Consumption and Saving Under Knightian Uncertainty

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Abstract

This paper studies consumption/saving problem under Knightian uncertainty in a two period setting. The multiple-priors utility model is adopted. The effects of income uncertainty and capital uncertainty on optimal savings are analyzed by deriving closed form solutions.

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1 Introduction

An agent's choice between saving and immediate consumption depends crucially on the future uncertainty. In the standard model, it is generally assumed that uncertainty is identical to risk. That is, a single probability measure on the state of the world is available to guide choice. For example, according to the rational expectations hypothesis, agents know precisely the objective probability law and their beliefs are identical to this probability law. Alternatively, according to the Bayesian approach, an agent's beliefs are represented by a subjective prior. By either approach, the situation of *uncertainty*, where information is too imprecise to be summarized adequately by probabilities, is ruled out. By contrast, Knight (1921) emphasizes the distinction between risk and uncertainty and argues that uncertainty is more common in decision-making.[1] For experimental evidence, the Ellsberg Paradox suggests that people prefer to act on known rather than unknown probabilities.[2] Ellsberg-type behavior contradicts the standard paradigm, i.e., the existence of any prior underlying choices.

To incorporate Knightian uncertainty, I adopt the multiple-priors utility model axiomatized by Gilboa and Schmeidler (1989) in a consumption/saving problem. To make analysis transparent, I focus on a two-period setting. In this model, the agent's beliefs about future events are represented by a set of priors. The set of priors captures both the degree of Knightian uncertainty and uncertainty aversion.[3]

The paper shows that optimal savings depend crucially on the source of uncertainty. Under income uncertainty, accumulated savings provide a buffer against this uncertainty. The agent can consume a certain amount out of savings when the future labor income is low. Such uncertainty is prevalent for wage and salary earners. By contrast, for self-employed persons, uncertainty is better described by coming from capital gain or loss. Under this uncertainty, the more one saves, the more one stands to lose. Giving up a unit of current consumption does not necessarily lead to a cer-

[1]Henceforth, I refer to such uncertainty as *Knightian uncertainty* or *ambiguity*.

[2]See Ellsberg (1961). One version of the story is as follows. A decision maker is a offerred a bet on drawing a red ball from two urns. The first urn contains exactly 50 red and 50 black balls. The second urn has 100 balls, either red or black, however the exact number of red or black balls is unknown. A typical agent chooses from the first urn rather than the second.

[3]For a formal definition of uncertainty aversion, see Epstein (1999) and Epstein and Zhang (2001).

tain increase in future consumption. Because of these differences, those two types of uncertainty result in very different impact on optimal consumption and saving.

Under income uncertainty, there is a separate component of precautionary savings that cannot be attributed to risk. This component is a first order function of the standard deviation of income and increases in uncertainty aversion or the degree of Knightian uncertainty. Moreover, it can arise even for quadratic utility. This is in sharp contrast to the standard model where precautionary saving is of second order and cannot arise for quadratic utility (e.g., Kimball (1990)).

Under capital uncertainty, the effect of Knightian uncertainty on optimal savings depends on two opposing income and substitution effects. Specifically, I consider the class of CRRA utility functions commonly used in the consumption/saving literature.[4] I show that if the intertemporal substitution parameter is bigger than one, an uncertainty averse agent saves less than an expected utility maximizer. Further, the agent saves less if he is more uncertainty averse or there is a higher degree of Knightian uncertainty. By contrast, one obtains the opposite conclusion if the intertemporal substitution parameter is less than one.

This paper is related to the vast literature on the consumption/saving problem.[5] The distinction between income risk and capital risk is emphasized by Sandmo (1970). He shows that increased riskiness of future income increases saving. He also shows that the effect of increased riskiness of returns on savings depends on the intertemporal substitution parameter in a way similar to that analyzed here (also see Levhari and Srinivasan (1969)). However, there is an important difference. In all examples studied below, I conduct comparative static analysis for the family of normal distributions with identical variances. As a result, riskiness is effectively fixed and hence all my comparative static results are driven exclusively by changing the degree of Knightian uncertainty.

The multiple priors utility model has been applied to finance in a number of papers.[6] None of them concerns the issues studied here. A related but different

[4]As is well known, for CRRA utility, the risk aversion parameter is the inverse of the intertemporal substitution parameter.

[5]See, for example, Phelps (1962), Leland (1968), Levhari and Srinivasan (1969), Sandmo (1970), Zeldes (1989), Caballero (1990), and Carroll (1997).

[6]See Epstein and Wang (1994, 1995), Chen and Epstein (2001), Epstein and Miao (2003), Kogan and Wang (2002), Cao et al (2002), Routledge and Zin (2003), and Miao and Wang (2003).

approach based on robust control theory is proposed by Hansen and Sargent and their coauthors (e.g., Anderson, Hansen and Sargent (2003) and Hansen and Sargent (2000)). They emphasize 'model uncertainty', which is also motivated in part by the Ellsberg Paradox. We refer readers to Epstein and Schneider (2002) for further discussion on these two approaches. Hansen et al (1999) apply the robust control approach to study the permanent income hypothesis and show that robustness can induce precautionary savings for quadratic utility. However, they do not distinguish between income uncertainty and capital uncertainty.

The paper proceeds as follows. Section 2 presents the model. The cases of income uncertainty and capital uncertainty are analyzed separately. Section 3 concludes.

2 The Model

Consider an agent's consumption and saving problem in a two period setting. Time zero is certain and uncertainty appears only in period 1. Fix a measurable state space $(\Omega, \mathcal{F})$ and a reference probability measure P on this space. In period zero, the agent has labor income y_0 and has no financial wealth. He decides how much to consume and how much to save. In period 1, the agent receives labor income y_1 as well as returns from saving. Denote the gross rate of return by R. Both y_1 and R may be random and defined on the probability space $(\Omega, \mathcal{F}, P)$.

It is clear that the agent faces the budget constraint:

$$c_0 + s = y_0, \;\; c_1 = Rs + y_1. \tag{1}$$

where s denotes savings and c_i denotes consumption in period $i = 0, 1$. This constraint can be rewritten as

$$c_1 = R\left(y_0 - c_0\right) + y_1. \tag{2}$$

The agent derives utility from consumption. The utility function is given by the multiple-priors utility model axiomatized by Gilboa and Schmeidler (1989):

$$U\left(c_0, c_1\right) = u\left(c_0\right) + \beta \min_{Q \in \mathcal{P}} E_Q\left[u\left(c_1\right)\right].$$

Here, $\beta \in (0, 1)$ is a discount factor and u is a vNM index. The special feature of this utility function is that the agent has a set of priors $\mathcal{P}$ over $(\Omega, \mathcal{F})$, instead of a single prior in the standard expected utility model. Intuitively, the multi-valued nature of $\mathcal{P}$ models Knightian uncertainty and the minimum delivers uncertainty aversion. Assume that $\mathcal{P}$ contains P, and is compact in the weak convergence topology. When $\mathcal{P} = \{P\}$, one obtains the standard expected utility model.

Gilboa and Schmeidler's (1989) axioms do not provide any structure on $\mathcal{P}$. In applications, a modeler usually has to impose some structure on $\mathcal{P}$ in order to obtain sharper results. One tractable specification is based on the entropy criterion. Formally, the set of priors is defined as

$$\mathcal{P}(P, \phi) = \left\{ Q \in \mathcal{M}(\Omega) : E_Q \left[\log \left(\frac{dQ}{dP} \right) \right] \leq \phi^2 \right\}, \ \phi > 0,$$

where $\mathcal{M}(\Omega)$ is the set of probability measures on Ω and dQ/dP denotes Radon-Nikodym derivative. This specification borrows from robust control theory (e.g., Anderson et al (2003), Hansen and Sargent (2000)). One interpretation related to statistics and econometrics is as follows. Interpret P as an approximating model. The model may be misspecified in the sense that there may be a set of models $\mathcal{P}(P, \phi)$. Each alternative in $\mathcal{P}(P, \phi)$ is evaluated according to the relative entropy index $E_Q \left[\log \left(\frac{dQ}{dP} \right) \right]$. This index is an approximation to the empirical log-likelihood ratio. The agent fears model misspecification and adopts robust decisions.

A special case studied by Kogan and Wang (2002) is as follows. Let P be the measure corresponding to a normal distribution with mean μ and variance σ^2. Let all probability measures in $\mathcal{P}(P, \phi)$ have normal distributions. Moreover, each measure Q in $\mathcal{P}(P, \phi)$ has a fixed variance σ^2 and a mean $\mu - v$ for some $v \in \mathbb{R}$. As shown in Kogan and Wang (2002), $\mathcal{P}(P, \phi)$ is isomorphic to the set

$$\mathcal{V}(\phi) = \left\{ v \in \mathbb{R} : \frac{1}{2} v^2 \sigma^{-2} \leq \phi^2 \right\}. \tag{3}$$

The parameter $\phi > 0$ models the degree of Knightian uncertainty. It can also be interpreted as an uncertainty aversion parameter. This specification will be adopted below.

Finally, the agent's decision problem can be described as

$$\max_{c_0, c_1 \geq 0} U(c_0, c_1) \tag{4}$$

subject to the budget constraint (2).

2.1 Income Uncertainty

I first consider the case of income uncertainty. That is, period one income y_1 is random, but the return R is constant. In order to obtain closed form solutions, I consider two utility specifications, exponential and quadratic vNM indexes, widely adopted in the consumption/saving literature.

For the exponential vNM index (CARA utility), the solution is summarized in the following proposition.

Proposition 1 *Under income uncertainty, if the vNM index is given by*

$$u(c) = -\frac{1}{\theta}e^{-\theta c}, \theta > 0.$$

then the optimal saving demand is given by

$$s^* = \frac{1}{1+R}y_0 + \frac{1}{\theta(1+R)}\log(\beta R) + \log\left(\max_{Q\in\mathcal{P}} E_Q\left[e^{-\theta y_1}\right]\right). \tag{5}$$

Proof. Substitute the budget constraint (2) into the utility function, one obtains

$$\begin{aligned} U(c_0, c_1) &= -\frac{1}{\theta}e^{-\theta c_0} + \beta\min_{Q\in\mathcal{P}} E_Q\left[-\frac{1}{\theta}e^{-\theta c_1}\right] \\ &= -\frac{1}{\theta}e^{-\theta c_0} + \beta\min_{Q\in\mathcal{P}} E_Q\left[-\frac{1}{\theta}e^{-\theta(y_1+R(y_0-c_0))}\right] \\ &= -\frac{1}{\theta}e^{-\theta c_0} - \beta\frac{1}{\theta}e^{-\theta R(y_0-c_0)}\max_{Q\in\mathcal{P}} E_Q\left[e^{-\theta y_1}\right] \end{aligned}$$

The first-order condition is given by

$$e^{-\theta c_0} = R\beta e^{-\theta R(y_0-c_0)}\max_{Q\in\mathcal{P}} E_Q\left[e^{-\theta y_1}\right].$$

From this equation one can solve for optimal consumption

$$c_0^* = \frac{R}{1+R}y_0 - \frac{1}{\theta(1+R)}\log(\beta R) - \frac{1}{\theta(1+R)}\log\left(\max_{Q} E_Q\left[e^{-\theta y_1}\right]\right).$$

The optimal saving is derived from $s^* = y_0 - c_0^*$. ■

One can rewrite (5) as

$$\begin{aligned}s^* &= \frac{1}{1+R}y_0 + \frac{1}{\theta(1+R)}\log(\beta R) + \frac{1}{\theta(1+R)}\log\left(E_P\left[e^{-\theta y_1}\right]\right) \\ &\quad + \left\{\log\left(\max_{Q\in\mathcal{P}} E_Q\left[e^{-\theta y_1}\right]\right) - \log\left(E_P\left[e^{-\theta y_1}\right]\right)\right\}.\end{aligned}$$

The first three terms on the right hand side constitute the optimal savings in the standard model. The last term gives extra savings due to ambiguity. Moreover, this term is bigger if the agent is more ambiguity averse in the sense that the set of priors is larger.

In order to obtain a sharper characterization, assume that y_1 is normally distributed with mean μ_y and variance σ_y^2. Then the reference measure P corresponds to the normal distribution $N\left(\mu_y, \sigma_y^2\right)$. Take the set of priors $\mathcal{P}$ to be all measures corresponding to the set

$$\mathcal{V}(\phi) = \left\{v \in \mathbb{R} : \frac{1}{2}v^2\sigma_y^{-2} \le \phi^2\right\}, \tag{6}$$

as described previously. Since under any measure $Q \in \mathcal{P}$, y_1 is normally distributed with some mean $\mu_y - v$ and variance σ_y^2, where $v \in \mathcal{V}(\phi)$, one can show that

$$\log\left(\max_{Q\in\mathcal{P}} E_Q\left[e^{-\theta y_1}\right]\right) = \log\left(\max_{v\in\mathcal{V}(\phi)} e^{-\theta(\mu_y - v) + \frac{1}{2}\theta^2\sigma_y^2}\right) = -\theta\left(\mu_y - \sigma\phi\right) + \frac{1}{2}\theta^2\sigma_y^2.$$

One can also derive

$$\log\left(E_P\left[e^{-\theta y_1}\right]\right) = -\theta\mu_y + \frac{1}{2}\theta^2\sigma_y^2.$$

Thus, the optimal saving rule is given by

$$s^* = \frac{1}{1+R}\left(y_0 - \mu_y\right) + \frac{1}{\theta(1+R)}\log(\beta R) + \frac{1}{1+R}\frac{\theta\sigma_y^2}{2} + \frac{1}{1+R}\phi\sigma_y.$$

The interpretation of the right hand side is transparent. The first term accounts for savings in anticipation of possible future declines in labor income. This is consistent with the permanent income hypothesis of Milton Friedman (1957). The second term accounts for savings (dissavings) due to impatience when the discount rate is lower than the interest rate or $\beta R > 1$ ($\beta R < 1$). The third term accounts for precautionary savings due to riskiness of labor income. It is proportional to the variance σ_y^2 of labor income and risk aversion parameter θ.

The special feature of my model is the presence of the last term. This term can be interpreted as precautionary savings due to Knightian uncertainty. It is proportional to the degree of Knightian uncertainty measured by the parameter ϕ. Moreover, this component of precautionary savings is first order in the sense that it is proportional to the standard deviation σ_y, instead of the variance σ_y^2 as in the standard model. In terms of testable implications, the model implies that a large component of the observed precautionary savings in the data may be attributed to Knightian uncertainty rather than risk.

In the standard model, the presence of precautionary savings are usually related to the third derivative of the vNM index (e.g., Leland (1968)). I now present a model with quadratic utility where precautionary savings can arise because of Knightian uncertainty.

Let the vNM index be

$$u(c) = -(b-c)^2, \; b > c,$$

where b is a bliss point. The agent's problem becomes

$$\begin{aligned} & \max_{c_0 \geq 0} \left\{ -(b-c_0)^2 + \beta \min_{Q \in \mathcal{P}} E_Q \left[-(b-c_1)^2 \right] \right\} \\ = \; & \max_{c_0 \geq 0} \left\{ -(b-c_0)^2 + \beta \min_{Q \in \mathcal{P}} E_Q \left[-(b-y_1-R(y_0-c_0))^2 \right] \right\}. \end{aligned}$$

I still assume that y_1 is normally distributed with mean μ_y and variance σ_y^2.

Proposition 2 *Assume* $\beta R = 1$ *and*

$$u(c) = -(b-c)^2, \; b > c.$$

Also assume the set of priors is described by (6). Then the optimal saving rule is given by

$$s^* = \frac{1}{1+R}(y_0 - \mu_y) + \frac{\sigma_y \phi}{1+R}.$$

Proof. First, one can derive that

$$
\begin{aligned}
& \min_{Q\in\mathcal{P}} E_Q\left[-\left(b-y_1-R\left(y_0-c_0\right)\right)^2\right] \\
= \;& \min_{Q\in\mathcal{P}} E_Q\left[-\left(b-R\left(y_0-c_0\right)-y_1+\mu_y-v-\mu_y+v\right)^2\right] \\
= \;& -\sigma_y^2-\max_{v}\left(b-R\left(y_0-c_0\right)-\mu_y+v\right)^2 \\
= \;& -\sigma_y^2-\left(b-R\left(y_0-c_0\right)-\mu_y+\sigma_y\phi\right)^2.
\end{aligned}
$$

Thus, the first order condition is given by

$$
\left(b-c_0\right)=\beta R\left(b-R\left(y_0-c_0\right)-\mu_y+\sigma_y\phi\right).
$$

When $\beta R=1$, the first-order condition can be simplified to yield optimal consumption

$$
c_0^*=\frac{R}{1+R}\left(y_0+\frac{\mu_y}{R}\right)-\frac{\sigma_y\phi}{1+R}.
$$

The expression for optimal saving follows from $s^*=y_0-c_0^*$. ■

The presence of the second term is due to Knightian uncertainty, which vanishes in the standard expected utility model (see Hall (1978)). Thus, different from the standard model, under Knightian uncertainty precautionary savings can arise even for quadratic utility.

2.2 Capital Uncertainty

In the previous section, uncertainty comes from the future labor income. I have shown that Knightian uncertainty induces a separate component of precautionary savings. I now consider the case where uncertainty comes from the return to saving, R.

For simplicity, assume that the agent does not receive any labor income in period 1. Then the agent has the budget constraint

$$
c_1=R\left(y_0-c_0\right).
$$

Let the vNM index be CRRA,

$$
u\left(c\right)=\frac{c^{1-\alpha}}{1-\alpha},
$$

where $\alpha > 0$ is the coefficient of relative risk aversion or the inverse of the coefficient of intertemporal substitution. Now the agent's problem is given by

$$\max_{c_0 \geq 0} \left\{ \frac{c_0^{1-\alpha}}{1-\alpha} + \beta \min_{Q \in \mathcal{P}} E_Q \left[\frac{(R(y_0 - c_0))^{1-\alpha}}{1-\alpha} \right] \right\}. \tag{7}$$

Note that the last term depends crucially on the parameter α. The solution is given in the following proposition

Proposition 3 *Assume*

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}.$$

Then the optimal saving rule is given by

$$s^* = \begin{cases} \frac{1}{1+\left(\beta \min_{Q\in\mathcal{P}} E_Q[R^{1-\alpha}]\right)^{-1/\alpha}} y_0, & \text{for } \alpha \in (0,1), \\ \frac{1}{1+\left(\beta \max_{Q\in\mathcal{P}} E_Q[R^{1-\alpha}]\right)^{-1/\alpha}} y_0. & \text{for } \alpha > 1, \\ \frac{\beta}{1+\beta} y_0 & \text{for } \alpha = 1. \end{cases}.$$

Proof. For $\alpha \in (0,1)$, the last term in (7) is given by

$$\min_{Q \in \mathcal{P}} E_Q \left[\frac{(R(y_0 - c_0))^{1-\alpha}}{1-\alpha} \right] = \frac{(y_0 - c_0)^{1-\alpha}}{1-\alpha} \min_{Q \in \mathcal{P}} E_Q \left[R^{1-\alpha} \right]$$

Thus, the first order condition is

$$c_0^{-\alpha} = \beta (y_0 - c_0)^{-\alpha} \min_{Q \in \mathcal{P}} E_Q \left[R^{1-\alpha} \right]$$

Simplifying yields optimal consumption.

$$c_0^* = \frac{(\beta \min_{Q\in\mathcal{P}} E_Q [R^{1-\alpha}])^{-1/\alpha}}{1 + (\beta \min_{Q\in\mathcal{P}} E_Q [R^{1-\alpha}])^{-1/\alpha}} y_0.$$

The optimal saving rule is

$$s^* = \frac{1}{1 + (\beta \min_{Q\in\mathcal{P}} E_Q [R^{1-\alpha}])^{-1/\alpha}} y_0.$$

When $\alpha > 1$,

$$\min_{Q\in\mathcal{P}} E_Q\left[\frac{(R(y_0-c_0))^{1-\alpha}}{1-\alpha}\right] = \frac{(y_0-c_0)^{1-\alpha}}{1-\alpha}\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right].$$

Thus the first-order condition is

$$c_0^{-\alpha} = \beta(y_0-c_0)^{-\alpha}\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right].$$

Simplifying yields optimal consumption

$$c_0^* = \frac{\left(\beta\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right]\right)^{-1/\alpha}}{1+\left(\beta\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right]\right)^{-1/\alpha}}y_0.$$

The optimal saving is

$$s^* = \frac{1}{1+\left(\beta\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right]\right)^{-1/\alpha}}y_0.$$

Finally, the case of $\alpha = 1$ follows immediately from the above calculation. ■

As in the standard model where the optimal saving rule is given by

$$s^{**} = \frac{1}{1+\left(\beta E_P\left[R^{1-\alpha}\right]\right)^{-1/\alpha}}y_0,$$

the optimal saving rule under Knightian uncertainty is also linear in wealth. Moreover, for a log utility agent it is independent of capital uncertainty. This is due to the additivity of a log function of products:

$$\log R(y_0-c_0) = \log R + \log(y_0-c_0).$$

The following two corollaries follow immediately from Proposition 3.

Corollary 4 *If $\alpha \in (0,1)$, then $s^* \leq s^{**}$. If $\alpha > 1$, then $s^* \geq s^{**}$.*

Corollary 5 *Suppose $\mathcal{P}_1 \subset \mathcal{P}_2$. Let s_1^* and s_2^* be the optimal saving corresponding to $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively. Then $s_1^* \geq s_2^*$ if $\alpha \in (0,1)$; and $s_1^* \leq s_2^*$ if $\alpha > 1$.*

These two corollaries imply that, in contrast to the case of income uncertainty, uncertainty aversion does not necessarily lead to higher savings under capital uncertainty. The parameter α is important for the comparison. Specifically, if $\alpha \in (0,1)$, an uncertainty averse agent saves less than an expected utility maximizer. Further, the agent saves less if he is more uncertainty averse or there is a higher degree of Knightian uncertainty. By contrast, if $\alpha > 1$, an uncertainty averse agent saves more than an expected utility maximizer. Moreover, the agent saves more if he is more uncertainty averse or there is a higher degree of Knightian uncertainty.

The intuition is similar to that discussed in Sandmo (1970). An increase in the degree of Knightian uncertainty makes the agent less inclined to expose his resources to the possibility of loss; hence the negative substitution effect on saving. On the other hand, higher uncertainty makes it necessary to save more in order to protect oneself against low levels of future wealth. This results in the positive income effect on saving. The overall effect depends on the intertemporal substitution parameter $1/\alpha$.

The analysis is more transparent if one adopts the entropy-based specification of the set of priors described before. Formally, let the reference measure for $\log(R)$ be the normal distribution with mean μ_R and variance σ_R^2. Also assume the set of priors is isomorphic to the set

$$\mathcal{V}(\phi) = \left\{ v \in \mathbb{R} : \frac{1}{2} v^2 \sigma_R^{-2} \leq \phi^2 \right\}. \tag{8}$$

Now, for $\alpha \in (0,1)$, one can show that

$$\begin{aligned} \min_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right] &= \min_{Q\in\mathcal{P}} E_Q\left[e^{(1-\alpha)\log R}\right] \\ &= \min_{v\in\mathcal{V}(\phi)} e^{(1-\alpha)(\mu_R - v)+\frac{1}{2}(1-\alpha)^2\sigma_R^2} \\ &= e^{(1-\alpha)(\mu_R-\sigma_R\phi)+\frac{1}{2}(1-\alpha)^2\sigma_R^2} \end{aligned}$$

It follows from Proposition 3 that the optimal saving rule is

$$s^* = \frac{1}{1 + e^{\frac{1-\alpha}{\alpha}\sigma_R\phi}\left(\beta e^{(1-\alpha)\mu_R+\frac{1}{2}(1-\alpha)^2\sigma_R^2}\right)^{-1/\alpha}} y_0.$$

It is clear that the saving demand is decreasing with the parameter ϕ.

Similarly, for $\alpha > 1$, one can show that

$$\max_{Q\in\mathcal{P}} E_Q\left[R^{1-\alpha}\right] = \max_{v\in\mathcal{V}(\phi)} e^{(1-\alpha)(\mu_R - v)+\frac{1}{2}(1-\alpha)^2\sigma_R^2} = e^{(1-\alpha)(\mu_R-\sigma_R\phi)+\frac{1}{2}(1-\alpha)^2\sigma_R^2}.$$

It follows from Proposition 3 that the optimal saving rate is given by

$$s^* = \frac{1}{1+e^{-\frac{\alpha-1}{\alpha}\sigma_R\phi}\left(\beta e^{(1-\alpha)\mu_R+\frac{1}{2}(1-\alpha)^2\sigma_R^2}\right)^{-1/\alpha}} y_0.$$

Thus, the saving demand is increasing with the parameter ϕ.

3 Conclusion

This paper analyzes consumption/saving decisions under Knightian uncertainty in a two-period setting. It is shown that income uncertainty and capital uncertainty have different effects on optimal savings. The model can be easily generalized to a dynamic setting. For example, one can adopt the recursive multiple-priors utility model studied in Epstein and Wang (1994) and axiomatized by Epstein and Schneider (2002). A closed form solution can still be derived for CARA and CRRA utility using similar methods to Phelps (1969), Levhari and Srinivasan (1969) and Caballero (1990). It can be shown that qualitative results do not change.

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