) > 0 = w_t(\zeta_t; \lambda_{t-1}, \mu_{t-1})$. Assume that a perturbation occurs in the form of a small reduction of ζ_t. Using the expression of u in (6) with no future period, one can see that the optimal response is still to place an order with probability 1.

Suppose now that there is arbitrage with $0 < \phi_t(\lambda_{t-1}, \mu_{t-1}) < 1 - \lambda_t$, and consider the difference between the payoff of an order and that of delay:

$$D(\zeta) = u_t(\zeta; \lambda_{t-1}, \mu_{t-1}) - w_t(\zeta; \lambda_{t-1}, \mu_{t-1}).$$

By definition of ϕ_t, $D(\zeta) < 0$ for all $\zeta \in (\phi_t, 1 - \lambda_{t-1}]$ and its derivative is not equal to 0 at the point $\zeta = \phi_t$. Hence the graph of $u_t(\zeta; \lambda_{t-1}, \mu_{t-1})$ cuts that of $w_t(\zeta; \lambda_{t-1}, \mu_{t-1})$ from above at $\zeta = \zeta_t$: if all currency holders but one reduce (increase) their probability of placing an order, the optimal response is to place an order with probability one (zero). The reaction is stabilizing.

4 Equilibrium properties

In the model, holding the currency at the beginning of period t is an option to buy. The expected gain from placing an order is U_t. The value of the option, which is the same as the value of the currency for a speculator, is therefore $1+Max(U_t, 0)$. The next result establishes a relation between the value of holding the asset and the value of holding the currency.

Lemma 4 *For any $(\lambda, \mu) \in (0,1) \times (0,1)$, the difference between the values of holding the asset and holding the currency is equal to*

$$V_t(\lambda, \mu) - \Big(1 + Max\Big(U_t(\lambda, \mu), 0\Big)\Big) = G\Big(\phi_t(\lambda, \mu), \lambda, \mu\Big),$$

where $G(\zeta, \lambda, \mu)$ is an increasing function of $\zeta \in (0, 1 - \lambda]$.

The monotonicity of G is intuitive. A higher value of ζ raises the probability of a devaluation, and the capital gain on the asset increases. The effect on the asset holders is greater than on the option holders because the gain of the option holders is dampened by the higher purchase price if there is no devaluation, and a lower probability of execution if the devaluation takes place.

Let us now compare the policy functions in two consecutive periods ϕ_t and ϕ_{t+1}. Suppose that for some value (λ, μ), $0 < \phi_{t+1}(\lambda, \mu) < 1 - \lambda$, and that there is an arbitrage between delay and no delay in period $t + 1$ if $(\lambda_t, \mu_t) = (\lambda, \mu)$. Assume that $\phi_{t+1} \geq \phi_{t+2}$ for all values of $(\lambda, \mu) \in (0, 1) \times (0, 1)$.

An intuitive argument indicates that if in equilibrium the payoff of an order is at least as high as that of delay in period $t+1$ for the initial condition $(\lambda_t, \mu_t) = (\lambda, \mu)$, then in period t for the same initial condition, $(\lambda_{t-1}, \mu_{t-1})$, the payoff of an order is at least as high as that of delay when $\zeta_t = \phi_{t+1}(\lambda, \mu)$.

The difference between the payoff of an order and that of delay is the sum of two terms. The first is the capital gain from a devaluation in period t, net of the purchase price if no devaluation takes place. This term depends only on the strategy ζ_t and the initial conditions which by assumption are the same in periods t and $t + 1$. The second term is the expected premium of holding the asset over the payoff of an order in the next period, $\delta E[V_{t+1} - Max(U_{t+1}, 0)]$, conditional on no devaluation in period t. By the induction hypothesis, $\phi_{t+1} \geq \phi_{t+2}$ and using Lemma 4, this term is higher in period $t + 1$ than in period $t + 2$ (for the same initial conditions). Since there is arbitrage in period $t+1$, the difference between the payoff of an order and that of delay must be non negative in period t. By definition of the policy function ϕ_t in (9), $\phi_t(\lambda, \mu) \geq \phi_{t+1}(\lambda, \mu)$. The argument which is detailed in the appendix leads to the following result.

Proposition 1 *Under Assumption 1, for any $(\lambda, \mu) \in (0,1) \times (0,1)$, $\phi_{t-1}(\lambda, \mu) \geq \phi_t(\lambda, \mu)$. For any t and $(\lambda, \mu) \in (0,1) \times (0,1)$, $\phi_t(\lambda, \mu)$ is non decreasing in the number T of periods of the game.*

The mass of speculators who attack the currency is an increasing function of the number of periods, *ceteris paribus*. Another description of the result is that the set of values (λ, μ) of the initial conditions at the beginning of a period t over which a speculative attack takes place is larger when the number of periods which follow period t is higher. More periods provide more opportunities for speculators to "communicate" through the market prices during an attack and thus a higher incentive to participate in a speculative attack.

Let $\phi_t^T(\lambda, \mu)$ be the policy function in period t when the game has T periods (not counting the last one). The subgame which begins in period t with conditions (λ, μ) and number of periods T is the same as that which begins in period $t-1$ with the same condition (λ, μ) and number of periods $T-1$:

$$\phi_t^T(\lambda, \mu) = \phi_{t-1}^{T-1}(\lambda, \mu).$$

Hence, $\phi_t^{T+1} = \phi_{t-1}^T \geq \phi_t^T$, by Proposition 1. If the number of periods in the game T increases ($T > t$), the sequence of policy functions $\phi_t^T(\lambda, \mu)$ is monotone increasing in T. Since ϕ_t^T is bounded by one, it tends to a limit. The equilibrium solution tends to a stationary solution when T tends to infinity.

Proposition 2 *For any fixed t and (λ, μ), if the number of periods T increases, the policy function $\phi_t^T(\lambda, \mu)$ does not decrease. If $T \to \infty$, $\phi_t^T(\lambda, \mu)$ tends to a limit $\phi(\lambda, \mu)$.*

5 An example with Gaussian noise

Assume that η has a normal distribution with variance σ_η^2. Let $\nu_t = Log(\mu_t/(1-\mu_t))$ be the Log likelihood ratio (LLR) between the two states. The Bayesian equation (5) takes the form

$$\nu_t = \nu_{t-1} + \lambda_t \frac{\theta_1 - \theta_0}{\sigma_\eta^2}\Big(y_t - \lambda_t \frac{\theta_1 + \theta_0}{2}\Big),$$

with $\lambda_t = \lambda_{t-1} + \zeta_t$, and $y_t = \lambda_t \theta + \eta_t$. The expected change of belief from period t to period $t+1$ is measured by

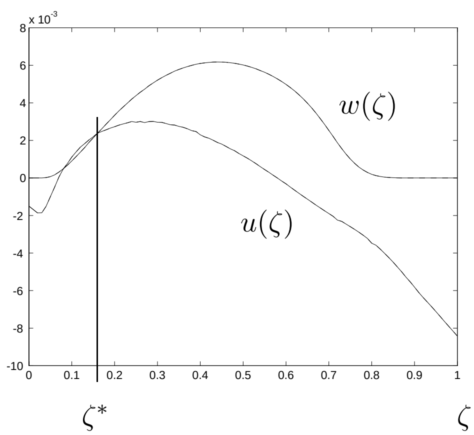
$$E[\nu_{t+1} - \nu_t] = \begin{cases} \dfrac{\Big(\theta_1 - \theta_0\Big)^2}{2\sigma_\eta^2}(\lambda_t + \zeta_t)^2 & \text{in state } \theta_1, \\ -\dfrac{\Big(\theta_1 - \theta_0\Big)^2}{2\sigma_\eta^2}(\lambda_t + \zeta_t)^2 & \text{in state } \theta_0. \end{cases} \tag{12}$$

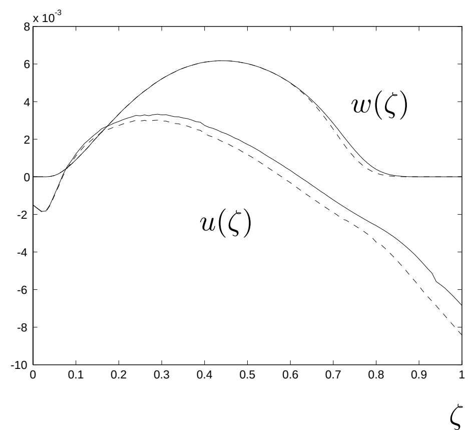
We have seen before that the signal to noise ratio in the demand y_t increases with the fraction of active speculators $\lambda_t + \zeta_t$. This property appears in the previous expression where the absolute value of the expected change of belief increases with the demand by speculators.

5.1 A numerical example

The graphs of the payoff of an order and that of waiting in the first period of a three period game ($T = 2$) are represented in Figure 1. By assumption, no agent holds the asset at the beginning of the game, $\lambda_1 = 0$, and the belief of the high state is $\mu_1 = 0.05$. The height of the band is $\gamma = 0.025$. The rate of the devaluation is $A = 0.275$. The parameter a is equal to 1 and the critical mass is therefore $\bar{X} = \gamma/a = 0.025$. The standard error of the noise trade is $\sigma_\eta = 0.08\bar{X}$. Hence, the probability α that a devaluation is triggered by the noise traders is less than 10^{-12}. The values of the actual masses of speculators in the two states are $\theta_0 = 0.7\bar{X}$ and $\theta_1 = 1.3\bar{X}$. The interest rate per period is 0.15 percent. The payoffs of an order and of delay are equal to $u_1(\zeta; 0, 0.05)$ and $w_1(\zeta; 0, 0.05)$, or $u(\zeta)$ and $w(\zeta)$ for short[12]. There are three equilibria, but the middle equilibrium value of λ is unstable. The two stable equilibria are the zero-equilibrium and an equilibrium in which a fraction of speculators purchase the asset. (For other values of the parameters, there may be more than one stable equilibrium where the mass of speculation is strictly positive).

When ζ is small, the value of a buy order is a decreasing function of ζ: a higher mass of speculation raises the price of the asset, but because this mass is still small, it does not provide much information on the state, and the gross payoff of an order remains low.





Dotted : $a = 1$ as in Figure 1.

Plain : $a = 0.94$.

Figure 1

The base case

Figure 2

Higher supply elasticity

[12] $u(\zeta)$ and $w(\zeta)$ are computed on a grid for $\zeta \in [0, 1]$ of width 0.02. The first step is the computation of $U_2(\lambda, \mu)$ and $V_2(\lambda, \mu)$ on a grid of values $(\lambda, \mu) \in [0, 1] \times [0, 1]$.

When the value of ζ, is sufficiently large, the price conveys an informative signal on the high state if that is the true state. The anticipation that such an information will be provided at the end of the first period and will generate a successful continuation of the attack in period 2 raises the value of a purchase order above the value of the option of delay. The gain of buying early compensates for the risk of finding out at the end of period 1 that the state is bad and enduring a capital loss in period 2.

5.2 Impact of parameters

The elasticity of supply by market makers

Suppose that in the first period only, a perturbation increases the elasticity of the supply $S(p) = (p-1)/a$. The value of a is lowered in the first period and unchanged in period 2 for the computation of U_2 and V_2, (for simplicity). The new graphs of u and w are presented in Figure 2 by the solid curves. The dotted curves are the same as in Figure 1.

To interpret the effect of a higher elasticity, recall that in the first period, either the mass of orders y_1 is greater than the critical mass $\bar{X}$ and a devaluation takes place, or $y_1 < \bar{X}$ in which case no devaluation takes place and agents learn from the observation of y_1 (which is equivalent to the observation of the price p_1).

An increase in the supply elasticity of the asset has two effects: it raises the critical mass $\bar{X}$, and it lowers the price p_1 for given y_1. The first of these two effects is significant only if the strategy to buy, ζ_1, is sufficiently large to trigger a devaluation in the first period. If ζ_1 is relatively small, the probability of a devaluation in period 1 is small and the first effect can be neglected. The second effect lowers the cost of an order for a given amount of information. Hence, the payoff of an order is increased while the payoff of delay is not changed significantly. The combination raises the equilibrium value of ζ^* as shown in Figure 2.

When the value of ζ is not small, the first effect may not be neglected. The higher value of the critical mass $\bar{X}$ generates a lower probability of devaluation in the first period. Hence the risk of missing the capital gain from the devaluation is lower and the payoff of delay is higher, as shown in Figure 2. (When ζ_1 is near one, the payoff of delay is always near zero).

Suppose now that the elasticity of supply is moderately lower in the first period (higher a). The previous arguments apply *a contrario*. The payoffs are represented in Figure 3. In the example of the figure, the equilibrium with a speculative attack and arbitrage disappears because of the higher cost of an order.

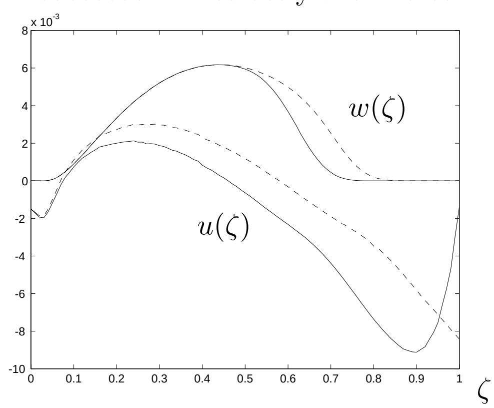
If the elasticity of supply is decreased from the base case by a larger amount, there is an equilibrium with a speculative attack in which all speculators buy without delay with $\zeta_1 = 1$ (Figure 4). Here the first effect which was described previously operates. A lower elasticity entails a lower critical mass and therefore a higher probability of devaluation in the first period[13].

Partially informed market makers

The previous discussion shows that the supply elasticity and the liquidity of the market have an ambiguous impact on the likelihood of a speculative attack.

[13] In the example, the mass of speculators in any state is higher than the critical mass if $1/a < 0.7$ or $a > 1.43$.

Thoughout the paper, market makers have *ex ante* a vanishingly small probability that there is a large mass of speculators. Suppose now for a moment that this *ex ante* belief is not negligible any more. When market makers see a price rise in the period, they revise upwards their probability of a high mass of speculators. This effect is equivalent to a downward shift of their supply. The supply schedule is therefore less elastic. In the examples of Figure 4, a small increase in the belief of market makers (probability of state θ_1) may prevent the occurence of a speculative attack because it raises the cost of communication between speculators through the market. A large increase in the belief facilitates a speculative attack which succeeds immediately with no communication between speculators.

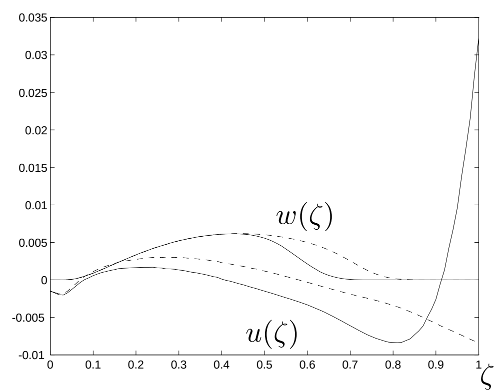


Dotted : $a = 1$ as in Figure 1.

Plain : $a = 1.2$.

Figure 3

Low supply elasticity



Dotted : $a = 1$ as in Figure 1.

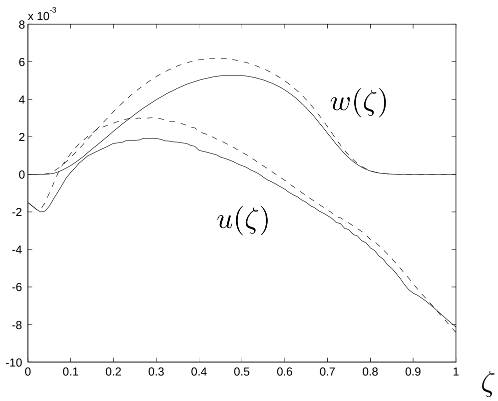
Plain : $a = 1.3$.

Figure 4

Lower supply elasticity

Noise traders

The noise traders, a standard feature of the model, affect the information content of the first period price. If the variance of noise trading is higher, the price in the first period is less informative (see equations (4) and (12)). When the state is high, agents are on average less informed about it at the end of the first period (for given ζ). This lowers the probability of a speculative attack in period 2, and therefore the incentive to attack in period 1. In Figure 5, the variance of the noise is higher than in the base case (Figure 1), for the first period only. There is no equilibrium with a speculative attack.

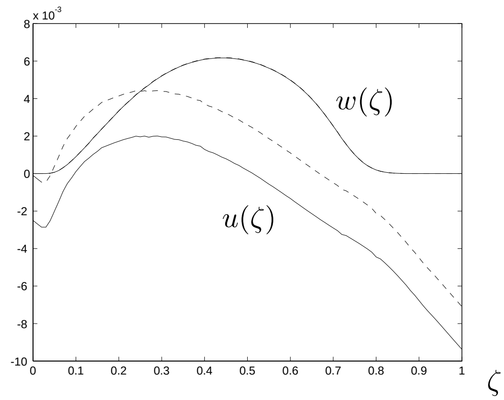


Dotted : $\sigma_\eta/\bar{X} = 0.08$ as in Figure 1.

Plain : $\sigma_\eta/\bar{X} = 0.1$.

Figure 5

Higher trade noise



Dotted : $r = 1 - \delta = 0.01$ percent.

Plain : $r = 0.25$ percent.

Figure 6

Low and high interest rate

The discount rate

The discount rate is defined as $r = 1 - \delta$. Suppose an attack takes place in the first period with an arbitrage between delay and no delay. The probability of a devaluation in the first period is very low. An order in the first period pays off because of the relatively low price in the first period and the expectation of a continuation of the attack with a devaluation in the next period. An important difference between the payoffs of an order and of delay is the foregone interest when the agent buys the asset in the first period. This effect is illustrated in

Figure 6 where the discount rate takes two values, $r = 0.01\%$ and $r = 0.025\%$. A higher discount rate lowers both the payoff of an order and that of delay, but the second effect is negligible[14].

6 Defense Policies

A defense policy is successful if it abolishes the coordination equilibrium with a speculative attack. Three types of policies are considered here: (i) widening the band of fluctuations, (ii) stabilizing the exchange rate through trading, and (iii) random interventions. For each policy it is assumed that the central bank cannot observe the state θ.

Widening the band of fluctuations

The widening of the band with a constant rate of devaluation A is equivalent to an increase in γ. In the present model, if $\gamma = A$ there is no capital gain if a devaluation takes place, hence no expected profit. If the attack fails however, there is a capital loss. Hence, there is no speculative attack in an equilibrium. By continuity, that property holds if $\gamma^* < \gamma \leq A$ for some γ^* $(0 < \gamma^* < A)$. A speculative attack can be prevented by a suitable widening of the band of fluctuations.

A relevant episode occurred at the end of July 1993 with the speculative attack against the French Franc which was part of the ERM. The regime had margins of 2.25 percent on each side of a reference level. After trying unsuccessfully to

[14] This is particularly clear for $\zeta = 0$: the payoff of delay is obviously 0. An agent who buys now pays 1 and gets roughly a present value of $\delta = 1 - r$ in the next period, as can be verified in Figures 5 and 6.

ward off the speculators through trading, the central banks of the monetary union raised the bands of fluctuations to 15 percent. The change of regime stopped the attack[15], as illustrated in Figure 1 of Obstfeld (1996).

Stabilizing the exchange rate through trade intervention

There are two types of trade interventions by the central bank, those which are *deterministic* and predictable, and those which are *random* and surprise the speculators. A trade policy which is determined by the exchange rate is predictable by rational agents. As an example, assume that the central bank supplies a quantity of foreign currency according to the linear rule

$$S_B = b(p - 1), \qquad \text{with } b > 0. \tag{13}$$

With $b > 0$, the central bank attempts to reduce the fluctuations of the exchange rate. Such a policy requires a positive level of reserves R. The problem of defense is interesting only if the reserves cannot prevent a speculative attack under perfect information, *i.e.* if $R < \theta_1 - \bar{X}$ (as in Obstfeld, 1996). Such a constraint imposes

[15] The last exchange rate between the DM and the FF before the change of regime was at the top of the band at 3.4304. The day after the change of regime (August 2), the rate increased to 3.5080 then fell to 3.46040 two days later. It then began to increase again and to hover around 3.50. However, by that time the information had probably changed. Agents expected the interest rate to be lowered in France to take advantage of the greater exchange rate flexibility and reduce unemployment. Eventually, expectations were mistaken: such a policy was not conducted by the central bank. After hovering between 3.48 and 3.55 until the beginning of December, the exchange rate decreased steadily during the last month of the year to end at 3.40. In agreement with the policy interpretation in this paper, after the exchange rate returned to its mid-band level, the central banks felt no need to reduce the bands back to the original value of 2.25 percent. The wider band contributed to the stabilization of the exchange rate. (For a discussion of the events see Buiter *et al.*, 1998).

a restriction on the stabilization policy. Since $S_B \leq R$ for all values of $p < 1+\gamma$, we must have $b \leq R/\gamma$.

Speculators with rational expectations anticipate the policy. We assume accordingly that they know the value of the policy parameter b and know that the total supply of foreign exchange is equal to $(p-1)/a'$ with $a' = 1/(b+1/a) < a$. The impact of the policy is the same as that of an increase in the supply elasticity of foreign exchange, or the liquidity of the market. We have seen in the previous section (Figure 2), that a higher elasticity of supply enlarges the domain of beliefs in which a speculative attack may take place. In the present model, a central bank which reduces the fluctuations of the exchange rate does not alter the function of the exchange rate as a coordination device, but it reduces the risk taken by speculators. Such a policy facilitates speculative attacks.

When the price of the foreign currency rises (because of a noise shock or an attack), a central bank which conducts a deterministic (and predictable) policy should not sell the foreign currency but it should *buy*.

Random intervention

Trades by the central bank which cannot be predicted by speculators have to be random. Assume that the central bank supplies a random amount R which is normally distributed $N(0, \sigma_R)$ and set before the opening of the market. Rational speculators know the parameter σ_R but cannot observe R. The random trading by the central bank adds noise and thus reduces the information content of the price in the first period. The intuition that the smaller information reduces the possibility of coordination between speculators is verified by numerical simulations.

In the example of Figure 5, the speculative attack is eliminated when $\sigma_R \geq 0.06\bar{X}$. For a policy of random interventions, some reserves are required (since the foreign currency has to be sold at times), contrary to the policy of deterministic trade. However, these reserves may be significantly smaller than what would be required under perfect information. This is an important implication of the present model for policy. By trading in a non predictable manner, the central bank can prevent speculators from coordinating an attack. The amount of reserves which are required for this policy can be smaller than what would be required if speculators had complete information on their total resources[16].

The interest rate

A standard defense policy is to raise the interest rate. This policy is illustrated in Figure 6. It is particularly powerful in the context of the model. For the parameters of the figure, an interest rate of 0.25 percent defends the regime. The policy is effective because it raises the cost of "communication" through the price at the beginning of an attack when no devaluation takes place yet. If the parameters of the model and the beliefs are such that there is an attack by all speculators which must succeed or fail in the first period, raising the interest rate is significantly less effective. This case is illustrated by Figure 4. Given the elasticity of supply in that figure, the interest rate has to be raised from 0.15 percent to 4 percent to prevent the speculative attack.

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[16] In Figure 5, the central bank needs a level of reserves equal to $R = 0.12\bar{X}$. (The normal distribution can be considered as an approximation of a distribution where the trade by the central bank is bounded). If all speculators have knowledge of the high state and attack, the central bank's reserves R are too low to fend off the attack: $\theta_1 - \bar{X} = 0.3\bar{X} > R$.

APPENDIX

Lemma 1

Assume that in period t, all asset holders sell and no agent buys the asset. If a devaluation occurs, it is triggered by the noise traders, with $\eta_t > \bar{X}$ (equation (1)). In that event, an asset holder is on the long side of the market and gets his order to be executed at the highest price in the band, $1 + \gamma$. If no devaluation occurs, the sell order is executed at the market price $p = 1 + a\eta_t$. The payoff of selling is therefore $\bar{V}$ with

$$\bar{V} = 1 + \gamma \int_{\eta > \bar{X}} dF(\eta) + a \int_{\eta < \bar{X}} \eta dF(\eta).$$

Since $\int \eta dF(\eta) = 0$ and $\gamma = a\bar{X}$,

$$\bar{V} = 1 - a \int_{\eta > \bar{X}} (\eta - \bar{X}) dF(\eta) = 1 - \beta.$$

Lemma 2

We first prove that if all asset holders sell in period t and no agent buys in period t or after, no asset holder wants to delay his sale in period t. Assume that one asset holder delays his sale until period $t + k$, $1 \leq k \leq T + 1 - t$. If a devaluation occurs in period $t + i$ with $i < k$, he sells in period $t + i + 1$ because of the discount factor. Hence, the delay until period $t + k$ is contingent on no devaluation before period $t + k$. Since no speculator holds the asset after period t, the demand for the asset is driven only by the noise traders and a devaluation occurs in period i only if no devaluation has occurred before and $\eta_i > \bar{X}$, with a probability $\alpha \geq 0$. If the agent sells after a devaluation, he gets $1 + A$. If he sells with no prior devaluation, he gets $\bar{V}$ (Lemma 1). The payoff of holding until period $t + k$ is therefore

$$\begin{aligned} v_k &= \delta\Big(\alpha(1+A) + ... + ((1-\alpha)\delta)^{k-1}\alpha(1+A) + (1-\alpha)((1-\alpha)\delta)^{k-1}\bar{V}\Big) \\ &< \frac{\delta\alpha(1+A)}{1-(1-\alpha)\delta} + \delta(1-\alpha)\bar{V}, \quad \text{with} \quad \bar{V} = 1-\beta. \end{aligned}$$

Delaying the sale is not optimal if $v_k < \bar{V}$. A sufficient condition is

$$\delta\alpha(1+A) < (1-\delta(1-\alpha))^2(1-\beta).$$

For any $(\delta, \beta^*) \in (0,1) \times (0,1)$, there is $\alpha_1^* > 0$ such that if $\alpha < \alpha_1^*$ and $\beta < \beta^*$ this condition holds.

We now prove that no agent would purchase the asset in any period i, $t \leq i \leq T+1$. From the first part of the proof, if an agent purchases the asset, he sells it the next period. His order is executed only if there is no devaluation in the same period. The payoff of such a strategy is

$$u = \int_{\eta < \bar{X}} (\delta\bar{V} - 1 - a\eta) dF(\eta).$$

Since $\bar{V} = 1-\beta$, $\int \eta dF(\eta) = 0$, and $a\int_{\eta > \bar{X}} \eta dF(\eta) = \beta + a\alpha\bar{X} = \beta + \gamma\alpha$, then

$$u = -(1-\alpha)(1-\delta)(1-\beta) + \alpha(\gamma+\beta).$$

For any $(\delta, \beta^*) \in (0,1) \times (0,1)$, there is $\alpha_2^* > 0$ such that if $\alpha < \alpha_2^*$ and $\beta < \beta^*$, $u < 0$. The value of α^* in the Lemma is $\alpha^* = Min(\alpha_1^*, \alpha_2^*)$.

Lemma 3

Let V^S be the payoff of an asset holder who sells without delay:

$$V^S = \int_{y_t > \bar{X}} (1+\gamma) dF_t + \int_{y_t < \bar{X}} (1 + a y_t) dF_t, \quad \text{with} \quad dF_t = dF(\theta, \eta; \mu_{t-1}).$$

Let V_t be the value of holding the asset until the end of the period:

$$V_t = \int_{y_t > \bar{X}} (1+A) dF_t + \delta \int_{y_t < \bar{X}} V_{t+1}\Big(\lambda, \mu_t(y_t, \mu_{t-1})\Big) dF_t.$$

$$V_t - V^S = (A-\gamma)\int_{y_t\,>\,\bar{X}} dF_t + \int_{y_t\,<\,\bar{X}} (\delta V_{t+1} - 1 - a y_t) dF_t.$$

Using the expression of U_t in (10) and (6),

$$(A-\gamma)\int_{y_t\,>\,\bar{X}} dF_t + \int_{y_t\,<\,\bar{X}} (\delta V_{t+1} - (1 + a y_t)) dF_t > U_t \geq 0.$$

Lemma 4

First, if there is a speculative attack ($\phi_t > 0$), using the definition of U_t in (10) and (6),

$$\begin{aligned} U_t &= (A-\gamma)\int_{y_t\,>\,\bar{X}} \frac{Max\Big((\bar{X} - \lambda_{t-1})\theta - \eta, 0\Big)}{\phi_t(\lambda_{t-1}, \mu_{t-1})\theta} dF_t \\ &\quad + \int_{y_t\,<\,\bar{X}} \Big(\delta\ V_{t+1} - (1 + a y_t)\Big) dF_t, \end{aligned}$$

Using (11), $V_t = 1 + U_t + G(\phi_t(\lambda_{t-1}, \mu_{t-1}), \lambda_{t-1}, \mu_{t-1})$, where the function G is independent of the period t:

$$\begin{aligned} G(\zeta, \lambda, \mu) &= A\int_{y\,>\,\bar{X}} \Big(1 - (\frac{A-\gamma}{A}) \frac{Max(\bar{X} - \lambda\theta - \eta, 0)}{\zeta\theta}\Big) dF(\theta, \eta; \mu) \\ &\quad + a\int_{y\,>\,\bar{X}} y dF(\theta, \eta; \mu), \quad \text{with} \quad y = \zeta\theta + \eta. \end{aligned}$$

Second, if $\phi_t(\lambda) = 0$, from Assumption 1, all asset holders sell: $V_t = \bar{V}$ and $U_t = 0$. We define $G(0, \lambda, \mu) = -\beta$.

If $\zeta > 0$, $G(\zeta, \lambda, \mu)$ is differentiable in ζ at $\phi(\lambda, \mu)$. Recall that $y = (\lambda + \zeta)\theta + \eta$. In the expression of G_ζ, the sum of the terms generated by the derivatives of the bounds of integration is equal to zero and

$$G_\zeta = (A-\gamma)\int_{y>\bar{X}} \frac{Max(\bar{X} - \lambda\theta - \eta, 0)}{\zeta^2\theta} dF > 0.$$

Proposition 1

We proceed by backward induction. In period $T+1$, $\phi_{T+1} \equiv 0$. Hence $\phi_T \geq \phi_{T+1}$. Assume that $\phi_{t+1} \geq \phi_{t+2}$.

Consider first the case where $\phi_{t+1}(\lambda,\mu) = 1-\lambda$. Since all currency holders in period $t+1$ place an order, there cannot be new orders in period $t+2$, and by Assumption 1, the game ends in period $t+2$. This equilibrium strategy in period $t+1$ is also an equilibrium strategy in period t (with the same (λ,μ)) and it is the highest equilibrium strategy. We have $\phi_t(\lambda,\mu) = \phi_{t+1}(\lambda,\mu) = 1-\lambda$.

Assume now that $0 < \phi_{t+1}(\lambda,\mu) < 1-\lambda$: there is an arbitrage in period $t+1$ between purchase and delay. We show that in period t, if $(\lambda_{t-1},\mu_{t-1})$ is identical to (λ,μ), then delay to order yields a lower payoff than no delay.

Recall the expression of u_t in (6), and of w_t in (7):

$$\begin{aligned} u_t(\zeta;\lambda_{t-1},\mu_{t-1}) \quad &= \quad (A-\gamma)\int_{y_t\,>\,\bar{X}} \frac{Max(\bar{X}-\lambda_{t-1}\theta-\eta,0)}{\zeta\theta} dF(\theta,\eta;\mu_{t-1}) \\ &+ \int_{y_t\,<\,\bar{X}} \Big(\delta\; V_{t+1}\big(\zeta+\lambda_{t-1},\mu_t\big) - (1+ay_t)\Big) dF(\theta,\eta;\mu_{t-1}). \end{aligned}$$

$$w_t(\zeta;\lambda_{t-1},\mu_{t-1}) = \delta\int_{y_t\,<\,\bar{X}} Max\Big(U_{t+1}\big(\zeta+\lambda_{t-1},\mu_t(y_t,\mu_{t-1})\big),0\Big) dF(\theta,\eta;\mu_{t-1}),$$

Applying Lemma 4 to V_{t+1} and U_{t+1}, we have

$$u_t(\zeta,\lambda,\mu)-w_t(\zeta,\lambda,\mu) = H(\zeta,\lambda,\mu)+\delta\int_{y\le\bar{X}} G\big(\phi_{t+1}(\zeta+\lambda,\mu'),\zeta+\lambda,\mu'\big)dF_\mu, \quad (14)$$

where $dF_\mu = dF(\theta,\eta;\mu)$, $y = (\lambda+\zeta)\theta+\eta$, μ' is the updated belief which depends on $(\mu,\zeta+\lambda,\eta)$ as specified in (5), and the function H is defined by

$$H(\zeta,\lambda,\mu) = (A-\gamma)\int_{y\,>\,\bar{X}} \frac{Max(\bar{X}-\lambda\theta-\eta,0)}{\zeta\theta} dF - \int_{y\,<\,\bar{X}} (1-\delta+ay)dF,$$

which is independent of t.

If $0 < \phi_{t+1}(\lambda,\mu) < 1-\lambda$, the payoff of an order and of delay are equal in period $t+1$. Hence,

$$0 = H(\phi_{t+1}(\lambda,\mu),\lambda,\mu) + \delta\int_{(\phi_{t+1}(\lambda,\mu)+\lambda)\theta+\eta\le\bar{X}} G\big(\phi_{t+2}(\zeta+\lambda,\mu'),\zeta+\lambda,\mu'\big)dF_\mu.$$

Consider the difference between the payoff of an order and that of delay in period t, for the same value (λ, μ) and the same strategy $\phi_{t+1}(\lambda, \mu)$. Substituting in (14),

$$u_t(\phi_{t+1}(\lambda, \mu); \lambda, \mu) - w_t(\phi_{t+1}(\lambda, \mu); \lambda, \mu) =$$

$$H(\phi_{t+1}(\lambda, \mu), \lambda, \mu) + \int_{(\phi_{t+1}(\lambda,\mu)+\lambda)\theta+\eta \leq \bar{X}} G\Big(\phi_{t+1}(\zeta + \lambda, \mu'), \zeta + \lambda, \mu'\Big) dF_{\mu}.$$

Since by assumption $\phi_{t+1} \geq \phi_{t+2}$ and G is increasing in its first argument (Lemma 4),

$$u_t(\phi_{t+1}(\lambda, \mu); \lambda, \mu) - w_t(\phi_{t+1}(\lambda, \mu); \lambda, \mu) \geq 0.$$

By definition of ϕ_t in (9), $\phi_t(\lambda, \mu) \geq \phi_{t+1}(\lambda, \mu)$.

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