Competing for Ownership*

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Abstract

We study how the internal organization of firms – specifically, the allocation of ownership of assets and the distribution of profit among the firm's managers – is determined in a competitive market. We ask how scarcity of assets, skills or liquidity in the market translates into ownership and control allocations within organizations. Firms will be more integrated when the terms of trade are more favorable to the short side of the market, when liquidity is unequally distributed among existing firms and when there is a positive uniform shock to productivity. The model identifies a price-like mechanism whereby local liquidity or productivity shocks propagate and lead to widespread organizational restructuring.

1 Introduction

In the neoclassical theory of the firm, market signals affect choices of products, factor mixes, and production techniques: if the price of output rises, quantity produced increases; if wages rise, fewer workers will be hired and relatively greater use will be made of machines. Firms' decisions in turn feed back to the market: increases in the number of goods produced will lower the product price, reductions in the number of workers hired will induce wages to

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fall. Thus the market provides a mechanism whereby shocks to a few firms – say, an improvement in their technology – propagate to the rest of the economy, inducing other firms to readjust their production plans. Because these feedback effects are so well established, the neoclassical firm remains the backbone of much of applied economic analysis.

The modern theory of the firm emphasizes contractual incompleteness, agency problems, and the resulting importance of organizational design elements such as task allocation, compensation schemes, asset ownership, and the assignment of authority and control (e.g., Jensen and Meckling, 1976; Grossman and Hart, 1986; Hart and Moore, 1990; Aghion and Bolton, 1992; Holmström and Milgrom, 1994). By introducing a rich set of new variables into economic analysis, it has made breakthroughs in our understanding of economic institutions as different as modern corporation and the sharecropped farm. Yet despite its original and still primary goal – clear since Coase (1937) asked his fundamental questions on the nature of the firm – of understanding firms that operate in market economies, rather little has been done to investigate the influence of the price mechanism on internal organizational decisions, much less on how those decisions feed back to the market and to other firms.

The purpose of this paper is to provide a simple framework for the analysis of this kind of interaction. We consider an economy in which pairs of production units – each consisting of a manager and a collection of assets – must produce together in order to generate marketable output. Firms comprising a pair of units are formed through a competitive matching process, in which contracts specifying the elements of the organizational design – an ownership structure and a profit sharing scheme – are determined. Our main concern is with how scarcity in the market affects the choice of organizational design and with how changes in the fundamentals of some firms can spill over, via a pecuniary externality, to economy-wide reorganizations.

In an individual production unit, an asset's contribution to profit depends on actions taken by the managers, which we decompose into two complementary components that we call *operation* and *logistics*. Operating effort can only be chosen by the manager initially associated with the asset and is not contractible. Logistical decisions are not contractible either, but the right to make them can be allocated via contract to either manager. For simplicity we assume that logistical choices (e.g., choosing the background music for a retail store) are costless. But while potentially beneficial for profits (some music is likely to induce consumers to make impulse purchases), those choices affect the private cost of operating decisions (such music may be unpleasant for the store's floor manager).

We identify ownership of an asset with the right to choose the logistical decision. Different ownership allocations generate different externalities, both positive and negative, among the firm's decision makers. If a manager retains ownership, he bears all the cost, but only shares in the benefit of the logistical decision, and therefore underprovides logistics. If he cedes ownership, his partner bears none of the cost but derives a positive benefit, and therefore she overprovides logistics.

This trade-off between retaining and ceding ownership of the asset will be a key consideration in the firm's organizational design. Of course, incentives in real firms are also affected by compensation schemes; we accomodate this by allowing for assignments of ownership to be bundled with shares of profit streams. Hence, the full contracting problem comprises a choice of ownership allocation and profit shares.

In the supplier market where firms compete for partners, we assume that the number of units on each "side" is unequal so that the terms of trade are determined by the willingness to pay of a marginal unit the long side: its manager will be indifferent between matching and staying out of the market. The terms of trade determined in the equilibrium of this market govern the division of surplus between managers, and this in turn determines the way those managers will organize their firms.

In our model, "liquidity," which refers to instruments such as cash that can be transferred costlessly and without any incentive distortions, is scarce. If it were not, it would suffice to focus on the contract that delivers the highest joint surplus to the managers: all firms would choose this organization and accomplish the surplus division with cash alone. This approach to predicting organizational design has been popular in much of the recent literature on the firm.

However, when liquidity is in short supply relative to the value of the transactions in question, firms will have to use the organizational variables – ownership allocations and sharing rules – to accomplish the surplus division commanded by the market. That profit shares serve as a means of surplus division is obvious enough, but they cannot do so neutrally because arbitrary divisions will adversely affect incentives. And the ownership allocation has a similar surplus-division role: awarding ownership to one manager gives him higher surplus, since it ensures that more of the ensuing logistical decisions will go in his preferred direction; the downside is that these decisions may pose significant costs on the other members of the firm.

Our model admits a continuum of ownership allocations as well as the usual continuum of sharing rules, and this feature facilitates studying how these two instruments covary in distributing surplus. Starting from the allocation in which managers get equal amounts of surplus, the first organizational variable to be distorted is the sharing rule; in this "refinance" regime, the firms remain non-integrated, and all that adjusts is the profit share. Ownership allocations are distorted only at more uneven surplus shares; in this "restructuring" regime, the manager receiving the preponderance of the surplus also owns most of the assets. In this regime, the share may not adjust at all in response to further surplus transfers, but if it does, it covaries positively with ownership. A manager for whom the market awards a large share of the surplus from production, and whose partner has little cash, will receive a large profit share and own most of the firm's assets.

The surplus that each partner obtains from a given contract is therefore a function of the characteristics of the relationship, in particular the production technology and the liquidity available. Higher productivity or more liquidity in the firm not only enlarges the feasible set but also "flattens" the frontier, that is, it increases the transferability of surplus. If productivity is high, managers have a high opportunity cost of failing to maximize profit; if firms have more liquidity, they can avoid using inefficient contractual instruments. Hence, a firm that receives a positive shock to its productivity or liquidity endowment will be able to accomplish surplus division more efficiently and reduce organizational distortions. We refer to this as the *internal effect.*

But such a shock may have much wider effects than on the firm that experiences it. The internal effect implies that a manager has effectively a higher "ability to pay" for a partner after a positive shock than before. He may therefore bid up the terms of trade in the supplier market: in order to meet the new price, firms which have not benefited from the shock will have to refinance and/or restructure. Thus the shock may have an *external effect*: "local" shocks may propagate via the market mechanism, leading to widespread reorganization.

The market equilibrium turns out to be amenable to a Marshallian supply-demand style of analysis, making the role of the external effect especially transparent. And while the internal effects of positive shocks to liquidity and technology are similar – they both decrease integration – the external effects are quite different. For instance, a uniform increase in the liquidity level of all agents lowers the degree of integration in all firms (the internal effect dominates the external effect). By contrast, a uniform shock to productivity increases the degree of integration in all firms (the external effect dominates the internal effect). As we show in Section 4, the model can capture quite simply the effects of more complex changes in the liquidity endowments or in productivity.

The external effect may also operate following a change in the relative scarcities of production units on the two sides. Suppose, for instance, that the short side of the market represents downstream producers and the long side their upstream suppliers. An increase in the supply of downstream units will raise the share of surplus accruing to upstream units: downstream managers will find their shares of profit lowered and upstream managers will own more assets.

Hence, the model offers a mechanism by which changes in traditional economic fundamentals – endowments, technology, or numbers participants – will manifest themselves as widespread reorganizations, sometimes in a direction opposite to what the internal effect would suggest.

In the next section we introduce the model and solve the basic contracting problem, which enables us to show how various divisions of the surplus between a pair of managers map into different organizational choices. This relationship is summarized in Sections 2.1.2 and 2.2 by Figures 1 and 2 and Proposition 5. The reader who is not interested in the details of the contracting problem may skip directly there before proceeding on to Sections 3 and 4, which discuss the market equilibrium and its comparative statics. Section 5 concludes with an informal discussion of other comparative-static results generated by the model and its extensions.

2 Model

In the economy we consider, profit is generated through the cooperation of two production units, one of each type $\theta = 1, 2$. Each unit consists of a risk-neutral manager and a collection of assets. Many interpretations are possible: the two types of manager might be supplier and manufacturer, and the assets plant and equipment; a chain restaurateur and franchising corporation (in which case some of the assets are reputational); or as a firm and its workforce, for which the assets might be interpreted as tasks.

Whatever the interpretation, we have in mind competitive outcomes, and so we suppose that there is a large number of both types of production unit: each side of the market is a continuum with Lebesgue measure, and we shall assume that the 2's are relatively scarce: the type 1's are represented by $i \in I = [0, 1]$ while the type 2's are represented by $j \in J = [0, n]$, where $n < 1$.

The i-th type-1 manager will have at her disposal a quantity $l(i) \geq 0$ of cash (or "liquidity") which may be consumed at the end of the period and which may be useful in contracting with managers of the opposite type; for the type 2's, the liquidity endowment is $\hat{l}(j)$. The indices i and j have been chosen in order of increasing liquidity; it is convenient to further assume that the corresponding liquidity distributions have strictly positive densities:

Assumption 1 The liquidity endowment functions $l(i)$ and $\hat{l}(j)$ are strictly increasing and continuous.

When discussing a generic production unit or its manager, we shall usually drop the indices.

2.1 The Basic Organizational Design Problem

Technology and Preferences. The collection of assets in the type-θ production unit is represented by a continuum indexed by $k \in [\theta - 1, \theta)$. An asset's contribution to profit is proportional to $q(k)\, e(k)$ where $q(k) \in [0, 1]$ is logistics and $e(k) \in \{0, 1\}$ is operating effort. While either manager is capable of choosing logistical decisions, operating effort $e(k)$ can only be chosen by the type-θ manager for $k \in [\theta - 1, \theta)$. The firm's profit will be proportional to the integral of the contributions of all assets in the pair of production units.

There is no logistics cost, but the (private) operating cost to the manager on asset k is $C(q(k), e(k)) = \frac{1}{2} q(k)^2 + ce(k)$,where $c \geq 0$.[1] This is the cost externality we alluded to: the cost to the manager operating the asset is increasing in $q(k)$, whether or not he has chosen it.

Consider for instance a two-man delivery service run comprised of a diminutive dynamo and a soulful brute. Delivery requires the brute to do the heavy lifting (e); driving the truck can be allocated to either man. The brute is intimidated by the dynamo's efficient driving (high q), which enhances profits by permitting many deliveries a day. If the brute drives, he is cautious, which though better for his stomach, reduces profits.

Or in a manufacturing enterprise, e could represent managerial attention devoted to overseeing assembly, supervising workers, and so on, while q could index choices of possible parts or material inputs, ordered by the value they contribute to the final product. Each input choice requires solving a number of manufacturing process problems; we are supposing that higher value inputs require greater learning and adaptation effort on the part of the manufacturer's management.[2]

[1] We could just as well suppose that $e(k)$ is continuous, with values taken in $[0, 1]$: with this specification of costs, interior values of e will never be chosen in equilibrium: see Appendix.

[2] In the 1960's, W. Corporation owned an electronic systems division that manufactured airplane cockpit voice recorders, and a composite materials division that made various compounds suitable for heat-resistant recording tape, a critical input for recorders. The electronic systems division had perfected a manufacturing process that used mylar tape, but W. ordered them to use a new metal-oxide tape developed by its materials division. The new tape was less flexible than mylar, and therefore subject to kinking and breakage, which raised manufacturing problems that required nearly a year of process redevelopment to resolve. A former manager of the systems division admits that had it been up to him, his division would have stuck with the mylar tape, simply because the experimentation and

Since manager 1 bears cost on $k \in [0,1)$ and manager 2 bears cost on $k \in [1,2)$, we write

$$\begin{aligned} C_1(q,e) &= \int_0^1 C(q(k), e(k))\, dk \\ C_2(q,e) &= \int_1^2 C(q(k), e(k))\, dk. \end{aligned}$$

The managerial decisions contribute to the firm's performance as follows. The firm either succeeds, generating profit $R > 1$, with probability $p(q,e)$; or it fails, generating 0, with probability $1 - p(q,e)$. Here $q : [0,2) \to [0,1]$ are the logistical decisions and $e : [0,2) \to \{0,1\}$ are the operating decisions. The success probability functional is

$$p(q,e) = \rho \int_0^2 e(k) q(k) dk,$$

where $\rho < \frac{1}{2}$ is a technological parameter. For the trade-off between monetary gains and private costs to be operative, that is to allow a variety of Pareto optimal contractual forms, ρR must not be too large (otherwise the monetary profit motive will make incentive provision trivial) nor too small with respect to c (otherwise it is impossible to provide incentives for operating effort).

Assumption 2 Define $A \equiv \rho R$. Then $A \leq \frac{1}{2}$ and $c \leq \frac{A^2}{8}$.

We shall find it convenient to measure a manager's payoff in terms of his *surplus,* which we define as the expected value of final income less private costs less initial liquidity.

Contracts. We make the following contractibility assumptions:

Assumption 3 (1) The decisions (q,e) are not contractible.
(2) The ability to choose $e(k)$ is not alienable.
(3) The *right to decide* $q(k)$ is both alienable and contractible.[3]
(4) The costs $C_i(q,e)$ are private and noncontractible.
(5) The realized profit is contractible.

retooling costs were not (and because of verifiability and incentive problems likely could never be) appropriately reimbursed, even if the metal oxide tape arguably had slightly better heat-resistance and recording properties.

[3]See Aghion et al. (2004) for similar assumptions. Given the timing of decisions in our model, we could just as well assume $e(k)$ is private information and that $q(k)$ is not observable to third parties. Even if it is reasonable to assume that $q(k)$ is revealed to the other manager after it is chosen, it is then too late to renegotiate or use a message game to enforce a decision: at that point, the two parties' preferences are independent of what has gone on before.

Represent the ownership allocation by a fraction ω of assets re-assigned to one of the managers (by the symmetry of the assets, there is no loss of generality in doing it this way).[4] The type-1 manager owns assets in $[0, 1-\omega)$, where $-1 < \omega \leq 1$, and the type-2 owns $[1-\omega, 2)$. Note that when 2 owns asset $k \in [0, 1)$, 2 chooses $q(k)$ while 1 chooses $e(k)$; hence changing the ownership allocation modifies both managers' the strategy sets.

When $\omega = 0$, each manager retains ownership of his original assets, and, following the literature, we refer to this situation as *non-integration.* As ω increases beyond 0, we have an increasing degree of integration (a growing fraction of the assets are owned by 2), until with $\omega = 1$ we have full integration. (The symmetric cases with $\omega < 0$ correspond to 1-ownership, but will not occur in any competitive equilibrium of our model, given the greater scarcity of 2's.) Since ω not only describes the ownership structure but also provides a scalar measure of the fraction owned by one party, we shall often refer to its (absolute) value as the *degree of integration* of the firm.

The managers sign a contract specifying the allocation of ownership and a sharing rule. Contracting is subject to the following two basic constraints.

Assumption 4 (Limited Liability) Incomes in all states must weakly exceed zero.

Assumption 5 (No Budget Breaker) The sharing rule pays a third party at least as much in the case of success as in the case of failure.

Assumption 4 is standard. Assumption 5 is made on plausibility as well as tractability grounds. It rules out contracts in which liquidity is pledged to a third party, to be forfeited in case of failure. Though such contracts would in principle strengthen the managers' incentives, they are somewhat implausible (because of third party incentives to sabotage the operation either on its own or via collusion with one of the managers). Moreover, ruling them out considerably simplifies the analysis of competitive equilibrium: the single-market supply-demand analysis we develop here would instead be rendered into a full-blown assortative matching problem with nontransferabilities (e.g., Legros and Newman, 1996, 2003), which nonetheless would yield similar conclusions.

These assumptions imply a simple characterization of the set of contracts. First, there is no use of outside finance. In principle, managers might decide

[4] This leaves out the logical possibility that the managers "swap" assets; in additon to ω, which indicates how many of 1's assets are shifted to 2, the contract would have an additional variable ψ indicating how many of 2's assets are shifted to 1. However, as we show in the appendix, under Assumption 2, asset swapping will never be Pareto efficient.

to borrow from the financial market in order to increase their liquidity at the time of contracting and therefore increase the lump sum transfer to the other manager. However, Assumptions 4 and 5 imply that such outside financing is strictly Pareto dominated by contracts that make no use of outside financing: if the upstream manager borrows, his incentives are weakened since he owes money to the lender, while for the downstream manager, having received the borrowed sum up front, there is no effect on his incentives; it would be better for the upstream manager simply to reduce his share of the profit, which has the same impact in the first instance on his incentives, but now strengthens those of his partner, thereby making both of them better off. (All proofs not appearing in the text are in the Appendix.)

Proposition 1 *Payoffs on the Pareto frontier are attained without the use of outside finance.*

Second, there is a *separation property*: in order to describe the set of utility levels that two managers can attain, it is enough to first calculate the set of payoffs that they could achieve if they had no liquidity and then to add to this a lump-sum division of their total liquidity. In other words, after signing, there will be a transfer of cash from one manager to the other; then decisions are taken, profit is realized, and shares distributed. The separation property says that the liquidity transfer has no effect on what is feasible with respect to the other aspects of the contract. Since there are only two profit levels, 0 and R, the sharing rule can be fully described by a single parameter s, $0 \leq s \leq 1$.

Lemma 2 *(Separation) Consider two managers 1 and 2 with total liquidity L. There is no loss of generality in restricting attention to contracts in which L is distributed in a lump sum fashion and in which manager 1 receives a fraction $(1-s)$ of the realized profit and manager 2 receives a fraction s of the realized profit.*

Solution of the game induced by (s, ω). Any contract (s, ω) induces a game between the managers in logistical and operating decisions, and to characterize the payoff possibilities, we seek the most efficient equilibrium for each choice of (s, ω).[5] For $\omega \geq 0$, the type-1 chooses $e_1(\cdot)$ and $q_1(\cdot)$ to

[5] We are making the assumption that all assets in the joint enterprise are "up and running," i.e., $e(k) = 1$ for all k. Although conceivably there are circumstances in which it would be desirable to shut down some of the assets in order to avoid imposing excessive costs on one of the managers, mild parameter restrictions will ensure such an arrangement is not Pareto optimal; details are in the appendix.

maximize

$$(1-s)A[\int_0^{1-\omega} e_1(k)q_1(k)dk + \int_{1-\omega}^1 e_1(k)q_2(k)dk + \int_1^2 e_2(k)q_2(k)dk]$$
$$-[\frac{1}{2}\int_0^{1-\omega} q_1(k)^2 dk + \frac{1}{2}\int_{1-\omega}^1 q_2(k)^2 dk + c\int_0^1 e_1(k)dk],$$

taking $e_2(\cdot)$, $q_2(\cdot)$ as given. Similarly, manager 2 chooses $e_2(\cdot)$ and $q_2(\cdot)$ to maximize

$$sA[\int_0^{1-\omega} e_1(k)q_1(k)dk + \int_{1-\omega}^1 e_1(k)q_2(k)dk + \int_1^2 e_2(k)q_2(k)dk]$$
$$-[\frac{1}{2}\int_1^2 q_2(k)^2 dk + c\int_1^2 e_2(k)dk],$$

Consider first situations where the manager both owns the asset and operates it. For $k \geq 1$, manager 2 chooses $e_2(k) = 1$ only if

$$sAq_2(k) - \frac{1}{2}q_2(k)^2 - c \geq 0$$

and will choose $q_2(k) = sA$ in this case. A necessary and sufficient condition for $e_2(k) = 1$ is then that $\frac{1}{2}s^2A^2 - c \geq 0$, or $s \geq \sqrt{2c}/A$. Similarly, for $k < 1-\omega$, manager 1 chooses $e_1(k) = 1$ and $q_1(k) = (1-s)A$ if and only if $1 - s \geq \sqrt{2c}/A$. The effect of these constraints is to bound feasible values of s away from 0 and 1. Define

$$\bar{s} \equiv 1 - \sqrt{2c}/A, \tag{1}$$

and note that by Assumption 2, $\bar{s} \geq \frac{1}{2}$ and therefore that the set $[1-\bar{s}, \bar{s}]$ is nonempty.

Next, for the assets $k \in [1-\omega, 1)$ that 2 owns but 1 operates, manager 1 sets $e_1(k) = 1$ only if $(1-s)Aq_2(k) \geq c$. Since 2 gets the benefit from a higher level of q on such assets, it is optimal for 2 to set $q_2(k) = 1$, its highest possible value, for all $k \in [1-\omega, 1)$. The incentive compatibility condition for $e_1(k) = 1$ reduces to $(1-s)A \geq c$, which is satisfied when the previous incentive compatibility conditions hold.[6]

Proposition 3 *Pareto efficient contracts satisfy* $s \in [1-\bar{s}, \bar{s}]$.

[6] By Assumption 2, $c < 1$; hence $\sqrt{2c} > c$ and the incentive compatibility condition $(1-s)A \geq \sqrt{2c}$ implies indeed $(1-s)A > c$ for any $s \in [1-\bar{s}, \bar{s}]$.

As we said earlier, part of the role of an organizational variable is to transfer surplus. We have already discussed the limits of this with respect to s. We now show that in the present model, ω broadly plays the surplus-transfer role as well. Indeed, from the individual point of view, owning more assets is better:

Proposition 4 *Given a sharing rule s, a manager's payoff is nondecreasing in the fraction of assets he owns.*

The argument is by revealed preference. The owner of an asset can always replicate the decisions that would be made on that asset if he didn't own it; (unlike in hold-up models, there is no strategic decision by his partner *before* the logistical decision is made that might make owning worse for him). The result then follows from the additivity of the payoffs over the assets.

Given a feasible contract (s, ω) 1's payoff taking into account the play of the induced game can be written

$$u_1(s,\omega) = \frac{1-s^2}{2}A^2 - c - \omega\frac{((1-s)A-1)^2}{2} \tag{2}$$

while 2 gets

$$u_2(s,\omega) = \frac{s(2-s)}{2}A^2 - c + \omega s A(1-(1-s)A), \tag{3}$$

The next step is to build up the payoff possibility frontier for any pair of type-1 and type-2 managers. Each point on the frontier will be generated by a different organizational arrangement, i.e., choice of sharing rule s and ownership structure ω. The separation result of Proposition 2 facilitates the derivation, since the complete frontier can be constructed by first considering the payoffs from all feasible organizations and then adding the liquidity in at the end.

2.1.1 Surplus maximizing contracts

Given the incentive problems arising from contractual incompleteness, it should come as no surprise that the first-best solution (in which $q(k) = A$ and $e(k) = 1$ for all k) cannot be attained. However, we can still ask what are the second-best contracts, given the constraints in contractibility. The total surplus generated by a contract (s, ω) is $W(s, \omega) = u_1(s, \omega) + u_2(s, \omega)$.

The optimal ownership structure trades off underprovision against overprovision of q. As we have said, having ownership of an asset one operates entails underprovision since $q(k) < A$, while ceding ownership entails over-provision by the new owner since $q(k) = 1 > A$. When the productivity parameter A is large enough, profit matters more than private costs to the managers, and total surplus is maximized by giving (nearly) full ownership to one manager.[7] This is simply a variant of the oft-noted point that ceding control over decisions to someone else may be a useful commitment device.

However, this case is of less interest here, partly because it overstates the benefits of integration (taken to its logical extreme, there ought to be only one giant firm in the economy), but mainly because there is little to analyze absent a trade-off between surplus generation and surplus division. As the type 2's are relatively scarce, the market will tend to assign the preponderance of surplus to them, and this would be accomplished by giving them ownership without much loss of efficiency. Moderate changes in market conditions would have no effect on the internal organization of the firm, and large changes, say by reversing the relative scarcity of 1's and 2's, would change the identity of the owners but not the form of organization.

Things are quite different, however, under Assumption 2. In this case, the costs generated by logistical decisions figure prominently enough in the managers' calculations to render the surplus production/surplus division trade-off nontrivial. The surplus-maximizing contract is $(1/2, 0)$, i.e., *non-integration with equal shares*. From the symmetry of the problem, this contract allocates equal surplus to the two parties, and we denote it $u_=$.

Absent sufficient liquidity, organizational choices will be made to accomplish the surplus division called for by the market, and in general this will entail a deviation from $(1/2, 0)$. In order to see precisely how this occurs, we shall need to derive the "pre-liquidity" Pareto frontier for the pair of managers, each point of which will correspond to a different organizational arrangement.

2.1.2 The Pre-liquidity Frontier U

We call $u_\theta(s, \omega)$ the type θ's surplus generated by the organization, or *generated surplus* for short, since it represents the surplus the type θ reaps from the organizational variables s and ω net of any ex-ante liquidity transfers.

The pre-liquidity frontier $\phi(u_1)$ is constructed by maximizing 2's surplus over s and ω subject to the guarantee of a surplus of u_1 to 1:

[7] We say "nearly," because putting $\omega = 1$, which in turn implies $s = \bar{s}$, typically violates manager 1's individual rationality.

$$\phi(u_1) \equiv \max_{s \in [1-\bar{s},\bar{s}],\omega} \frac{s(2-s)}{2} A^2 - c + \omega s A (1-(1-s)A) \tag{4}$$

$$\text{s.t. } \frac{1-s^2}{2} A^2 - c - \omega \frac{(1-(1-s)A)^2}{2} \geq u_1$$

We denote the set of payoffs $(u_1, \phi(u_1))$ by U. We shall describe the solution to this problem here; details are in the Appendix. Starting at the 45°-line, where $\phi(u_1) = u_1 = u_=$ with $s = 1/2$ and $\omega = 0$, decreasing u_1 and therefore increasing 2's surplus can be accomplished through two instruments, namely *refinancing* (increasing s) and *restructuring* (increasing ω). For small deviations from equal payoffs, the best way to transfer surplus is via refinance alone: though this will distort incentives on the q's, the surplus loss is only second order, and this is preferable to shifts of ownership, which result in large changes in the q's on the assets that change ownership, and therefore first order losses in surplus. Eventually, though, one begins to use restructuring: there is a share level $s^* > 1/2$ above which it is optimal to raise ω as well as s when u_1 falls, *provided s doesn't already equal $\bar{s}$.*[8] Above $\bar{s}$, only restructuring is available as an instrument, so that if $\bar{s} < s^*$, one uses either refinancing or restructuring, never both together. We will focus on this parametric case .

Assumption 6 $s^* > \bar{s}$.[9]

Proposition 5 *Under Assumption 6, the solution to problem (4) is characterized by two intervals of 1's generated surplus u_1, $[0, \underline{u}]$ and $(\underline{u}, u_=]$, such that*

(i) The type-2's profit share is constant ($s = \bar{s}$) and the degree of integration ω is linear and strictly decreasing in u_1 on $[0, \underline{u}]$;

(ii) There is nonintegration ($\omega = 0$) and s is strictly decreasing and strictly concave in u_1 on $[\underline{u}, u_=]$;

(iii) The Pareto frontier ϕ is linear on $[0, \underline{u}]$, strictly concave on $[\underline{u}, u_=]$, and concave on $[0, u_=]$;

(iv) The total surplus W is increasing concave in u_1 on $[0, u_=]$.

It is straightforward to check that $\underline{u} = A\sqrt{2c} - 2c$, $u_= = \frac{3}{8}A^2 - c$.

[8] The cutoff value s^* is the unique level of s for which both first order conditions of a relaxed version of (4) – in which the constraint $s \in [1-\bar{s}, \bar{s}]$ is ignored – are satisfied as equalities at $\omega = 0$.

[9] The restriction is that c is not too small in terms of A, specifically that $c > \frac{1}{2}\left[\frac{2}{3}A + \frac{1}{6} - \frac{1}{6}\sqrt{(1+8A-8A^2)}\right]^2$.

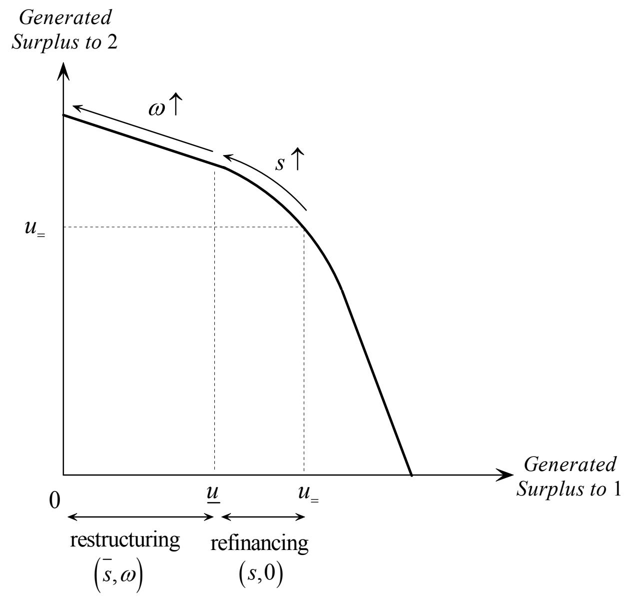

Figure 1: Two regimes of reorganization

Thus in the upper half quadrant, the frontier can be divided into two "zones": the "refinance" zone, in which movements along the frontier are accomplished via changes in s alone, and the "restructuring" zone where it is ω that varies. Notice that as 1's payoff decreases, the number of assets 2 owns (weakly) increases. At the same time total surplus is decreasing; thus it is fair to say that here *reallocations of ownership are used to transfer surplus, not merely to generate it.*

Assumption 6 has the analytic advantage that it keeps the frontier function ϕ concave and the degree of integration ω convex.[10] While we shall focus on this case, most of our results do not depend on this simplification, and we discuss the alternate case in the Appendix. A further strengthening of Assumption 6 is to set parameters so that $\bar{s} = 1/2$ (this is equivalent to $c = A^2/8$); then $\underline{u} = u_=$, and the frontier in each half quadrant is linear.

2.2 Completing the Picture: Adding Liquidity

The endowments of a matched pair of managers with liquidities $(l, \hat{l})$ expands their joint surplus possibilities relative to those generated by the set

[10] When $s^* < \bar{s}$, we show in the Appendix that the solution to (4) is characterized by three intervals $[0, \underline{u}]$, $(\underline{u}, u^*]$, $(u^*, u_=]$ such that (1) $s = \bar{s}$ and ω is strictly decreasing in u_1 in $[0, \underline{u}]$; (2) s and ω are strictly decreasing in u_1 in $(\underline{u}, u^*]$; and (3) $\omega = 0$ and s is strictly decreasing on $(u^*, u_=]$. While ϕ is still concave locally in the two extreme intervals, it is convex when $u_1 \in (\underline{u}, u^*]$.

of contracts (s, ω) by allowing for one-for-one utility transfers between these managers (as we have observed, organizational and financial changes do not allow for such efficient transfers except perhaps locally). But by Proposition 2 the liquidity levels do *not* affect the surpluses generated from the contracts (s, ω) themselves. Thus, modifying the frontier $\phi(\cdot)$ constructed so far to take account of the liquidity endowments of the managers is quite simple: one need only add it to a line segment whose endpoints are $(-l, l)$ and $(\hat{l}, -\hat{l})$, since the line segment describes all possible liquidity transfers between the managers.

Let U be the set of payoffs (u_1, u_2) on the pre-liquidity frontier and $\hat{U} = U - \Re_+$ its comprehensive extension. Denoting the , the set of lump-sum transfers for $(l, \hat{l})$ by $T\left(l, \hat{l}\right) = \left\{(t_1, t_2) : t_1 + t_2 = 0, -l \leq t_1 \leq \hat{l}\right\}$, the set of feasible surplus payoffs for a partnership $(i, j) \in I \times J$ is $\hat{U} + T\left(l\left(i\right), \hat{l}\left(j\right)\right)$. Figure 1 is an illustration of this construction for the special case in which $\bar{s} = \frac{1}{2}$, so that U is piecewise linear. Note that in the upper half quadrant, the set of feasible surpluses is equivalently described by $\hat{U} + T\left(l\left(i\right), 0\right)$, that is the liquidity of type 2 does not matter.

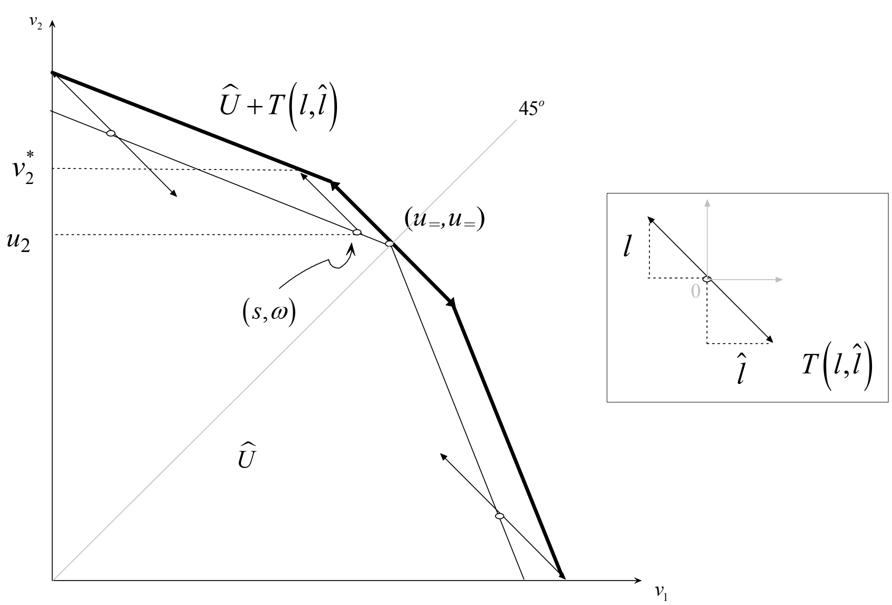

Figure 2: Feasible set

Notice that in the special case that agents with sufficient levels of liquidity (in particular, $l \geq u_=$ for the type-1) achieve full transferability: the Pareto frontier is linear with unit slope magnitude. In this case, no matter what the division of surplus might be, the production units always remain non-integrated with equal profit shares accruing to each manager; the desired surplus division is accomplished ex-ante with a liquidity transfer. The study of organizational arrangements in the special case of full transferability reduces to the calculation of what maximizes total surplus.

But in the general case, where liquidity is scarce, partnerships with different levels of liquidity will choose different organizational forms and achieve different levels of total surplus, given a fixed level going to the type 2's. This is the "internal" liquidity effect: in general, more is better in the sense that the firm can generate higher surplus. With liquidity in short supply, there is a trade-off between surplus division and surplus production.

3 Market Equilibrium

Since the problem of market equilibrium involves "matching" the type 1's and type 2's into partnerships of two (with some of the 1's necessarily left unmatched), we have an assignment game with nontransferable utility. In thinking about equilibrium in this matching market it is convenient to use the core as a solution concept. This requires a partition of the set of agents into coalitions that share surplus on the Pareto frontier and are stable in the sense that no new firm could form and strictly improve the payoffs to its members.

We need only concern ourselves with coalitions that are singletons and pairs (which we call "firms") consisting of one type 1 production unit $i \in I$ and one type 2 production unit $j \in J$. Since there is excess supply of type 1 production units, there is at least a measure $1 - n$ of type 1 managers who do not find a match and who therefore obtain a surplus of zero. Stability requires that no unmatched type 1 manager can bid up the surplus of a type-2 manager while getting a positive surplus. Necessary conditions for this are that all type 2 managers are matched and that they have a generated surplus not smaller than $u_=$. As we have shown, for such generated surpluses, the 2's liquidity does not matter. Thus all 2's are equally good as far as a 1 is concerned and they must therefore receive the same surplus.[11]

This "equal treatment" property for the 2's is an important simplification relative to most assignment models in which there is heterogeneity on both sides of the market.[12] It enables us to identify the set of firms F with

[11] If in firm (i, j) type 2 j has a strictly larger surplus than type 2 j' in the firm (i', j'), the firm (i, j') could form and both i and j' could be better off since the Pareto frontier is strictly decreasing.

[12] As we pointed out above, this is where the no-budget-breaking assumption comes in. Without it, the separation property does not hold and it will not be possible to treat the

the index of the type 1 manager in the firm, and we refer to "firm i" to indicate that the firm consists of the i-th type 1 production unit and a type 2 manager.[13]

Definition 6 *An equilibrium consists of a set of firms $F \subset I$ with Lebesgue measure n, a surplus v_2^* received by the type 2 managers, and a surplus function $v_1^*(i)$ for type 1 managers such that:*

(ii) (feasibility) For all $i \in F$, $(v_1^*(i), v_2^*) \in \hat{U} + T(l(i), 0)$. For all $i \notin F$, $v_1^*(i) = 0$.

(ii) (stability) For all $i \in I$, for all $j \in J$, for all $(v_1, v_2) \in \hat{U} + T(l(i), 0)$, either $v_1 \leq v_1^*(i)$ or $v_2 \leq v_2^*$.

Since the type-2 managers have the same equilibrium payoff, we can reason in a straightforward demand-and-supply style by analyzing a market in which the traded commodity is the type 2's. We construct the demand as follows. The amount of surplus a 1 is willing *and able* to transfer to a 2 depends on how much liquidity he has. The most he would offer of course is the entire maximum surplus $2u_=$, which he could do provided his liquidity exceeds $u_=$. A 1 with zero liquidity can offer $\phi(0)$. In general, agent i with $l(i) < u_=$ can offer $\phi(l(i)) + l(i)$, since this gives her zero surplus.[14] Since the frontier has slope magnitude less than unity above the 45^o-line, this effective *willingness to pay* $w(i)$ is nondecreasing in i; since l is increasing in i, we have a (weakly) downward sloping "demand" schedule given by

$$w(i) = \min\{2u_=, \phi(l(1-x)) + l(1-x)\},$$

where x is the quantity of 2's. The supply is vertical at n, the measure of 2's. Equilibrium is at the intersection of the two curves: this indicates that n of the 1's are matched, as claimed above, and that the marginal 1 is receiving zero surplus.

Proposition 7 *The equilibrium set of firms is $F = [1-n, 1]$ and the equilibrium surplus of type 2 managers is*

$$v_2^* = \min\{2u_=, \phi(l(\bar{\imath})) + l(\bar{\imath})\},$$

where $\bar{\imath} = 1 - n$ is the marginal type 1 manager .

2's the same.

[13]For each firm $i \in F$ corresponds a type 2 manager $j(i)$; this matching function must be measure consistent; see Legros and Newman (2003).

[14]The pair chooses an organizational form that generates $(l, \phi(l))$, and since type-1 transfers l to the 2 we obtain a surplus of 0 for 1 and $\phi(l) + l$ for 2.

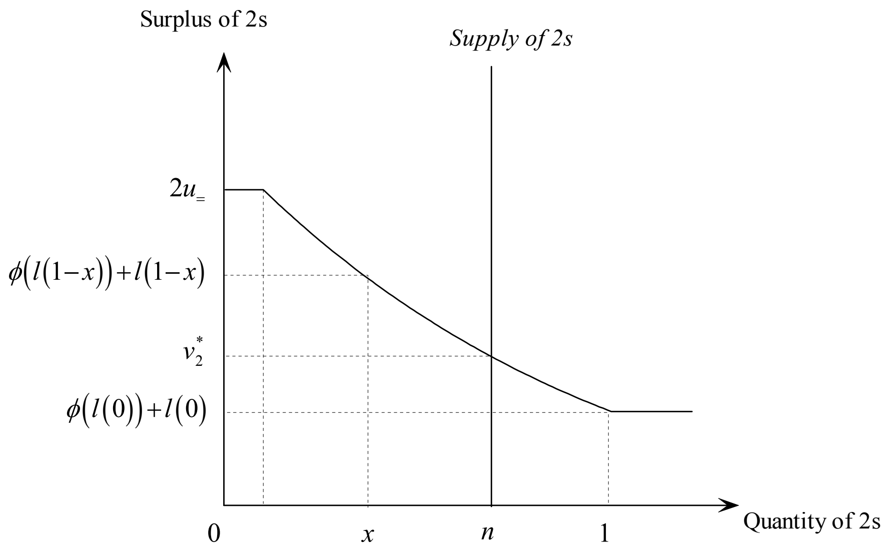

Figure 3: The supplier market

If $l(\bar{\imath}) \geq u_=$, efficiency is obtained since each matched type 1 is able to pay $u_=$ to the type 2 manager; note that in this case the equilibrium surplus of all type 1 managers is zero. We will consider below situations in which $l(\bar{\imath}) < u_=$.

The type 1 manager with liquidity $l(\bar{\imath})$ has a surplus of 0; his *generated surplus* however is $u_1(l(\bar{\imath}), v_2^*) = l(\bar{\imath})$. An inframarginal type 1 manager with liquidity $l > l(\bar{\imath})$ will be able to generate a higher surplus for himself since he can transfer more liquidity than the marginal type 1. If $l \geq v_2^* - u_=$, the inframarginal type 1 has a generated surplus of $2u_= - v_2^*$ and the contract is the efficient contract $s = \frac{1}{2}, \omega = 0$. If $l < v_2^* - u_=$, type 1 has a generated surplus of $u_1(l, v_2^*)$ satisfying $\phi(u_1(l, v_2^*)) + l = v_2^*$.

Properties of the generated surplus are easily derived from Proposition 5(iv).

Lemma 8 *The generated surplus $u_1(l, v_2^*)$ of an inframarginal type 1 is decreasing concave in v_2^* and increasing concave in l.*

Proposition 5(i)-(ii) and Lemma 8 show that there is a simple relationship between the generated surplus and the contractual terms, in particular the structure of ownership. Indeed, from small changes in the generated surplus accruing to the 1 will *either* result in changes in s or in ω, but not both simultaneously. Since the generated surplus $u_1(l, v_2^*)$ is increasing in l, there will be a critical liquidity level $L(v_2^*)$, increasing in v_2^*, that separates firms in the "refinance zone" from those in the "restructuring zone": above $L(v_2^*)$, firms are nonintegrated and only profit shares vary with l, while below $L(v_2^*)$, profit shares are fixed at $\bar{s}$ and variation in l leads to variation in ownership structure. Since $L(v_2^*)$ is increasing in v_2^*, the higher is the surplus accruing to the 2's, the fewer firms will be nonintegrated.

Lemma 9 *(i) If $l(\bar{\imath}) \geq \underline{u}$ all firms choose nonintegration.*

(ii)If $l(\bar{\imath}) < \underline{u}$, define $L(v_2^*)$ by $\phi(\underline{u}) + L(v_2^*) = v_2^*$. Firms with type 1 liquidity of $l \in [l(\bar{\imath}), L(v_2^*)]$ choose integration contracts $(\bar{s}, \omega)$, ω decreasing in l. Firms with type 1 liquidity of $l > L(v_2^*)$ choose nonintegration contracts $(s, 0)$, s decreasing in l.

Thus the model captures two aspects of the influence of market conditions on internal organization: not only do firms respond to v_2^* in making organizational decisions, but the way they respond to internal shocks (e.g., small liquidity windfalls) will also depend on v_2^*. When 2's command high payoffs, small increases in a 1's liquidity will be likely to result in a restructuring, specifically a (partial) reacquisition of ownership by the 1, whereas when the 2's are less well compensated the same shock will more likely simply lead to a greater profit share for the 1. Of course, to study these effects systematically, we must take account of the fact that v_2^* itself is endogenous, which we do in the next section.

4 Comparative Statics of Market Equilibrium

In equilibrium, there will typically be variation in organizational structure across firms, and this is accounted for by variation in their characteristics. In particular, "richer" firms are less integrated, accrue smaller shares of profit to the type-2, and generate greater surplus for the managers. [15] Similar results have been found in the literature on financial contracting (Jensen and Meckling 1976, Aghion and Bolton 1992): more liquidity inside the firm improves the efficiency of contracting. We refer to this as an *internal* effect.

But more liquidity overall can also lead to *more* integration: if the marginal firm's liquidity increases, v_2^* rises, possibly by more than an inframarginal firm's gain in liquidity. As a result, the inframarginal firm may become more integrated, and indeed it is possible that the economy's average level of integration may increase via this *external* effect.

[15] Holmström and Milgrom (1994) emphasize a similar cross-sectional covariation in organizational variables. In their model, the variation reflects differences in technology but not differences in efficiency, since all firms are surplus maximizing. Here by contrast, the variation stems from differences in liquidity and reflects differences in organizational efficiency.

We shall consider three types of shocks that may lead to reorganizations in the economy: changes in the relative scarcity of the two types, changes in the distribution of liquidity and changes in the technological parameter A.

4.1 Relative Scarcity

In order to isolate the "external effect" our first comparative statics exercise involves changes in the tightness of the supplier market, i.e., in the relative scarcities of 1's and 2's.

Suppose that the measure of 2's increases, for instance from entry of downstream producers into the domestic market from overseas. Then just as in the standard textbook analysis, we represent this by a rightward shift of the "supply" schedule: the "price" of 2's decreases. Indeed, as n increases the marginal liquidity of type 1 decreases since $l(\bar{\imath})$ is decreasing with n. What of course is different from the standard textbook analysis is that this change in price entails (widespread) refinancing or corporate restructuring.

Let $F(n)$ be the set of firms when there is a measure n of type 2 firms. As n increases to $\hat{n}$, there is an equilibrium set $F(\hat{n})$ where $F(n) \subset F(\hat{n})$; that is after the increase in supply, new firms are created but we can consider that previously matched managers stay together. From Lemma 8, the generated surplus of *all* type 1 managers in firms in $F(n)$ increases. Managers in a firm in $F(n)$ will either refinance (decrease s) or restructure (decrease ω) in response to the reduction in the equilibrium value of v_2^*. The analysis is similar in the opposite direction: a decrease in the measure of 2's leads to an increase in v_2^*. Thus, we have

Proposition 10 *In response to an increase in the measure of 2's, the firms remaining in the market become (weakly) less integrated and profit shares accruing to the 2's weakly decrease. Total surplus generated by each of these firms does not decrease.*

It is worth remarking that if the relative scarcity changes so drastically that the 2's become more numerous, then 1's get the preponderance of the surplus and tend to become the owners; the analysis is similar to what we have seen, with the role of 1's and 2's reversed. The point is that the owners of the integrated firm gain control because they are scarce, not because it is efficient for them to do so: in this sense, organizational power stems from market power.

As an application, if we interpret the assets as tasks or duties, the model can suggest a simple explanation for the "empowerment of talent" that has been noted by several authors (see Marin and Verdier, 2003; and references therein). Here empowerment means giving the highly skilled and professional workers decision rights over more of these tasks, i.e., more discretion. A large literature in labor economics has shown that in the last thirty years the demand for skilled workers in North America and Western Europe has outstripped the (nonetheless growing) supply. Interpreting the 1's in our model as the corporate demanders of talent and the 2's as the talented workforce, relative rightward shifts in demand mean more surplus to the type 2's, which will manifest itself variously as bigger cash payments, greater shares of profit (use of bonus schemes or possibly stock options), and greater "empowerment," often in combination. As long as firms' liquidity is restricted (relative to the scale of operations), tighter labor markets mean more control by these workers, not merely higher wages.

However, this story is heuristic: increases in demand for the talented workforce most likely emanate from entry of new firms (which in turn entails a change in the liquidity distribution among the active firms) and from increases in productivity (e.g., "skill-biased technical change"). Thus, a general analysis of the effects of changes in relative scarcity requires separate consideration of the effects of changes in liquidity and productivity; we provide this in the next two subsections.

4.2 Liquidity Shocks

Evaluating changes in the liquidity distribution is complicated by the presence of two countervailing effects. First, as we have noted, there is an *internal effect:* if the liquidity of an individual type 1 increases, he can "afford" a more efficient organization, which typically entails an increase in his share of profit and the fraction of assets he owns. But increasing liquidity also increases the 1's effective willingness to pay, so if a distributional change increases the value of the *marginal* liquidity, it creates an *external effect* via an increase in v_2^* that results in efficiency-reducing restructuring and refinancing in potentially *all* the relationships.

In light of Proposition 7, we ignore the distribution of type 2 agents. The dependence of the organizational variables on the type 1 liquidity l and the equilibrium surplus v_2^* is summarized in the following simple corollary of Proposition 5 and Lemma 8.

Lemma 11 *Under Assumption 6:*
(i) The share s is nondecreasing in v_2^* and nonincreasing in l
(ii) The degree of integration ω is nondecreasing convex in v_2^*, nonincreasing convex in l.
(iii) The total surplus W is decreasing concave in v_2^* and increasing concave

in l.

Equipped with this result, we can derive simple comparative statics. We focus on the aggregate degree of integration in the market, but will also summarize the effects on the average sharing rule and the aggregate surplus generated by the economy.

Suppose the initial liquidity endowment is $l(i)$ and that the economy receives a "shock" that transforms $l(i)$ into $\psi(l\,(i))$; the shock function $\psi\,(\cdot)$ is assumed continuous and increasing. We wish to compare the degree of integration before and after the shock. Let $\omega\,(l, v_2^*)$ be the degree of integration in a firm with a type 1 manager having liquidity l when the equilibrium surplus to 2 is v_2^*.

The change in average degree of integration is

$$\int_{\bar{\imath}}^{1} \omega\left(\psi(l(i)), v_2^*(\psi(l\,(\bar{\imath})))\right) di - \int_{\bar{\imath}}^{1} \omega\left(l(i), v_2^*\left(l\,(\bar{\imath})\right)\right) di, \tag{5}$$

where $\psi(l\,(\bar{\imath}))$ and $l\,(\bar{\imath})$ are the respective marginal liquidity levels and the notation $v_2^*(\cdot)$ reflects the dependence of the 2's equilibrium surplus on the marginal liquidity.

We now study some special cases that place more structure on the problem.

4.2.1 Positive Shocks to Liquidity

Suppose that the shocks $\psi(l) - l$ are both positive and nondecreasing in l. Note that a *uniform* shock in which every type 1 receives the same increase to his endowment is a special case. So is a multiplicative shock in which the percentage increase to the endowment is the same for all 1's. The impact of this shock is to increase both the "purchasing power" of the type 1's, which, via the internal effect, reduces the degree of integration, but also to increase the equilibrium surplus to 2, which, via the external effect, has the opposite effect. However, it is a simple matter to demonstrate that in this case, the internal effect dominates: more liquidity implies less integration. Heuristically, the change in v_2^* is $\phi'(l(\bar{\imath})) + 1$ times the change in $\bar{\imath}$'s liquidity; since $-1 < \phi' < 0$, this is smaller than the liquidity increase and thus $\bar{\imath}$ can cover the new price and still buy back some assets; all $i > \bar{\imath}$ have at least as large an increase in their endowments and can therefore do the same. Of course, negative, nonincreasing shocks yield the opposite changes in surplus and organization.

Proposition 12 *Under positive, nondecreasing, shocks to the liquidity distribution of type 1:*

(i) the aggregate degree of integration decreases;

(ii) the profit shares between 1 and 2 become more equal;

(iii) total welfare rises.

To maintain this conclusion, the proviso that the shocks are monotonic can be relaxed, but not arbitrarily. Positive shocks alone are not enough, and having more liquidity in the economy may actually imply that there is higher overall degree of integration. Intuitively, if the positive shock hits only a small neighborhood of the marginal type 1, the price v_2^* will increase and the inframarginal unshocked firms will choose to integrate more in response to the increase in v_2^*. In the Appendix, we provide an example to demonstrate

Proposition 13 *There exist first order stochastic dominant shifts in the distribution of type-1 liquidity that lead to more integration and lower surplus in the aggregate.*

We turn now to consider other types of distributional changes.

4.2.2 Inequality and the Integration-Minimizing Distribution

It is helpful to compare distributions with a common marginal liquidity level, as this restricts attention to the internal effect. Thus in this subsection we compare two endowment functions $l(i)$ and $\psi(l(i))$ that are equal for the marginal type-1, i.e. $l(\bar{\imath}) = \psi(l(\bar{\imath}))$.

Suppose first that l and $\psi \circ l$ have a single crossing property at $l(\bar{\imath})$: $\psi(l(i)) < l(i)$ for $i < \bar{\imath}$ and $\psi(l(i)) > l(i)$ for $i > \bar{\imath}$. Since all *matched* 1's have greater liquidity and the equilibrium surplus v_2^* is by construction fixed, the generated surplus to 2 falls in every firm and the economy becomes less integrated. If one supposes further that $\int_0^1 l(i)di = \int_0^1 \psi(l(i))di$, then in fact the new liquidity distribution (which is essentially the inverse of the liquidity endowment function) is riskier then the old one in the sense of second order stochastic dominance (equivalently, it is more unequal in the sense of Lorenz dominance). This is an instance in which *increasing inequality may lower integration and raise efficiency.*

Now maintain the common marginal liquidity assumption, and denote the inverses of the restrictions of $l(\cdot)$ and $\psi(l(\cdot))$ to $[\bar{\imath}, 1]$ as $\bar{l}^{-1}$ and $\overline{(l \circ \psi)}^{-1}$: these are just the conditional distributions of liquidity above $l(\bar{\imath})$. Suppose that $\bar{l}^{-1}$is more unequal than $\overline{(l \circ \psi)}^{-1}$. Then because ω is convex in l (Lemma 11) and v_2^* is the same for both distributions, there is less integration under the new distribution.

This suggests the opposite of the previous conclusion: *increasing inequality may raise integration and lower efficiency.* These two results are easily reconciled: while the single-crossing result refers to the distribution for the economy as a whole, the second result refers to the distribution *only among the existing firms.*

If one is interested in the optimal distribution of liquidity for the economy as a whole, it is clear that one wants the marginal liquidity as low as possible, so as to minimize the equilibrium price. But from the previous result, the distribution among the firms must be as egalitarian as possible. And finally, one wants to maximize the liquidity of the inframarginal firms. Taking these three factors into account, along the with the fact that the liquidity of the 2's has no effect on organization or efficiency, one arrives at the following

Proposition 14 *Under Assumption 5, a distribution of liquidity that minimizes the aggregate level of integration and maximizes the aggregate surplus consists of two atoms:* $1-n$ *of the type 1's and all of the 2's get zero; the remaining* n *type 1's each get* $1/n$ *times the mean liquidity.*

This likely is a very unequal distribution indeed.[16] From the empirical point of view the important distinction is between overall inequality and inequality among the selected sample of matched firms, which in this model at least, can work in opposite directions.

4.2.3 A Global Condition when the Frontier is Linear

When the frontier ϕ is linear in the upper half quadrant, as it will be in the parametric case mentioned above in which $\bar{s} = 1/2$, we are able to obtain global necessary and sufficient conditions for aggregate organizational and surplus comparisons. This case provides a clean separation of the internal and external effects of changes in liquidity distributions.

Proposition 15 *Assume that* ϕ *is linear above the* 45^o*-line with slope* $-\alpha$ *(*$\alpha < 1$*). Consider two continuous endowments* l *and* $\psi(l)$*, with marginal liquidity levels* $l(\bar{\imath})$ *and* $\psi(l(\bar{\imath}))$ *and conditional mean liquidity levels* μ *and* $\hat{\mu}$ *on* $[l(\bar{\imath}), l(1)]$ *and* $[\psi(l(\bar{\imath})), \psi(l(1))]$ *respectively. Total welfare improves (and average integration decreases) when the distribution changes from* l *to* $\psi(l)$ *if and only if*

$$\hat{\mu} - \mu \geq (1-\alpha)\left(\psi(l(\bar{\imath})) - l(\bar{\imath})\right).$$

[16] This distribution doesn't satisfy Assumption 1, of course, but equilibrium is perfectly well defined nonetheless. It is true that there is an indeterminacy in the value of v^* with this distribution; the optimum is achieved at the lowest value, which is $\phi(0)$.

If the marginal agent's liquidity increases ($\psi(l(\bar{\imath})) > l(\bar{\imath})$), for instance, the average level of integration falls only if the mean liquidity increases enough. Otherwise, the other agents' increased ability to transfer surplus, and thereby reduce distortionary reassignments of ownership, will be offset by the increase in the required level of surplus transfer. The condition in Proposition 15 also underscores the role of the degree of inefficiency in transferring surplus via ownership structure rather than via monetary transfers. As α increases (for instance, with increases in A), the inefficiency (as measured by $(1-\alpha)$) decreases, and for a given change in the marginal agent's liquidity, the condition on the change in mean liquidity becomes less stringent.

4.3 Productivity Shocks

The external effect outlined in the previous section offers a propagation mechanism whereby local shocks that affect only a few firms initially may nevertheless entail widespread reorganization. Empirically this implies that to explain why a particular reorganization happens, there is no need to find a smoking gun in the form of a change *within* that organization: instead the impetus for such change may originate elsewhere in the economy. The same logic applies to other types of shocks, most prominently among them innovating productivity shocks. These are often thought to be the basis of large-scale reorganizations such as merger waves (Jovanovic and Rousseau, 2002).

We model a (positive) productivity shock or technological innovation as an increase in A. We suppose the shock inheres in the type 1's. Suppose that in the initial economy, all firms have the same technology; after a shock, a subset of them, an interval $[i_0, i_1]$, have access to a better technology (for them, $\hat{A} > A$). We restrict ourselves to considering "small" shocks in the sense that Assumptions 2 and 5 continue to hold for $\hat{A}$, so that we can still use Proposition 5.

Raising A modifies the game that managers play given a contract (s, ω) : it is clear from (2) and (3) that both managers obtain a larger surplus from a given contract. Hence the feasible set expands and the type-1's willingness to pay also increases. What is perhaps less immediate is that there is also more transferability within the firm.

Lemma 16 *(i) A positive productivity shock increases $\underline{u}$, increases $u_=$ on the pre-liquidity frontier, raises $\bar{s}$ and raises $\bar{\omega}$, the maximum individually rational level of ω.*

(ii) There is more transferability in the sense that for a contract (s, ω) the slope of the frontier is steeper in the region $u_2 \geq u_1$ when A increases.

Let $\phi(\cdot; A)$ be the function defining the Pareto frontier. The maximum willingness to pay $w(i)$ depends now on the technology available inside the firm,

$$w(i) = \begin{cases} \min\{2u_{=}(A), \phi(l(i); A) + l(i)\} & i \notin [i_0, i_1] \\ \min\left\{2u_{=}\left(\hat{A}\right), \phi\left(l(i); \hat{A}\right) + l(i)\right\} & i \in [i_0, i_1]. \end{cases}$$

Let

$$\begin{aligned} \pi &: [0,1] \to [0,1] \\ \pi(i) &\geq \pi(\hat{\imath}) \Leftrightarrow w(i) \geq w(\hat{\imath}). \end{aligned}$$

be a reordering of the indexes of type 1 managers that is consistent with the reordering on willingness to pay induced by the shock. The marginal type 1 agent is i_π such that the Lebesgue measure of the set $\{i : w(i) \geq w(i_\pi)\} = n$, and the set of equilibrium firms is $F = \{i : \pi(i) \geq \pi(i_\pi)\}$.

Lemma 16(ii) implies that – for a *fixed* equilibrium surplus for 2 – a shocked firm integrates less since it is able to transfer surplus in a more efficient way. Hence when the 2s' equilibrium surplus is fixed, positive technological shocks lead to less integration in the economy. However, when the 2s' equilibrium surplus increases, there is a force toward more integration. Unshocked firms certainly integrate more; for shocked firms, we show below that while they benefit internally from the technological shock, the countervailing effect of an increase in the 2s' equilibrium surplus dominates. The net effect is towards more integration *for all firms in the economy* if the marginal firm is a shocked firm.

Proposition 17 *(i) (Inframarginal shocks) If $i_\pi = 1 - n$ and $w(i_\pi) = v_2^*(A)$, then $F = [1-n, 1]$, the shocked firms become less integrated and the 2's shares fall, while the unshocked firms remain unaffected*

(ii) (Marginal shocks) If $1-n \in [i_0, i_1]$, and $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1 + \varepsilon)$, then $i_\pi = 1 - n$, $F = [1-n, 1]$, the equilibrium price $v_2^*\left(\hat{A}\right)$ increases and all firms, shocked and unshocked, integrate more.

(iii) If there is a uniform shock to the technology ($i_0 = 0, i_1 = 1$), then $i_\pi = 1 - n$, $F = [1-n, 1]$ and each firm integrates more.

Thus the effect of small positive productivity shocks depends on what part of the economy they affect. If they occur in "rich" firms (case (i)), only the innovating firms are affected, and they become less integrated. But innovations that occur in "poor" firms (case (ii)) may affect the whole economy: even firms that don't possess the new technology become more integrated.

The first result suggests that "local" reorganizations involving established firms originate within those firms, and is consistent with the view (e.g., Rajan and Wulf, 2003) that recent technological advances may be responsible for a palpable "flattening" of corporate hierarchies, at least if we interpret this as reduced integration. On the other hand, since new technologies are often introduced by new, small firms – the very ones that are likely to be liquidity poor – the second result suggests that widespread reorganizations (such as merger waves) are more likely to be set off by the entry of new firms embodying these new technologies. This interpretation supports the view (Jovanovic and Rousseau, 2002) that technological advances are responsible for the recent wave of mergers, i.e., increased integration! The point is that technological advances can have both effects: the origins of the innovations are crucial to determining how reorganization plays out in the economy.

Proposition 17 (iii) emphasizes that in contrast to reduced integration, more equal shares and greater efficiency after a positive uniform liquidity shock, a uniform positive productivity shock will have the *opposite* effects. In this sense the external effect of productivity shocks is more powerful than that for liquidity shocks.

This result also helps confirm the conjecture at the end of Section 4.1 that increased demand for skilled workers would lead to their empowerment. The growth in demand for skilled workers alluded to in Section 4.1 is often attributed to "skill-biased technical change." In terms of our model, with the 2's interpreted as the skilled labor force, this corresponds to a widespread increase in A, and the result implies that the skilled will get more control.

5 Discussion

If one asks the question "who gets organizational power in a market economy?," one is tempted to answer "to the scarce goes the power." There is a tradition in the business sociology literature (reviewed in Rajan and Zingales 2001) which ascribes power or authority to control of a resource that is scarce within the organization. Similar claims can be found in the economic literature (Hart and Moore, 1990; Stole and Zweibel, 1996). Our results suggest that organizational power may emanate from scarcity *outside* the organization, i.e., from market power. But this result has to be qualified somewhat: Proposition 13 suggests that having more liquidity may actually cause one to *lose* power, via what we have called the external effect of shocks to fundamentals. Similarly, the possessors of a new technology, if they are inframarginal, will gain ownership (Proposition 17 (i)), but if they are marginal may lose it. This is evidence of the importance of market effects for the allocation of power inside firms and more generally of their importance for the study of organizations.

We now discuss some other implications of the model.

5.1 Interest Rate

We have assumed that the interest rate (the rate of return on liquidity) is exogenous and is not affected by changes in the liquidity distribution or the technology available to firms. One can easily extend the model to allow for liquidity that yields a positive return though the period of production. Because liquidity in this model is used only as a means of surplus transfer, and not as a means to purchase new assets, the effects of this can be somewhat surprising. Raising this interest rate means that liquidity transferred at the beginning of the period has a higher value to the recipient than before: formally, the effect is equivalent to a multiplicative positive shock on the distribution of liquidity, and by Proposition 12, firms will integrate less if the interest rate increases, and will integrate more if the interest rate decreases. If liquidity transfers made in the economy affect the interest rate, then increases in the aggregate level of liquidity, by lowering interest rates, may constitute a force for integration above and beyond that suggested by the example in Proposition 13. These observations suggest that the relationship between aggregate liquidity and aggregate performance is unlikely to be straightforward; whether the potentially harmful organizational consequences would counter or even outweigh the traditional real investment responses is a question for future research.

5.2 Product Market

If we imagine all the firms sell to a competitive product market, then the selling price inheres in R, which we have thus far viewed as exogenous (for instance the supplier market is contained in a small open economy, with prices determined in the world market). But if instead price is determined endogenously in the product market, then shocks to product demand will change the price, which has the effect of changing A for all firms. Suppose the price increases. Then from Proposition 17(iii) in the analysis of productivity shocks, all firms become more integrated.

Next, notice that expected output is proportional to A for nonintegrated firms and $\bar{s}A + (1-\omega)(1-\bar{s})A + \omega$ for integrated ones. Integrated firms produce more than nonintegrated ones, and since $\bar{s}$ increases with A, as does average ω, aggregate output rises in response to an increase in A. Thus, if product price rises, so does output, and we conclude that the product supply curve is upward sloping. An increase in consumer demand therefore raises equilibrium price: increasing demand results in greater integration.

What is more, the product market price effect now means that more local shocks will result in widespread reorganization: more than just the very poorest firms in the economy may be "marginal." To see this, suppose a number of perfectly nonintegrated firms innovate. With fixed prices, these firms produce more output, but nothing further happens. With endogenous prices, the increased output in the first instance lowers product price; all other firms in the economy treat this exactly like a (uniform) negative productivity shock: they all become less integrated. Thus product market price adjustment has a kind of "amplification" effect on organizational restructuring.[17]

Previous work has analyzed how the intensity of product market competition may act as an incentive tool for managers.[18] In this literature the set of firms and their internal organization are exogenous. Here we wish to emphasize a causal relation in the opposite direction that becomes apparent once organization is allowed to be endogenous: organizations may affect product market prices, even when there is perfect competition. As discussed in Legros and Newman (2004), the fact that the product market – even a competitive one – can be affected by the internal organization decisions of firms has implications for consumer welfare, the regulation of corporate governance, and competition policy.

6 Appendix I: Proofs

6.1 Proof of Proposition 1

Suppose that manager 1 borrows B from a third party who transfers it to 2 and that the contract specifies an additional lump sum transfer $t_1 \geq 0$ to 2 and uses share s and ownership structure ω, with resulting equilibrium (q, e). Since the creditor must make nonnegative profits, he must get a payoff of D when profit is R (and 0 when profit is 0), $p(q,e)\,D \geq B$. Resulting payoffs to 1 and 2 are

[17] Of course the effect is self-limiting because as they become less integrated, they lower their output, causing the price to go up again. As shown in Legros-Newman (2004), these product market effects can be more pronounced in models that rely on somewhat different trade-offs in their basic organizational model than the one considered here.

[18] See Hart (1983), Scharfstein (1988) and Schmidt (1997).

$$
\begin{aligned}
u_1 &= -t_1 + p(q,e)\left[(1-s)R - D\right] - C_1(q,e) \\
u_2 &= B + t_1 + p(q,e)\, sR - C_2(q,e)
\end{aligned}
$$

Consider the contract without borrowing consisting of the same transfer of t_1 and a share to 2 of $sR + D$ and to 1 of $(1-s)R - D$ where D assumes the value it did in the first contract. If $p(e,q)$ were still the equilibrium probability of success, this contract pays both partners exactly the same as the first contract. But since 2's ex-post share is now larger, he raises $q_2(k)$ for $k \in [1,2]$; by revealed preference he is better off, and 1, by virtue of the increase in the success probability is also better off. Any borrowing is Pareto dominated by a contract that involves no borrowing.

6.2 Proof of Lemma 2

Consider a contract (S,ω) where the sharing rule gives absolute contingent consumptions to manager 2 of $S(R)$ and $S(0)$; by budget balance, manager 1 gets contingent consumption of $R + L - S(R)$ and $L - S(0)$; by limited liability $S(0) \in [0,L]$ and $S(R) \in [0, R+L]$. Let $p(q,e)$ be the resulting probability of success in the equilibrium of the game induced by the contract (S,ω). In choosing q and e, each manager considers his marginal share, that is $S(R) - S(0)$ for manager 2 and $R - (S(R) - S(0))$ for manager 1. Utility payoffs are then

$$
\begin{aligned}
u_1 &= p(q,e)\left[R - (S(R) - S(0))\right] + L - S(0) - C_1(q,e) \\
u_2 &= p(q,e)\left[S(R) - S(0)\right] + S(0) - C_2(q,e).
\end{aligned}
$$

Suppose first that the marginal shares are non negative for both managers, that is that $R \geq S(R) - S(0) \geq 0$. Consider the new contract in which the two managers first share the total liquidity L so that manager 2 gets $S(0)$ and manager 1 gets $L - S(0)$ and then agree to a contract $(\hat{s},\omega)$ where $\hat{s} \in [0,1]$ is a fixed share of profit defined by $\hat{s} = \left[S(R) - S(0)\right]/R$. Then the marginal shares are the same for each manager and, since the ownership structure has not changed, the equilibrium of the induced game is the same.

We show now that the assumption of non negative marginal shares is without loss of generality, because any contract with a negative marginal share is Pareto dominated by a contract with non-negative marginal shares.

Suppose by contradiction that $S(R) - S(0) < 0$. Then, for any value of q, $e_2(k) = 0$ for $k \in [1,2)$ since the marginal return on operation is negative for manager 2. Choosing $q(k) > 0$ for $k \in [1,2)$ does not increase expected profit but increases $C_2(q,e)$; it is Pareto optimal to set $q(k)=0$ for all $k\in[1,2)$, which can be implemented for instance by having manager 2 retain ownership of his assets. Expected profit depends only on decisions made for $k\in[0,1)$ and total surplus is $p(q,e)R+L-C_1(q,e)$; this is maximized by choosing $q^{FB}(k)=A$ and $e^{FB}(k)=1$ for each $k\in[0,1)$. However, if manager 1 owns asset k, he chooses $q(k)>q^{FB}(k)$ since $R-(S(R)-S(0))>R$; if manager 2 owns asset k, he chooses $q(k)=0<q^{FB}(k)$ since he wants to minimize $p(q,e)$. Let q^*,e^* denote the equilibrium choices for q,e and let u_i^* be the expected utility level of manager i. Substituting the expression $u_2^*=p(q^*,e^*)[S(R)-S(0)]+S(0)$ into the expression for u_1, we obtain,

$$\begin{aligned} u_1^* &= p(q^*,e^*)R+L-u_2^*-C_1(q^*,e^*) \\ &< p(q^{FB},e^{FB})R+L-u_2^*-C_1(q^{FB},e^{FB}) \end{aligned} \tag{6}$$

where for $k\in[1,2)$, $e^*(k)=q^*(k)=0$ and for $k\in[0,1), e^*(k)=q^*(k)=0$ if manager 2 owns k and $q^*(k)=A\frac{R-(S(R)-S(0))}{R}$, $e^*(k)=1$ if manager 1 owns k; the strict inequality in (6) follows $(q^*,e^*)\neq(q^{FB},e^{FB})$.

Consider a new contract: managers 1 and 2 own the assets they operate ($\omega=0$) and the sharing rule $\hat{s}$ is chosen to satisfy $\hat{s}(R)=\hat{s}(0)=u_2^*$; note that since $0\leq u_2^*<S(0)\leq L$, $\hat{s}$ satisfies limited liability. Since he has a zero marginal share, manager 2 still chooses $\hat{q}(k)=\hat{e}(k)=0$ on his assets and his expected utility is still $\hat{u}_2=u_2^*$. For manager 1, the expected payoff is $\hat{u}_1=\max_{q,e}p(q,e)R+L-u_2^*-C_1(q,e)$: manager 1 chooses the first best level $q^{FB}(k)=A$ and $e^{FB}(k)=1$ on $k\in[0,1)$. Therefore, from (6), $\hat{u}_1>u_1^*$, and the new contract Pareto dominates the previous contract. The case $S(R)-S(0)>R$ is treated similarly. This concludes the proof.

6.3 Derivation of Pre-Liquidity Pareto Frontier and Proof of Proposition 5

Recall that payoffs are (2) and (3) or

$$\begin{aligned} u_1(s,\omega) &= \frac{1-s^2}{2}A^2-c-\omega\frac{((1-s)A-1)^2}{2} \\ u_2(s,\omega) &= \frac{s(2-s)}{2}A^2-c+\omega sA(1-(1-s)A), \end{aligned}$$

It is immediate that u_1 is a linear decreasing function of ω and u_2 is a linear increasing function of ω.

Let λ be the Lagrange multiplier on the constraint involving 1's payoff in problem (4) of $\max\{u_2(s,\omega):u_1(s,\omega)=u\}$. Note that in the problem we have ignored the incentive constraint that $s \in [1-\bar{s}, \bar{s}]$. The first-order conditions are

$$s \left\{ \begin{array}{c} = 1-\bar{s} \\ \in (1-\bar{s}, \bar{s}) \\ = \bar{s} \end{array} \right\}$$

$$\Rightarrow (1-\omega)(1-2s)A^2 + \omega A + sA^2 - \lambda((1-\omega)(1-s)A^2 + \omega A - (1-2s)A^2) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0 \tag{7}$$

$$\omega \left\{ \begin{array}{c} = 0 \\ \in (0,1) \\ = 1 \end{array} \right\}$$

$$\Rightarrow sA - s(1-s)A^2 - \lambda(\frac{(1-s)^2A^2}{2} - (1-s)A + \frac{1}{2}) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0 \tag{8}$$

Let $\Lambda(s,\omega)$ be the value of λ for which (7) holds with an equality and let $K(s)$ be the value of λ for which (8) holds with an equality. After some simple algebra,

$$\begin{aligned} \Lambda(s,\omega) &= \frac{(1-\omega-s+2\omega s)A+\omega}{(s-\omega+\omega s)A+\omega} \\ K(s) &= \frac{2sA}{1-(1-s)A}. \end{aligned} \tag{9}$$

We note that $K(s)$ is increasing in s and that $\Lambda(s,0) = \frac{1-s}{s}$ is decreasing in s. Therefore, there exists a unique value of s, that we denote by s^* for which $K(s) = \Lambda(s,0)$.[19] When $s < s^*$, $\Lambda(s,0) > K(s)$ and therefore when $\lambda = \Lambda(s,0)$, we have indeed $\lambda > K(s)$ and therefore $\omega = 0$ by (8). The frontier has slope $-\Lambda(s,0)$; since $\Lambda(s,0)$ is decreasing with s, the frontier is concave. In this regime, $\omega = 0$ and $s = \sqrt{1 - \frac{2(u_1+c)}{A^2}}$ and s is decreasing and concave in u_1.

Two cases are of interest.

Case $s^* < \bar{s}$. Noting that $\Lambda(s,\omega)$ is increasing in ω, it is necessary that $s > s^*$ and $\omega > 0$ in order to have $K(s) = \Lambda(s,\omega)$; keeping $s = s^*$ while increasing ω would lead to a contradiction since $\Lambda(s^*,\omega) > \Lambda(s^*,0) = K(s^*)$ and therefore if $\lambda = \Lambda(s^*,\omega)$, $\lambda > K(s^*)$ and we should have $\omega = 0$. Replicating this argument, s and ω must jointly increase in order to satisfy the

[19] One can check that $s^* = \frac{1}{3A}\left(A + \frac{1}{2}\sqrt{8A - 8A^2 + 1} - \frac{1}{2}\right)$.

equality $K(s) = \Lambda(s,\omega)$. This continues until $s = \bar{s}$, after which the frontier becomes linear. Note that when $s \in (s^*, \bar{s})$ the slope of the frontier is given by $\frac{du_2}{du_1} = -K(s)$ and since $K(s)$ is increasing in s, $-K(s)$ is decreasing in s, which shows that the frontier is convex.

Case $\mathbf{s}^* \geq \bar{\mathbf{s}}$. choosing $\omega = 0$ and $s = \bar{s}$ implies that $\Lambda(\bar{s}, 0) > K(\bar{s})$. If 1 must get a lower surplus, ω must increase and the Pareto frontier is linear with slope equal to $\frac{\partial u_2/\partial\omega}{\partial u_1/\partial\omega} = 2\frac{\bar{s}A(1-(1-\bar{s})A)}{((1-\bar{s})A-1)^2} = \frac{2(A-\sqrt{2c})(1-\sqrt{2c})}{A(1-\sqrt{2c})^2}$. When $\omega = 0$, $\underline{u} = u_1(\bar{s}, 0) = \frac{1-\bar{s}^2}{2}A^2 - c,$. or using (1), $\underline{u} = \frac{2\bar{s}}{1-\bar{s}}c = A\sqrt{2c} - 2c$.

Hence, when $u_1 \in [0, \underline{u}]$, $s = \bar{s}$ and $u_1(\bar{s}, \omega)$ is linear and decreasing in ω, which proves that ω is linear and decreasing in u_1. When $u_1 \in [\underline{u}, u_=]$, the frontier has slope $\Lambda(s, 0)$, decreasing in s, and the frontier is concave. However we cannot conclude immediately that the frontier is globally concave because there is a kink at $(\underline{u}, \phi(\underline{u}))$. Indeed, the absolute value of the right derivative is $\Lambda(\bar{s}, 0)$ while the absolute value of the left derivative is $K(\bar{s})$. The frontier is globally concave if $K(\bar{s}) < \Lambda(\bar{s}, 0)$. Since K is increasing in s and since $s^* < \bar{s}$, $K(\bar{s}) < K(s^*)$. Since $\Lambda(s, 0)$ is decreasing in s, $\Lambda(s^*, 0) < \Lambda(\bar{s}, 0)$. Now, by definition of s^*, $K(s^*) = \Lambda(s^*, 0)$; hence $K(\bar{s}) < \Lambda(\bar{s}, 0)$ and the frontier is concave as claimed.

6.4 Implications of Relaxing Assumption 6

The only results this change affects are those that relied on concavity of the frontier and convexity of ω, namely the discussion in section 4.2.2 leading up to Proposition 14. Relaxing Assumption 6 doesn't change the fact that giving 2 more surplus entails giving him greater ownership, so that changes in market conditions that give 2's more surplus will continue to increase the degree of integration. Also the slope remains less than unity in magnitude, increasing 2's generated surplus still lowers total surplus.

6.5 Proof of Lemma 9

(i) The marginal firm chooses non integration since the generated surplus of the type 1 manager is $\min\{l(\bar{\imath}), u_=\} > \underline{u}$; from Proposition 5, all inframarginal firms choose also nonintegration.

(ii) The marginal firm chooses integration since the generated surplus of the marginal type 1 is less than $l(\bar{\imath}) < \underline{u}$. The cutoff value $L(v_2^*)$ is well defined and corresponds to an inframarginal agent: by Proposition 7 $v_2^* = \phi(l(\bar{\imath})) + l(\bar{\imath})$ and $v_2^* - \phi(\underline{u}) = \phi(l(\bar{\imath})) - \phi(\underline{u}) + l(\bar{\imath})$; since ϕ is decreasing $\phi(\underline{u}) < \phi(l(\bar{\imath}))$ and therefore $v_2^* - \phi(\underline{u}) > l(\bar{\imath})$.

6.6 Proof of Propositions 12

To see this, it is enough to show that $\omega\left(\psi(l(i)), v_2^*(\psi(l\left(\bar{\imath}\right)))\right) \leq \omega\left(l(i), v_2^*\left(l\left(\bar{\imath}\right)\right)\right)$ for all i. Now ω depends on v_2^* and l only via its dependence on the generated surplus u_1. In firm i the generated surplus is $\hat{u}_1$ after the shock and solves

$$\begin{aligned} \phi\left(\hat{u}_1\right) &= \min\{u_=, v_2^*(\psi\left(l\left(\bar{\imath}\right)\right)) - \psi(l(i))\} \\ &= \min\left\{u_=, \phi(\psi\left(l\left(\bar{\imath}\right)\right)) + \psi\left(l\left(\bar{\imath}\right)\right) - \psi(l(i))\right\}. \end{aligned}$$

Before the shock it was u_1 solving

$$\begin{aligned} \phi\left(u_1\right) &= \min\{u_=, v_2^*(l\left(\bar{\imath}\right)) - l(i)\} \\ &= \min\left\{u_=, \phi\left(l\left(\bar{\imath}\right)\right) + l\left(\bar{\imath}\right) - l\left(i\right)\right\}. \end{aligned}$$

We note that $\phi\left(\psi\left(l\left(\bar{\imath}\right)\right)\right) - \phi\left(l\left(\bar{\imath}\right)\right) \leq 0 \leq \left[\psi\left(l\left(i\right)\right) - l\left(i\right)\right] - \left[\psi\left(l\left(\bar{\imath}\right)\right) - l\left(\bar{\imath}\right)\right]:$ the left hand side is nonpositive since ϕ is decreasing and $\psi\left(l\left(\bar{\imath}\right)\right) \geq l\left(\bar{\imath}\right)$, while the right hand side is nonnegative because the shocks are nondecreasing. Rewriting this expression, as $\phi\left(\psi\left(l\left(\bar{\imath}\right)\right)\right) + \psi\left(l\left(\bar{\imath}\right)\right) - \psi\left(l\left(i\right)\right) \leq \phi\left(l\left(\bar{\imath}\right)\right) + l\left(\bar{\imath}\right) - l\left(i\right)$ then implies $\min\{u_=, v_2^*(\psi\left(l\left(\bar{\imath}\right)\right)) - \psi(l(i))\} \leq \min\{u_=, v_2^*(l\left(\bar{\imath}\right)) - l(i)\}$. Therefore, $\phi\left(\hat{u}_1\right) \leq \phi\left(u_1\right)$ and $\hat{u}_1 \geq u_1$. It follows from Proposition 5 that the firm will restructure and choose a lower value of ω. This proves (i); (ii) and (iii) are direct consequences of (i) and Proposition 5.

6.7 Proof of Proposition 13

It is enough to provide an example. Consider the liquidity distribution $l\left(i\right) = i$ where $i \in \left[0, 1\right]$; and suppose that $n = 1$, that is that the marginal liquidity is 0. The contract for the marginal firm is therefore an integration contract with $s = \bar{s}$ and $\omega < 0$; from Proposition 5 the frontier is linear and can be written $\phi\left(u_1\right) = -\alpha u_1 + \phi\left(0\right)$, where $\alpha \in \left(0, 1\right)$.[20] To simplify assume that $1 < \underline{u}$; this will insure that when the equilibrium surplus is $v_2^* = \phi\left(0\right)$ the firm with $i = 1$ chooses an integration contract.

>From Proposition 9, the equilibrium surplus is $v_2^*\left(0\right) = \phi\left(0\right)$. Let $\varepsilon < 2\underline{u}$ and define $\psi\left(l\right)$ by

$$\psi\left(l\right) = \begin{cases} \delta, \text{if } l \leq \delta \\ l \text{ if } l \geq \delta. \end{cases}$$

[20] $\alpha < 1$ follows concavity of ϕ and the fact that total surplus is $\phi\left(u\right) + u$ and is maximum at $u = u_=$.

$\psi(l)$ is increasing and continuous. The marginal liquidity is now $\psi(0) = \delta$ and the new equilibrium surplus is $v_2^*(\delta) = \phi(\delta) + \delta$. The generated surpluses in a firm with type 1 of index i is before the shock (distribution l)

$$u_1(i) : \phi(u_1)(i) = \phi(0) - i$$

and after the shock (distribution ψ)

$$\hat{u}_1(i) : \phi(\hat{u}_1(i)) = \phi(\delta) + \delta - \psi(i).$$

Firms with $i \geq \delta$ have the same liquidity but a higher equilibrium surplus accrues to type 2, and by Lemma 8 $\hat{u}_1 < u_1$. Precisely,

$$\begin{aligned} \phi(\hat{u}_1(i)) - \phi(u_1(i)) &= \phi(\delta) - \phi(0) + \delta \\ &\Leftrightarrow \\ u_1(i) - \hat{u}_1(i) &= \frac{1-\alpha}{\alpha}\delta \end{aligned}$$

For firms with $i < \delta$, $\psi(l(i)) = \delta$, and

$$\begin{aligned} \phi(\hat{u}_1(i)) - \phi(u_1(i)) &= \phi(\delta) - \phi(0) + i \\ &\Leftrightarrow \\ u_1(i) - \hat{u}_1(i) &= -\delta + \frac{i}{\alpha}. \end{aligned}$$

Therefore, for all firms $i \geq \alpha\delta$ the generated surplus decreases and these firms are more integrated. For firms with $i < \alpha\delta$, the generated surplus increases and these firms are less integrated.

For the linear part of the frontier, the generated surplus is also a linear function of ω (see 2) and we can write $\omega(i) = -\beta u_1(i) + \omega(0)$, where $\beta > 0$. Hence, the change in the degree of integration is $\hat{\omega}(i) - \omega(i) = \beta(u_1(i) - \hat{u}_1(i))$, and in the aggregate,

$$\begin{aligned} \int_0^1 (\hat{\omega}(i) - \omega(i))\, di &= \beta\left[\int_0^\delta \left(-\delta + \frac{i}{\alpha}\right) di + \int_\delta^1 \frac{1-\alpha}{\alpha}\delta di\right] \\ &= \beta\frac{\delta}{\alpha}\left(1 - \alpha - \frac{\delta}{2}\right). \end{aligned}$$

Hence as long as $\delta < 2(1-\alpha)$, average integration increases in the economy.

Note that $\psi(l)$ does not satisfy Assumption 1 since it is constant on $l \leq \delta$. However, by continuity, there exists $\tilde{\psi}(l)$ satisfying Assumption 1 for which average integration increases.

6.8 Proof of Proposition 15

The pre-liquidity frontier is $\phi(u_1) = -\alpha u_1 + \phi(0)$, where $\alpha = \frac{2A}{2-A}$ (substitute $\bar{s} = \frac{1}{2}$ in (2)-(3)). The equilibrium surplus is $v_2^* = (1-\alpha) l(\bar{\imath}) + \phi(0)$ and welfare is $W = \phi(u_1) + u_1 = (1-\alpha) u_1 + \phi(0)$. The generated surplus is $u_1(l, v_2^*)$ solving $\phi(u_1) = v_2^* - l$; hence, $u_1 = -\frac{1-\alpha}{\alpha} l(\bar{\imath}) + \frac{l}{\alpha}$, and therefore welfare in a firm with type 1 manager i is

$$W_i(l) = \phi(0) + \frac{1-\alpha}{\alpha} l(i) - \frac{(1-\alpha)^2}{\alpha} l(\bar{\imath})$$

and total surplus is $W(l) = \int_{\bar{\imath}}^1 W_i(l)\, di$. Hence, change of welfare when going from l to $\psi \circ l$ is

$$W(\psi \circ l) - W(l) = \int_{\bar{\imath}}^1 \frac{1-\alpha}{\alpha} (\psi(l(i)) - l(i)) - n\frac{(1-\alpha)^2}{\alpha} (\psi(l(\bar{\imath})) - l(\bar{\imath}))$$

Noting that the conditional means are $\mu = \int_{\bar{\imath}}^1 l(i) \frac{di}{n}$ and $\hat{\mu} = \int_{\bar{\imath}}^1 \psi(l(i)) \frac{di}{n}$ we have

$$W(\psi \circ l) - W(l) \geq 0 \Longleftrightarrow \hat{\mu} - \mu \geq (1-\alpha)(\psi(l(\bar{\imath}) - l(\bar{\imath}))$$

as claimed. Since the degree of integration is linear in the generated surplus, integration decreases when the condition holds.

6.9 Proof of Lemma 16

Going back to Proposition 5 the Pareto frontier of the feasible set is characterized by levels of generated surpluses $\underline{u}(A) = A\sqrt{2c} - 2c$ and $u_{=}(A) = \frac{3}{8}A^2 - c$ such that :

For $u_1 \in [0, \underline{u}(A)]$, the contract is an integration contract $(\bar{s}, \omega)$. Hence from (2) and $\bar{s} = 1 - \frac{\sqrt{2c}}{A}$, we obtain after some computations,

$$\begin{aligned} u_1(\bar{s}, \omega; A) &= A\sqrt{2c} - 2c - \omega\frac{\left(1-\sqrt{2c}\right)^2}{2} \\ u_2(\bar{s}, \omega; A) &= \frac{A^2}{2} - 2c + \omega(A - \sqrt{2c})\left(1 - \sqrt{2c}\right) \end{aligned} \tag{10}$$

Note that the maximum ω consistent with 1's individual rationality is

$$\bar{\omega}(A) = 2\frac{A\sqrt{2c} - 2c}{\left(1-\sqrt{2c}\right)^2}$$

which is increasing in A. From Proposition 5 $\underline{u}(A), u_{=}(A)$ are both increasing in A. Note that for a given value of ω, the absolute value of the slope of the frontier is $\left|\frac{du_2/d\omega}{du_1/d\omega}\right| = \frac{2(A-\sqrt{2c})}{1-\sqrt{2c}}$ which is increasing in A.

For $u_1 \in [\underline{u}(A), u_{=}(A)]$, the contract is a non-integration contract $(s, 0)$ and

$$\begin{aligned} u_1(s, 0; A) &= \frac{1-s^2}{2}A^2 - c \\ u_2(s, 0; A) &= \frac{s(2-s)}{2}A^2 - c. \end{aligned} \tag{11}$$

Here the slope of the frontier is $\frac{du_2/ds}{du_1/ds} = -\frac{1-s}{s}$ and independent of A.

6.10 Proof of Proposition 17

$>$From Proposition 5, a pair of payoffs (u_1, u_2) on the pre-liquidity frontier is generated by a unique contract (s, ω). Let $(s(l, v_2^*; A), \omega(l, v_2^*; A))$ be the contract chosen in a firm with technology A when 1 has liquidity l and 2 must obtain v_2^*; with this contract the generated payoffs solves $\phi(u_1; A) = v_2^* - l$.

Remark 18 *Note that there are other possibilities beside the two considered in the Proposition.*

Case 1: A first possibility is $i_1 < 1-n$, that is, shocked firms were not matched in the initial economy but because $w(i_1) > v_2^*(A)$, some of these firms will be matched. In this case, the set of "new entrants" are firms with $i \in [i_\pi, i_1]$ while the set of "old firms" are those with index $i \geq k$, where $k \geq 1-n$ satisfies $w(k) = w(i_\pi)$ and $i_1 - i_\pi = k - (1-n)$ (hence firms $i \in [i_\pi, i_1]$ "replace" firms $i \in [1-n, k]$).[21] *Since $w(i_\pi) > v_2^*(A)$, the degree of integration in old firms. For new firms, the question is whether the increase in price $w(i_\pi) - w(1-n)$ is large enough to overcome the internal effect of technology shock pushing towards less integration.*

Case 2: Another possibility is $1-n \in (i_0, i_1)$ and $w(1-n) > \lim_{\varepsilon\downarrow 0} w(i_1+\varepsilon)$. Then there exists $k > i_1$ such that $w(k) = w(1-n)$, and either $i_\pi \in (i_1, k]$ or $i_\pi \in [i_0, 1-n)$. In either case, if $l(i_\pi)$ is low enough, the increase in equilibrium surplus to the 2 may be small enough that the internal effect dominates and shocked firms integrate less.

[21] The existence of such values of $\bar{\imath}$ and k is insured if indeed $\bar{\imath} \in (i_0, i_1)$. By assumption $w(i_1) > w(1-n)$. If $w(i_0) \geq w(1-n)$, but $w(1-n+\lambda) < w(i_0)$, where $\lambda = i_1 - i_0$ is the measure of shocked type 1 firms; the marginal type is then $\bar{\imath} = 1-n+\lambda < i_0$ which contradicts our assumption. If $w(i_0) \geq w(1-n)$ we need therefore that $w(1-n+\lambda) \geq w(i_0)$, in which case there exists k such that $k = 1-n+i_1-\bar{\imath}$ and $w(k) = w(\bar{\imath})$. If $w(i_0) < w(1-n)$, there exists $i \in (i_0, i_1)$ such that $w(i) = w(1-n)$ and we can replicate the previous argument with $\lambda = i_1 - i$.

We continue with the proof of the Proposition.

(i) (Inframarginal shocks) If $i_\pi = 1 - n$ *and* $w(i_\pi) = v_2^*(A)$, *then the shocked firms become less integrated and the 2's shares fall, while the unshocked firms remain unaffected*

This is a direct consequence of Lemma 16(ii) and Proposition 5.

(ii) (Marginal shocks) If $1-n \in [i_0, i_1]$, *and* $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1 + \varepsilon)$, $i_\pi = 1 - n$, $F = [1-n, 1]$, *the equilibrium price* $v_2^*\left(\hat{A}\right)$ *increases and all firms, shocked and unshocked, integrate more.*

If $1 - n \in (i_0, i_1)$ and $w(1-n) \leq \lim_{\varepsilon \downarrow 0} w(i_1 + \varepsilon)$, $1 - n$ minimizes $w(i)$ over $i \geq 1 - n$; therefore $i_\pi = 1 - n$ and $F = [1-n, 1]$.

>From Lemma 16, $\phi(u; A)$ is increasing in A for any value of u. Since $v_2^*\left(\hat{A}\right) - v_2^*(A) = \phi\left(l(1-n); \hat{A}\right) - \phi(l(1-n); A)$, $v_2^*(A)$ is increasing in A; it follows from Proposition 5 that all unshocked firms $[i_1, 1]$ integrate more.

If the firm $1 - n$ did not integrate before the shock (that is chose $\omega = 0$ and $s < \bar{s}$), all $i > 1-n$ firms also chose not to integrate since ω is decreasing in the liquidity of type 1. Hence, it is immediate that an increase in A can only lead to more integration.

Consider now the case where firm $1 - n$ integrated before, that is chose a contract $(\bar{s}, \omega)$, $\omega > 0$. If i_1 chose initially a contract $(s, 0)$, there exists $k \in (1-n, i_1)$ choosing the contract $(\bar{s}, 0)$ and all firms with $i < k$ integrate $(\omega > 0)$ and all firms with $i > k$ do not integrate; firms with $i > k$ will necessarily integrate more after the shock. Hence without loss of generality assume that all firms $i \in [1-n, i_1]$ integrated before the shock, i.e., that the equilibrium contracts lead to surpluses on the linear part of the frontier. By Proposition 5, the pre-liquidity Pareto frontier is given by the map

$$\phi(u_1) = \phi(0; A) - \alpha(A) u_1, \tag{12}$$

where $\alpha(A) = 2\frac{A-\sqrt{2c}}{1-\sqrt{2c}}$ is increasing in A. Let $u_1(i; A)$ be the equilibrium generated surplus of type 1 when the technology is A and let $\omega(i; A)$ be the degree of integration chosen by firm i in equilibrium. Recall that, $\phi(u_1(i; A)) = v_2^*(A) - l(i)$. Since $1 - n$ is the marginal type 1,

$$u_1(1-n; A) = l(1-n), \tag{13}$$

$$\begin{aligned} v_2^*(A) &= \phi(l(1-n); A) + l(1-n) \\ &= \phi(0; A) - \alpha(A)\, l(1-n; A) - l(1-n). \end{aligned} \tag{14}$$

and therefore using (12) and (14) we have

$$\alpha(A)\left[u_1(i) - u_1(1-n)\right] = l(i) - l(1-n). \tag{15}$$

Since $\alpha(A)$ is increasing in A, it follows from (15) that $u_1(i) - u_1(1-n)$ must decrease after the shock.

The generated surplus is linear in ω (see (10)) and we have:

$$u_1(i;A) - u_1(1-n;A) = \frac{\left(1-\sqrt{2c}\right)^2}{2}\left(\omega(1-n;A) - \omega(i;A)\right) \tag{16}$$

and (15)-(16) imply

$$\omega(1-n;A) - \omega(i;A) \text{ is decreasing in } A. \tag{17}$$

By (13) and (10),

$$\omega(1-n;A) = 2\frac{A\sqrt{2c} - 2c - l(1-n)}{\left(1-\sqrt{2c}\right)^2}$$

is clearly increasing in A; it follows from the previous observation that $\omega(1-n;A) - \omega(i;A)$ $\omega(i;A)$ is also increasing in A, therefore by (17), $\omega(i;A)$ is increasing in A for all $i > 1-n$.

(iii) If $i_0 = 0$ and $i_1 = 1$, the arguments for (ii) apply since $1-n$ is still the marginal type 1 manager.

7 Appendix II: Parameter Restrictions

Here we provide sufficient conditions on the parameters that yield the concave frontiers – along with the simple description of organizational forms – described in the text. We consider them roughly in order of increasing strength.

In order for there to be a trade-off between surplus division and surplus production (and thus a role for the market to determine internal organization), we need

- nonintegration with $s = 1/2$ produces more surplus than letting one party own all the assets and receiving all the profit (this being the most efficient given full ownership). Simple calculations reveal that the necessary and sufficient condition for this is that $A \leq 2 - \sqrt{2}$; however this is stronger than necessary, since if $c > 0$, this isn't even feasible, and in any case isn't individually rational for the partner ceding ownership. It's easier elsewhere just to take $A \leq 1/2$.

- A feasibility requirement that all assets can be operated is that the maximum interim rational share is at least 1/2 (otherwise both partners cannot be incentive compatible); this is essentially a requirement that c not be "too big": $\bar{s} \geq 1/2 : \sqrt{2c} \leq A/2$ or $c \leq A^2/8$; this is not quite sufficient to guarantee that all assets *will* be operated; see below.

- No shut down (see below): it is an interesting logical possibility, which may have empirical counterpart; that some of the firms' assets will be shut down in order to compensate a manager who is ceding ownership; though worthy of further research, we choose here to rule it out in order to focus on other issues. What is required is a simple condition on the costs of operating assets that is approximately the same as the previous one, but neither implies nor is implied by it

- no swapping (see below): another interesting logical possibility: managers "swap" assets as a commitment device. This makes sense if productivity is far more important to payoffs than costs, and turns out to be ruled out by the other assumptions (in particular the first and third, which together guarantee that the slope of the frontier above the 45°-line is less than unity in magnitude).

- concavity of the frontier: this facilitates some of the aggregate computations in Section 4.2.2, but does not otherwise affect most of the conclusions. What is required here is that ω and s do not truly covary, for which the necessary and sufficient condition is that $s^* \geq \bar{s}$: a simple computation shows that this is equivalent to $\sqrt{2c} \geq \frac{2}{3}A + \frac{1}{6} - \frac{1}{6}\sqrt{(1 + 8A - 8A^2)}$, in other words, c is "not too small."

It is this last condition that is really most stringent, but it should be clear that its role is more of expositional rather than conceptual importance.

7.1 No Shutting Down of Assets

As in the text, we continue to maintain the assumption $A \leq 1/2$. If $\hat{s} \geq s > \bar{s}$, any asset owned by 1 will be shut down; if $s > \hat{s}$, then all of the type-1 assets are shut down. Thus for a contract (s, ω) the payoffs will be

$$u_1 = \begin{cases} (1-s)A[sA + \omega] - \frac{\omega}{2} - \omega c, \ \hat{s} \geq s > \bar{s} \\ (1-s)sA^2 - \frac{\omega}{2}, \ s > \hat{s} \end{cases}$$

$$u_2 = \begin{cases} sA[\frac{sA}{2} + \omega] - c, \ \hat{s} \geq s > \bar{s} \\ \frac{s^2A^2}{2} - c, \ s > \hat{s} \end{cases}$$

If $\bar{s} < s < \hat{s}$, **then** $\omega = 0$. Solve the Pareto problem $\max_{s,\omega} u_2$ s.t. $u_1 \geq v$ with multiplier λ and obtain from the first-order conditions

$$\omega \left\{ \begin{array}{c} = 0 \\ \in (0,1) \\ = 1 \end{array} \right\} \text{as } \lambda \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \frac{sA}{c + \frac{1}{2} - (1-s)A}$$

$$s \in (1/2, \hat{s}) \Longrightarrow \lambda = \frac{\omega + sA}{\omega - (1-2s)A}$$

Since $A \leq c + \frac{1}{2}$, $\frac{\omega + sA}{\omega - (1-2s)A} > \frac{sA}{c+\frac{1}{2}-(1-s)A}$ for any $\omega \in [0,1]$, so we must have $\omega = 0$.

If $s = \hat{s}$ **then** $\omega = 0$. Varying ω above 0 here generates a frontier of slope magnitude less than 1 ($= 2\hat{s}A = 2(A-c) < 1$, since $A \leq c + \frac{1}{2}$), while varying s above $\hat{s}$ with $\omega = 0$ generates a steeper frontier (slope $= \frac{s}{2s-1} > 1$), so Pareto dominates $\omega > 0$ with $s = \hat{s}$

If $s > \hat{s}$, **then** $\omega = 0$. This is immediate by inspection: 2's payoff is independent of ω in this case, and he therefore does not benefit from $\omega > 0$, while 1 is hurt.

We conclude that if s is large enough that any 1-assets are shut down, they will all be owned by 1 ($\omega = 0$), and the ($s > \bar{s}$)-frontier is continuous.

To ensure that no assets are shut down, we simply impose that the best payoff to 2 on the ($s > \bar{s}$)-frontier is smaller than his best payoff on the ($s \leq \bar{s}$)-frontier**.** That this suffices depends on noting that the ($s > \bar{s}$)-frontier begins below the ($s \leq \bar{s}$)-frontier and is smooth, with negative slope greater than unity in magnitude, and yielding its maximum payoff to 2 at $s = 1$, while the ($s \leq \bar{s}$)-frontier has slope less than unity. The maximal payoff to 2 on the ($s > \bar{s}$)-frontier is $\frac{A^2}{2} - c$, where 1 gets 0. On the ($s \leq \bar{s}$)-frontier, the maximal payoff to 2 obtains when 1 gets zero, which entails that $(1-\bar{s})A[\bar{s}A + \omega] - \frac{\omega}{2} - \omega c = 0$, or $\omega = \frac{\bar{s}(1-\bar{s})A^2}{\frac{1}{2}+c-(1-\bar{s})A}$. In this case, 2 obtains $\frac{\bar{s}^2A^2}{2} + \frac{\bar{s}(1-\bar{s})A^2(\frac{1}{2}+c-(1-\bar{s})A-\bar{s}(1-\bar{s})A^2)+\bar{s}^2(1-\bar{s})A^3}{\frac{1}{2}+c-(1-\bar{s})A} - c$, which exceeds $\frac{A^2}{2} - c$ if and only if $\bar{s}^2 A \frac{1-(1-\bar{s})A}{\frac{1}{2}+c-(1-\bar{s})A} \geq \frac{1-\bar{s}}{2}$, or, using the definition of $\bar{s}$, $4\bar{s}^2 A \geq (1-\bar{s})(1-(1-\bar{s})A)$. In terms of c and A, this reduces to $4A^2 - (1+8A)\sqrt{2c} + 10c \geq 0$.

Thus shutting down assets is Pareto dominated when $4A^2 - (1+8A)\sqrt{2c} + 10c \geq 0$ and $A \leq \frac{1}{2} + c$, the Pareto frontier is therefore as described in the text.

There is a positive measure of the parameter space satisfying all of the above conditions. In particular, the case $c = A^2/8$ with A large enough is included, and thus admits the expositionally useful case in which the Pareto frontier is piecewise linear with the constant share $\bar{s} = 1/2$.

7.2 No Swapping of Assets

Asset swapping is a means of effectively committing the managers to high levels of q. This commitment is only worthwhile if productivity is sufficiently high relative to costs; our parametric case of interest rules this out.

To see this, note that if assets are to be swapped, we can characterize the situation via two ownership parameters ψ and ω : manager 1 owns $k \in [0, 1-\omega)$ and $k \in [1, 2-\psi)$, and 2 owns $k \in [1-\omega, 1)$ and $[2-\psi, 2]$, where $\psi \in [0, 1]$ and now $\omega \in [0, 1]$ instead of $[-1, 1]$.

Given a contract (s, ψ, ω) with utility allocation above the 45°-line (we restrict attention to this case; the other one is similar), it is straightforward to check that the payoffs are now (assuming $s \leq \bar{s}$; if $\bar{s} < s \leq \hat{s}$, the $(1-\omega)$ terms vanish, and if $s > \hat{s}$, so do the ω terms)

$$u_1 = (1-s)A[\frac{(1-\omega)(1-s)A}{2} + \psi s A + (1-\psi) + \omega] - \frac{\omega}{2} - c$$
$$u_2 = sA[\frac{\psi s A}{2} + (1-\psi) + \omega + (1-\omega)(1-s)A] - \frac{1-\psi}{2} - c$$

Total surplus is

$$W = \psi s^2 A^2 (\frac{1+\psi}{2}) + (1-\psi)A + \omega A + (1-\omega)(\frac{1-s^2}{2})A^2 - \frac{1-\psi}{2} - \frac{\omega}{2} - 2c$$

Asset swapping then entails $\psi < 1$ (with $\omega \geq 0$), and we need to rule it out. Observe that both u_2 and W are increasing in ψ ($\frac{\partial u_2}{\partial \psi} = \frac{1}{2} + \frac{s^2 A^2}{2} - sA = \frac{1}{2}(1-sA)^2 > 0$; $\frac{\partial W}{\partial \psi} = s(1-\frac{s}{2})A^2 - A + \frac{1}{2} > 0$ for $s \geq 1/2$ and $A < 2/3$), while u_1 is decreasing ($\frac{\partial u_1}{\partial \psi} = (1-s)A(sA-1) < 0$); this is true even if $s > \bar{s}$. From the previous derivations, we know that the Pareto frontier above the 45°-line for the set of contracts restricted by $\psi = 1$ has slope magnitude less than one.

Take an arbitrary contract (s, ψ, ω) with $u_1(s, \psi, \omega) \leq u_2(s, \psi, \omega)$. Then $u_1(s, 1, \omega) \leq u_2(s, 1, \omega)$ as well. Let U be the restricted utility possibility set above the 45°-line, that is points generated by the set of restricted contracts (or Pareto inferior points generated from a restricted contract plus free disposal).

For any contract $(s, 1, \omega)$ in the restricted set of contracts with utilities above the 45°-line, the set

$$P(s, \omega) = \{(u_1, u_2)|u_1 \leq u_2, u_1 \geq u_1(s, 1, \omega), u_2 \leq u_2(s, 1, \omega), \\ u_1 + u_2 \leq u_1(s, 1, \omega) + u_2(s, 1, \omega)\}$$

lies in U, since U's frontier has slope less than 1. Moreover, $(u_1(s, \psi, \omega), u_2(s, \psi, \omega)) \in P(s, \omega)$ by construction. Thus, (s, ψ, ω) is in U, and is therefore generated by or is Pareto inferior to some contract $(s', 1, \omega')$.

Implicit in this are parametric restrictions, of course, the same ones used to generate a frontier slope less than one, i.e., Assumption 2.

7.3 Continuous operating decisions

We show here that allowing for continuous operating decisions with linear cost doesn't change anything. Suppose $e(k) \in [0, 1]$ instead of $\{0, 1\}$, all k, and keep the cost generated by asset k equal to $\frac{1}{2}q(k)^2 + ce(k)$. First consider assets that 1 operates. If he also owns them, then in order to implement an interior $e(k)$, we must have $e(k)q_1(k)A(1 - s) - e(k)c = 0$ or $q_1(k) = c/(1 - s)A$. But then 1's payoff is $e(k)(1 - s)Aq_1(k) - \frac{1}{2}q_1^2(k) - e(k)c$,which upon substitution leads to the negative payoff $-\frac{1}{2}q_1^2(k) = -\frac{c^2}{(1-s)^2A^2}$; 1 would do better to pick $e(k) = q_1(k) = 0$, and better still to pick $e(k) = 1$, $q_1(k) = (1 - s)A$ (provided $s < \bar{s}$; if not, 0 is optimal). Thus interior e's are not implementable for assets owned by 1.

For assets owned by 2, the only way to sustain an interior $e(k)$ is to have $(1 - s)A = c$; since $e(k) > 0$, $q_2(k) = 1$. The payoff to 2 is strictly increasing in $e(k)$, while 1 is indifferent, so $e(k) = 1$ is Pareto optimal.

The argument for assets operated by 2 is similar.

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