Optimal Gradualism*

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Abstract

This paper studies how gradualism affects the welfare gains from trade, technology, and reforms. When people face adjustment frictions, gradual shocks create less adverse distributional effects in the short run. We show that there are welfare gains from inducing a more gradual transition via temporary taxes on trade and technology, and provide formulas for the optimal path for taxes. Our formulas account for the possibility that reallocation effort responds to policy, and for the existence of income taxes and assistance programs. Using these formulas, we compute the optimal temporary taxes needed to mitigate the distributional consequences of rising import competition from China and the deployment of automation technologies substituting for routine jobs. Our formulas can also be used to compute the optimal timing of economic reforms or trade liberalizations, and we apply them to study Colombia's trade liberalization in 1990—a prominent example where optimal policy called for a more gradual reform.

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Technological progress, trade, and economic reforms generate periods of adjustment during which some people fall behind, lose their jobs, experience wage declines, and see their livelihoods disrupted.[1] Even if technology and trade are positive developments in the long-run, dealing with short-run disruptions for workers remains an important policy concern, especially in the wake of rapid changes in the economy.[2]

Existing evidence points to large disruptions. Autor et al. (2014) document that an average US worker in an industry exposed to Chinese import competition experienced a cumulative income loss equal to half their annual earnings in 1990 over the 1992–2007 period relative to unexposed workers. Cortes (2016) shows that US workers who in 1985 held routine jobs—those that can be more easily automated—experienced a subsequent decline in wages of 16% by 2007 relative to similar workers in other occupations.

How should policy respond during these adjustment periods? Do short-run disruptions imply that more gradual advances in technology and trade are preferable?

This paper shows that short-run disruptions create potential gains from gradualism and justify temporary taxes on new technologies and trade or embracing gradual reforms. Our main contribution is to provide formulas for the optimal path for taxes on new technologies and trade that capture the gains from gradualism. We evaluate these formulas in a calibrated version of our model that matches the empirical estimates of Autor et al. (2014) for trade and Cortes (2016) for the automation of routine jobs. Our formulas call for temporary taxes on trade and automation technologies of 10%–15% phased out over time. We also use our formulas to study Colombia's trade liberalization in 1990 and show that optimal policy called for a more gradual reform.

We derive these formulas in a model where workers are displaced by technology or trade. Ex-ante identical workers are allocated across islands a-la Lucas and Prescott (1974). Some islands represents jobs automated by new technologies (e.g., welding or data-entry clerks) or segments of industries disrupted by international trade (e.g., low-cost apparel or household electronics). At time t_0, a new technology arrives, capable of replacing workers in these

[1] For evidence in the context of trade, see Autor and Dorn (2013); Autor et al. (2014). For evidence in the context of automation technologies, see Cortes (2016); Adão et al. (2021); Acemoglu and Restrepo (2020, 2022). Finally, see Goldberg and Pavcnik (2005) for evidence on how dismantling trade protection reduces the relative wages of workers in exposed industries.

[2] In the US, industrial robots installations and imports from China tripled in a few years (see Autor et al., 2013; Acemoglu and Restrepo, 2020, respectively), and the share of e-commerce in retail went from 0.6% to 10% from 1999 to 2019 (see US Census, 2022). As Erik Brynjolfsson and Andrew McAffe put it in *The Second Machine Age*, "People are falling behind because technology is advancing so fast and our skills and organizations aren't keeping up" (Brynjolfsson and McAfee, 2014). Managing short-run disruptions is also a key policy concern when it comes to policy reforms (see, for example, Rodrik, 1995).

islands by producing the same output at lower costs. These costs decline over time as technology improves exogenously, capturing advances in automation (as in Acemoglu and Restrepo, 2022) or improvements in Chinese exporters' productivity (as in Caliendo et al., 2019; Galle et al., 2022). Real wages at disrupted islands fall over time and real wages at other islands increase. As in Alvarez and Shimer (2011), workers reallocate at a rate $\alpha > 0$, which represents the time it takes to find new jobs or acquire skills required in other jobs. The transition features a temporary decline in real wages and consumption for disrupted workers and higher real wages for all in the long run.[3]

Using this model, we provide analytical answers to the two questions above:

Given a path for technological progress, should the government induce a more gradual adjustment via temporary taxes on new technologies?

Using a variational approach, we derive formulas for the optimal tax path on new technologies and show that the optimum involves a temporary increase in taxes that is phased-out over time. Temporary taxes are optimal even if technology, trade, and reforms make everyone better off in the long run. Taxes on new technologies should be higher when disrupted workers experience a large drop in consumption during the transition, which is itself linked to the decline in income documented by Autor et al. (2014) for the China Shock and Cortes (2016) for the automation of routine jobs.

Our formulas account for the possibility that taxing new technologies might generate adverse incentives for reallocation, which creates a motive for a faster phase out of the initial tax. Our formulas also extend to an scenario in which the planner can reform the income tax system or create temporary assistance programs to ease the transition for disrupted households. We characterize the optimal generosity of these programs during the transition and show that taxes on new technologies are still optimal in the short run.[4] The logic is the same as in Naito (1999) and Costinot and Werning (2022): taxing new technologies is valuable because it assists workers affected by technological disruptions or globalization and not those who reduced their work effort to exploit the tax system.

[3]Our model is designed to capture the effects of labor-replacing technologies or technologies that work by substituting workers at some of their existing roles. We see trade, offshoring, and automation technologies working in this way. These technologies have the potential to reduce wages of displaced workers and raise wages of all other workers. Other developments, such as factor-neutral improvements in technology, or technologies that directly complement skilled workers do not fit our description.

[4]If island-specific lump-sum transfers or island-specific wage subsidies were available, redistribution can be done without distorting production. This is a direct implication of the Second Welfare Theorem or the taxation principles in Diamond and Mirrlees (1971). In practice, these additional (and more desirable policy tools) might be limited, since identifying workers whose livelihoods were disrupted by technology and trade as opposed to regular economic churn and sectoral fluctuations might be challenging.

Conditional on government policy, does the economy benefit from more gradual technological advances along the transition?

Here too, we use a variational approach to compute the welfare gains from moving to a counterfactual world where advances in technology are more gradual. We refer to these as the gains (or costs) from *technological gradualism.* The gains from technological gradualism can be positive when technology advances rapidly and displaced workers experience large drops in consumption. However, with optimal taxes in place, there are no gains from technological gradualism and faster technological progress is always welcomed. Thus, an economy with optimal gradual policy in place gains more from technological change.

Applications: We apply our framework to study the automation of routine jobs, the China Shock, and Colombia's 1990 trade liberalization. We calibrate the model to match the evidence in Cortes (2016) on the automation of routine jobs and in Autor et al. (2014) on the China Shock. The evidence points to limited opportunities for reallocation and implies low values for the reallocation rate α of 2.7% per year for workers in routine jobs and 1.8% per year for the China Shock. In addition, we back out the underlying path for technology from data on occupational wages or import shares.

We find that the automation of routine jobs had a negative welfare effect of 6%–8% on workers in disrupted islands, depending on assumptions about initial assets and capacity to borrow. This is driven by a short-run income decline of 12% from 1985 to 2000, which recovers by 2025. From an utilitarian perspective, these short-run disruptions justify an optimal tax on automation technologies of 10%–12.5% over 1985–1995, which is then phased out and reaches a level of 4% by 2020 in the least gradual scenario.

The China Shock had a negative welfare effect of 15%–19% on workers in disrupted islands, depending on assumptions about initial assets and capacity to borrow, though these account for only 1.6% of the US workforce. From an utilitarian perspective, these disruptions justify an optimal tax on Chinese imports of 10%–15% over 1991–2000, which is then phased out and reaches a level of 3% by 2020 in the least gradual scenario.

In both applications, we estimate no gains from more technological gradualism, even absent taxes on technologies or trade. This points to an important distinction: the fact that taxing trade and automation technologies in the short run is optimal does not imply that society benefits from moving to a world where exogenous advances in trade and automation are slower. This is because taxing technologies generates a more gradual path for wages and additional tax revenue; while a more gradual technology path reduces tax revenue.

In a final application, we use our formulas to compute the optimal trade liberalization policy for Colombia. In 1990, Colombia embarked in a rapid and ambitious program of trade liberalization, reducing effective tariffs by 37.5% in two years. We calibrate our model to match the immediate increase in imports following the reform and the drop in wages in previously protected industries estimated by Goldberg and Pavcnik (2005), which points to small values of α of 3% per year. Optimal policy calls for a more gradual reform, with tariffs remaining at a fourth of their initial level by 2000—10 years after the reform started. Reallocation rates of 20% per year—one order of magnitude higher than what we estimate—are needed to justify Colombia's swift drop in tariffs.

Related literature: Our optimal tax formulas relate to the tariff formula in Grossman and Helpman (1994) and the formula for optimal taxes on new technologies in Propositions 1 and 3 of Costinot and Werning (2022). Grossman and Helpman (1994) and Helpman (1997) focus on redistribution via tariffs across workers specialized in different industries. Our optimal tax formula shares the same structure as theirs and generalizes it to a dynamic environment where workers reallocate over time.[5]

Our characterization of optimal taxes builds on Costinot and Werning (2022) and extends their variational arguments to a dynamic setting.[6] Despite methodological similarities, the problem solved by Costinot and Werning differs from ours. They are interested in *pre-distribution*: taxing technology to reduce permanent inequality between ex-ante different people. Technologies that reduce wages at the bottom of the income distribution relative to the top have "tagging" value. Taxing these technologies achieves a better distribution of income than using income taxes alone, an insight that goes back to Naito (1999). The problem we study is complementary. In our model, winners and losers are ex-ante identical and new technologies are taxed to ease the transition. This is why our formulas for optimal taxes are linked to the short-run decline in income for exposed workers (i.e., the regressions in Autor et al., 2014; Cortes, 2016, used here), and do not depend on how robots or trade affect wages at different points of the income distribution (i.e., the quantile regressions in Acemoglu and Restrepo, 2020; Chetverikov et al., 2016, used by Costinot and Werning). The formulas in Costinot and Werning prescribe long-run taxes on the basis of pre-distribution and our formulas prescribe short-run taxes to ease transitions.

Our paper also contributes to a recent literature on the optimal taxation of automation motivated by distributional considerations (Thuemmel, 2018; Guerreiro et al., 2021; Donald,

[5]The formulas also differ in that Grossman and Helpman assume a quasi-linear aggregator across islands and their weights emerge from lobbying and not necessarily from welfare considerations.

[6]Variational arguments have been used extensively to characterize properties of optimal income tax schedules (Saez, 2001; Tsyvinski and Werquin, 2017; Saez and Stantcheva, 2016).

2022) or inefficiencies (Acemoglu et al., 2020; Beraja and Zorzi, 2022). Thuemmel (2018); Guerreiro et al. (2021) show that non-zero taxes on robots are justified even when income taxes are available as an additional tool for redistribution, in line with Naito (1999).[7] Like us, Guerreiro et al. (2021) emphasize that optimal taxes on robots are positive along the transition and zero in the long run, when affected cohorts of workers retire from the labor market. Beraja and Zorzi (2022) also argue for temporary taxes on automation technologies, though in their case taxation is motivated by efficiency considerations: firms do not internalize the fact that their decision to automate push displaced workers against their borrowing constraint, which generates excessive automation.[8] We contribute to this literature by providing intuitive and general formulas for optimal taxes on automation technologies that provide a tight link between the theory and the empirical evidence and identify the key features of the data that inform optimal taxes. We also show that the same formulas can be applied to understanding how trade competition should be handled during a transition period and how economic reforms should be conducted.

Finally, we contribute to the literature on optimal timing of reforms and trade liberalization, going back to Mussa (1984) and with contributions by Edwards and van Wijnbergen (1989); Karp and Paul (1994); Rodrik (1995); Bond and Park (2002); Chisik (2003). Mussa argued that "a general case for gradualism in trade liberalization can be based on a desire to limit the income and wealth losses sustained by owners of resources initially employed in protected industries," which is the rationale for gradualism studied in this paper.

Roadmap: Section 1 introduces our model and characterizes the transitional dynamics following advances in trade or labor-replacing technologies. Section 2 derives formulas for optimal taxes and the gains from technological gradualism. Sections 3, 4, and 5 apply our framework to the automation of routine jobs in the US, the China Shock, and Colombia's trade liberalization. Proofs and derivations are in the Appendix.

1 A Model of Economic Disruptions

Status quo: Consider an economy with a mass 1 of households and a set of islands $x \in \mathcal{X}$. Each island produces a good $y_{x,t}$, which combines with the output of other islands into a

[7] A complementary line of work studies the compensation of displaced workers via changes in income taxes (see Antràs et al., 2017; Tsyvinski and Werquin, 2017) and derives formulas for welfare that account for distributional considerations when these optimal taxes are in place.

[8] In their extensions, Beraja and Zorzi consider the role of redistribution and show that this leads to higher taxes on new technologies, which is in line with our results.

final numeraire good y_t according to a constant returns to scale production function

$$y_t = f\left(\{y_{x,t}\}_{x\in\mathcal{X}}\right).$$

Initially, islands produce these goods with labor, so that $y_{x,t} = \ell_{x,t}$, where $\ell_{x,t}$ denotes the mass of workers in island x. We assume that the initial allocation of workers $\ell_{x,0}$ across islands before the shock equates wages to a common level $\bar{w}$. This can be thought of as the steady state of the reallocation process introduced below.

The disruption: At time $t = 0$ a new technology arrives. For a subset of *disrupted islands* $x \in \mathcal{D} \subset \mathcal{X}$, it becomes possible to produce their goods using a new technology embodied in capital $k_{x,t}$. New capital can be produced from $1/A_{x,t}$ units of the final good, where $A_{x,t}$ is the productivity of the new technology, which increases over time and converges to A_x.

Following the arrival of new technologies, the production of $y_{x,t}$ becomes

$$y_{x,t} = \begin{cases} \ell_{x,t} + k_{x,t} & \text{if } x \in \mathcal{D} \\ \ell_{x,t} & \text{if } x \notin \mathcal{D}. \end{cases}$$

The disruption leads to permanent changes in island wages $w_{x,t}$, which prompt workers to reallocate with Poisson probability $\alpha_x > 0$ to an island of their choice.

Taxes: The government sets taxes $\tau_{x,t}$ on new technologies, raising revenue

$$T_t = \sum_{x\in\mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}},$$

which gets redistributed in a lump sum way. We first consider a baseline version of our model where the government has no other tools for redistribution or assistance, and study these tools in our extensions.

Households: Households are ex-ante identical in their labor productivity and are all endowed with 1 unit of labor. Before the shock, all households in island x hold assets $a_{x,0}$. After the shock, they make consumption and saving decisions to maximize

$$U_{x,0} = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \cdot u(c_{x,t}) \cdot dt\right] - \kappa(\alpha_x)$$

subject to some budget constraints that we left unspecified, but that could capture various scenarios, ranging from hand-to-mouth, to perfect risk sharing within islands.

For our purposes, it suffices to work with households indirect utility function

$$U_{x,0} = \mathcal{U}\left(\{w_{x',t} + T_t\}_{x' \in \mathcal{X}}, a_{x,0}; \alpha_x\right) - \kappa_x(\alpha_x),$$

which gives the maximum expected utility for a household in island x facing a path of incomes $\{w_{x',t} + T_t\}_{t=0}^{\infty}$ across islands and that reallocates at a rate α_x.[9]

The term $\kappa_x(\alpha_x)$ captures reallocation costs. We consider two cases. With *exogenous reallocation,* $\alpha_x > 0$ is fixed and we set $\kappa_x(\alpha_x) = 0$. With *endogenous reallocation,* α_x is chosen by households to maximize $\mathcal{U}\left(\{w_{x',t} + T_t\}_{x' \in \mathcal{X}}, a_{x,0}; \alpha_x\right) - \kappa_x(\alpha_x)$.

1.1 Transitional Dynamics and Equilibrium

We impose two assumptions on f, which are satisfied by commonly used aggregators.

Assumption 1 (Symmetry) *For all islands $x', x'' \notin \mathcal{D}$ and any island $x \in \mathcal{D}$, we have*

$$\frac{\partial^2 f}{\partial y_x \partial y_{x'}} = \frac{\partial^2 f}{\partial y_x \partial y_{x''}}.$$

This assumption ensures that technology benefits non-disrupted islands equally. It implies a common wage w_t at non-disrupted islands along the transition. This allows us to study redistribution between non-disrupted and disrupted islands—the winners and losers of trade, technological progress, or reforms—and abstract from redistribution between winners (i.e., software engineers benefiting more than economists from the automation of sales jobs).

Let $c^f(p)$ denote the unit cost function associated with the aggregator f. With some abuse of notation, we denote by $c^f(\{w_x\}_{x \in \mathcal{D}}, w)$ the price of the final good when the price of the island x output is w_x for $x \in \mathcal{D}$ and w for other islands. Also, we denote by c^f_x and c^f_w the partial derivatives of this cost function with respect to w_x and w.

Assumption 2 (Adoption) *For any vector of wages with $w_x < \bar{w}$ for $x \in \mathcal{D}$ and a wage $w > \bar{w}$ in non-disrupted islands such that $c^f(\{w_x\}_{x \in \mathcal{D}}, w) = 1$, we have*

$$\frac{c^f_x\left(\{w_x\}_{x \in \mathcal{D}}, w\right)}{c^f_w\left(\{w_x\}_{x \in \mathcal{D}}, w\right)} > \frac{c^f_x\left(\{\bar{w}\}_{x \in \mathcal{D}}, \bar{w}\right)}{c^f_w\left(\{\bar{w}\}_{x \in \mathcal{D}}, \bar{w}\right)}$$

[9] Our formulation assumes that either we are in a small open economy and the interest rate is fixed, or households are hand to mouth and do not save nor borrow. This is why the interest rate does not affect indirect utilities.

This assumption ensures that new technologies are adopted in all disrupted islands (so long as the after-tax cost of the new technology is below the initial market wage). It prevents adoption in one island from reducing demand in other disrupted island.

Assumptions 1 and 2 hold when there is a single disrupted and a single non-disrupted island, but also when there are many islands whose outputs are combined via a constant-elasticity of substitution aggregator, f.

The following propositions characterize the transitional dynamics of the economy in terms of wages and employment across islands.

PROPOSITION 1 *Suppose that Assumptions 1 and 2 hold and that $\bar{w} > (1 + \tau_{x,t})/A_{x,t}$, so that the new technologies are adopted at time 0. Wages are given by*

$$w_{x,t} = \begin{cases} (1 + \tau_{x,t})/A_{x,t} & \text{if } x \in \mathcal{D} \\ w_t & \text{if } x \notin \mathcal{D}, \end{cases}$$

where the common wage w_t in non-disrupted islands satisfies

$$1 = c^f\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right).$$

Along the transition, island employment is given by $\ell_{x,t} = e^{-\alpha_x t} \cdot \ell_{x,0}$ for $x \in \mathcal{D}$ and $\ell_t = 1 - \sum_{x\in\mathcal{D}} e^{-\alpha_x t} \cdot \ell_{x,0}$ for the remaining islands; output is given by

$$y_t = \ell_t \cdot \frac{1}{c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)};$$

and new technology utilization at island $x \in \mathcal{D}$ is given by

$$k_{x,t} = \ell_t \cdot \frac{c^f_x\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)}{c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)} - \ell_{x,t} > 0.$$

PROPOSITION 2 *Suppose Assumptions 1 and 2 hold.*

- *If households are insured against the island disruption, and there is risk sharing inside islands, $U_{x,0} = U_0$ and $c_{x,t} = c_t$ across all islands.*
- *If households had the same assets before the shock and they are not insured against the disruption, then $a_{x,0} = a_0$ and $U_{x,0} < U_0$.*
- *If households are hand-to-mouth, then $a_{x,0} = 0$ and $U_{x,0} < U_0$.*

2 Optimal Policy and the Gains from Gradualism

We maintain Assumptions 1 and 2 and consider the case where households are uninsured against the disruption, so that there is a role for policy.

We evaluate policies using a symmetric welfare function $W_0 = \sum_{x\in\mathcal{X}} \int_{h\in x} \mathcal{W}\left(U^h_{x,0}\right) \cdot dh$, where $U^h_{x,0} = U_{x,0}$ is the expected lifetime utility of household h in island x after the shock and $\mathcal{W}$ is an increasing and concave function. The per-capita Pareto weights (or social marginal welfare weights) for households from island x are therefore

$$g := \mathcal{W}'(U_0) \geq 0 \text{ for } x \notin \mathcal{D} \qquad\qquad g_x := \mathcal{W}'(U_{x,0}) \geq 0 \text{ for } x \in \mathcal{D}.$$

This implies that $g_x \geq g$ capturing the incentives to compensate losers from the disruption.[10]

Let $\mathcal{U}$ and $\mathcal{U}_x$ denote indirect utilities for households who were initially at non-disrupted islands and disrupted island x, respectively:

$$U_0 = \mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty}), \qquad\qquad U_{x,0} = \mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha_x) - \kappa_x(\alpha_x).$$

Of all households from island $x \in \mathcal{D}$, a fraction $P_{x,t} = e^{-\alpha_x \cdot t}$ will still work in the disrupted island at time t, consume $c_{x,t}$, and have a marginal utility of consumption at time t of $\lambda_{x,d,t} = e^{-\rho t} \cdot u'(c_{x,t})$. The remaining $1 - P_{x,t}$ households will have reallocated by that time, with a fraction $\alpha_x \cdot e^{-\alpha_x \cdot t_n}$ reallocating to a non-disrupted island at time $t_n \in [0, t]$. We denote their consumption at time t by $c_{x,t_n,t}$, and denote the average marginal utility of consumption for these households at time t by $\lambda_{x,n,t} = \mathbb{E}[e^{-\rho t} \cdot u'(c_{x,t_n,t}) | t_n \leq t]$. Households at non-disrupted islands do not reallocate, face no uncertainty, consume c_t and have a marginal utility of consumption $\lambda_t = e^{-\rho t} \cdot u'(c_t)$ at time t.

We first provide a general lemma that characterizes the change in welfare resulting from a variation in taxes and technology utilization. This lemma relates to Lemma 1 in Costinot and Werning (2022), but differs in that it accounts for the fact that variations in taxes affect households' incomes at all future states. It also builds on variational arguments from Saez (2001); Saez and Stantcheva (2016); Tsyvinski and Werquin (2017).

Lemma 1 (Variations Lemma) *Consider a variation in taxes that induces a change in wages $dw_t, dw_{x,t}$, technology $dk_{x,t}$, tax revenue dT_t, and reallocation effort $d\alpha_x$. This vari-*

[10] Our formulas for optimal taxes apply more generally when using *generalized social marginal welfare weights* that depend on broader ethical and political considerations (as in Saez and Stantcheva, 2016). Both features can be captured by having g_x and g depend on additional arguments.

ation changes tax revenue and social welfare by

$$(1) \quad dT_t = -\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot dw_{x,t} + \sum_x \tau_x \cdot \frac{dk_{x,t}}{A_{x,t}}$$

$$(2) \quad dW_0 = \int_0^\infty \Bigg[\sum_{x \in \mathcal{D}} \ell_{x,0} \cdot g_x \cdot \Big(P_{x,t} \cdot \lambda_{x,d,t} \cdot (dw_{x,t} + dT_t) + (1 - P_{x,t}) \cdot \lambda_{x,n,t} \cdot (dw_t + dT_t) \Big) + \ell_0 \cdot g \cdot \lambda_t \cdot (dw_t + dT_t) \Bigg] \cdot dt.$$

The lemma shows that the benefits of distorting technology depend on the λ's—the marginal utilities of consumption for disrupted and non-disrupted households. This object is rarely observed or estimated. Most empirical papers estimate the income drop experienced by households in disrupted jobs or industries but do not estimate the resulting drop in consumption or the increase in their marginal utility of consumption.

When taking our model to the data, our approach is to calibrate the model to match the income paths estimated in Autor et al. (2014) and Cortes (2016) and infer the path for marginal utilities of consumption by considering four scenarios that summarize households' consumption behavior and available tools for sharing risks during the transition:

I. Hand-to-mouth (transition risk and no borrowing): this scenario assumes households are hand-to-mouth. This implies $\lambda_{x,d,t} = e^{-\rho t} \cdot u'(w_{x,t} + T_t)$, $\lambda_{x,n,t} = e^{-\rho t} \cdot u'(w_t + T_t)$, and $\lambda_t = e^{-\rho t} \cdot u'(w_t + T_t)$. In this scenario, households cannot borrow to smooth their consumption along the transition and face the risk of transitioning late.

II. No borrowing and no transition risk: this scenario assumes no borrowing from other islands or foreigners. However, we allow households to engage in risk sharing within their island to share the risks of transitioning late. Equivalently, one could think of this as a case where each household owns a mass 1 of units of labor, which it then retools at a rate α to be used in other islands, so that it faces no uncertainty. In both cases, we obtain $\lambda_{x,d,t} = \lambda_{x,n,t} = e^{-\rho t} \cdot u'(P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t)$, and $\lambda_t = e^{-\rho t} \cdot u'(w_t + T_t)$.

III. Borrowing with transition risk: this scenario assumes that households can borrow at an exogenous interest rate r but face the risk of transitioning late. In disrupted islands, households problem can be summarized by the following system of HJB equations

$$\begin{aligned}\rho v_x(a,t) - \dot{v}_x(a,t) &= \max_c u(c) + \frac{\partial v_x(a,t)}{\partial a} \cdot (ra + w_{x,t} - c) + \alpha_x \cdot (v(a,t) - v_x(a,t)), \\ \rho v(a,t) - \dot{v}(a,t) &= \max_c u(c) + \frac{\partial v(a,t)}{\partial a} \cdot (ra + w_t - c).\end{aligned}$$

Here, $v_x(a,t)$ is the value function of households in disrupted islands at time t with assets a (and taking α_x as given), and $v(a,t)$ is the value function of households in non-disrupted islands with assets a. This problem can be solved numerically using the tools from Achdou et al. (2021), and produces paths for $\lambda_{x,d,t} = e^{-\rho t} \cdot u'(c_{x,t})$, $\lambda_{x,n,t} = \mathbb{E}[e^{-\rho t} \cdot u'(c_{x,t_n,t}) | t_m \leq t]$ and $\lambda_t = e^{-\rho t} \cdot u'(c_t)$ as functions of the stream of incomes in disrupted and non-disrupted islands, the reallocation rate α_x, and initial asset holdings. Appendix A.5 provides details.

IV. Borrowing with no transition risk: this scenario assumes ex-post complete markets. That is, households can freely save and borrow at exogenous interest rate r and share transition risks within their islands. Households' problem becomes

$$\max \int_0^\infty e^{-\rho t} \cdot u(c_{x,t}) \cdot dt \quad \text{s.t.: } 0 \leq \int_0^\infty e^{-rt} \cdot \Big[P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t - c_{x,t} \Big] \cdot dt + a_{x,0}.$$

which implies $\lambda_{x,d,t} = \lambda_{x,n,t} = e^{-rt} \cdot u'(c_{x,0})$ and $\lambda_t = e^{-rt} \cdot u'(c_0)$.

2.1 Optimal Policy with Exogenous Reallocation

The next Proposition provides our first formula for optimal taxes. In this and the following sections, we use the partial derivative $\partial \ln w_x / \partial \ln z$ to denote the effect of a log change in quantity z on the log of the wage $w_x = \partial f / \partial y_x$ holding all other quantities constant.

PROPOSITION 3 *With exogenous reallocation effort, a necessary condition for an optimal tax sequence is that*

$$\tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1 \right) \cdot \left(- \frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}} \right), \tag{3}$$

where $m_{x',t} = k_{x',t} / A_{x',t}$ denotes expenditure on $k_{x',t}$, and $\chi_{x,t}$ is the per-capita social value of increasing income in island x at time t, computed as

$$\chi_{x,t} = \begin{cases} g_x \cdot \lambda_{x,d,t} & \text{if } x \in \mathcal{D} \\ \dfrac{1}{\ell_t} \cdot \left(\displaystyle\sum_{x \in \mathcal{D}} \ell_{x,0} \cdot (1 - P_{x,t}) \cdot g_x \cdot \lambda_{x,n,t} + \ell_0 \cdot g \cdot \lambda_t \right) & \text{otherwise} \end{cases}.$$

Moreover, the formula in equation (3) implies zero long-run taxes $\lim_{t\to\infty} \tau_{x',t} = 0$.

The derivation follows Costinot and Werning (2022). The idea is that, at an optimum, a variation that changes $k_{x',t}$ should not change welfare. Using Lemma 1 to evaluate this variation yields the optimal tax formula in equation (3).

Equation (3) says that, at an optimum, the marginal cost of reducing $k_{x',t}$—the foregone tax revenue on the left-hand side—equals the distributional gains from curbing automation or trade—the right-hand side.[11] These distributional gains and the tax on automation and trade depend on:

1. **The influence of technology on wages:** Taxes should be higher when technology has a sizable negative impact on the wage of disrupted islands, as captured by the elasticities $-\partial \ln w_{x,t}/\partial k_{x',t}$. In this case, reducing the use of the technology is an effective tool to redistribute income towards households in disrupted islands.

2. **Differences in marginal utilities and aversion to inequality:** Taxes depend on the gap in marginal social values $\chi_{x,t}$ between disrupted and non-disrupted islands, which capture incentives for redistribution. This gap is large when disrupted households reduce their consumption during the transition. This depends on the income decline experienced by disrupted households, informed by the evidence in Cortes (2016) for routine jobs and Autor et al. (2014) for the China Shock, and households' ability to smooth consumption.

3. **Time horizon:** Taxes should be higher in the short run. This is because a variation that increases income at disrupted islands in the short run benefits a large fraction of disrupted households. In the long run, taxes are zero because workers reallocate.[12]

2.2 Endogenous Reallocation

We extend Proposition 3 to the case with endogenous reallocation. With endogenous reallocation, policy variations change reallocation rates by $d\alpha_x$. The effect of $d\alpha_x$ through

[11] Our formula applies to an economy with no ex-ante inequality between or within islands. This helps us isolate the incentives to induce gradual transitions to mitigate the cost of the disruption for affected households. Appendix A.4 extends some of our results to an economy with ex-ante inequality across households within and between islands. Under mild assumptions, inequality within islands does not affect our tax formulas. Inequality across islands creates incentives for higher taxes on technology if disrupted islands had lower consumption (in line with the logic of pre-distirbution in Costinot and Werning (2022)).

[12] This is also true in a model where technology does not lead to the full substitution of labor in disrupted islands. In this case, the new steady state involves fewer workers in disrupted islands and an equal wage across islands. A zero long-run tax maximizes this common wage.

households' transition probabilities is second order, because households internalize this benefit. However, changes in reallocation bring general equilibrium effects on factor prices (for a given level of technology utilization) and revenue that might improve welfare.

To account for these GE effects, we need additional notation. Let $\mathcal{U}_{x,\alpha} = \partial\mathcal{U}_x/\partial\alpha_x$ denote the utility gains of changing the reallocation rate for a household in island x. Households' choice of α_x satisfies the first-order condition $\mathcal{U}_{x,\alpha} = \kappa'_x(\alpha_x)$. Let $\mathcal{U}_{x,\alpha,d,t} \cdot dt$ denote the marginal effect of changes in income at time t in island x on $\mathcal{U}_{x,\alpha}$ and $\mathcal{U}_{x,\alpha,n,t} \cdot dt$ denote the marginal effect of changes in income at time t in non-disrupted islands on $\mathcal{U}_{x,\alpha}$. In general, $\mathcal{U}_{x,\alpha,d,t}$ is negative and $\mathcal{U}_{x,\alpha,n,t}$ positive, reflecting the disincentives of policies that tax new technology on reallocation efforts. Define $\varepsilon_{x'',x} \cdot \delta$ as the rate of change in $\alpha_{x''}$ when $\mathcal{U}_{x,\alpha}$ changes by δ. The cross partials $\varepsilon_{x'',x}$ depend on the curvature of the cost function $\kappa_x(\alpha_x)$ and the way in which reallocation away from island x affects factor prices and tax revenue, shaping incentives for reallocating away from island x''. When $\varepsilon_{x'',x} = 0$ for all x'', x we are back in the exogenous reallocation case.

Proposition 4 provides formulas for optimal taxes in terms of the disincentive effects $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$ and the cross partials $\varepsilon_{x,x''}$. The Appendix provides formulas for these objects in terms of primitives.

Proposition 4 *When effort is endogenous, a necessary condition for an optimal tax sequence is that*

$$\tau_{x',t} = \sum_{x\in\mathcal{X}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi^{end}_{x,t}}{\bar{\chi}^{end}_t} - 1\right) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}\right), \tag{4}$$

where the χ^{end}'s are now given by

$$\chi^{end}_{x,t} = \begin{cases} \chi_{x,t} + \dfrac{1}{\ell_{x,t}} \cdot \displaystyle\sum_{x''\in\mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot \mathcal{U}_{x,\alpha,d,t} & \text{if } x \in \mathcal{D} \\ \chi_{x,t} + \dfrac{1}{\ell_t} \cdot \displaystyle\sum_{x,x''\in\mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot \mathcal{U}_{x,\alpha,n,t} & \text{otherwise,} \end{cases}$$

and μ_x is the social value per displaced worker of increasing the reallocation rate α_x:

$$\mu_x = \int_0^\infty (-s \cdot e^{-\alpha_x s}) \cdot \left[\sum_{x''\in\mathcal{X}} \ell_{x'',s} \cdot (\chi_{x'',s} - \bar{\chi}_s) \cdot \frac{\partial w_{x'',s}}{\partial \ell_{x,s}}\right] \cdot ds. \tag{5}$$

The formula for optimal taxes shares the same structure as before. All that is needed is redefining the χ's, so that the social marginal value of increasing future income at different islands accounts for its effect on reallocation rates and the social benefit of reallocation (namely, higher wages at disrupted islands and higher tax revenues). This leads to lower optimal taxes on automation and trade and a more rapid phase out.[13]

In some relevant cases, the disincentive effects of taxing automation and trade justify *subsidizing* these technologies in the medium run. For example, when households are hand to mouth, we have $\mathcal{U}_{x,\alpha} = \int_0^\infty e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot [u(w_t + T_t) - u(w_{x,t} + T_t)] \cdot dt$, $\mathcal{U}_{x,\alpha,d,t} = -(t \cdot P_{x,t}) \cdot \lambda_{x,d,t}$, and $\mathcal{U}_{x,\alpha,n,t} = (t \cdot P_{x,t}) \cdot \lambda_{x,n,t}$. Social marginal values of increasing incomes at disrupted and non-disrupted islands become

$$\chi_{x,t}^{\text{end}} = \begin{cases} \chi_{x,t} - \dfrac{1}{\ell_{x,t}} \cdot \sum\limits_{x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot (t \cdot P_{x,t}) \cdot \lambda_{x,d,t} & \text{if } x \in \mathcal{D} \\ \chi_{x,t} + \dfrac{1}{\ell_t} \cdot \sum\limits_{x,x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot (t \cdot P_{x,t}) \cdot \lambda_{x,n,t} & \text{otherwise,} \end{cases}$$

For small t, $\chi_{x,t}^{\text{end}}$ is higher in disrupted islands, and short-run taxes are optimal. For intermediate t's we could have $\chi_{x,t}^{\text{end}} < 0$ in disrupted islands due to the disincentive effects on reallocation effort and a subsidy to $k_{x,t}$ would be justified.

In sum, with endogenous reallocation effort, short-run taxes protect losers. Medium-run subsidies (or a more rapid phase out) provides back-loaded incentives for reallocation. Zero long-run taxes or subsidies continue to be optimal, since $t \cdot P_{x,t} \to 0$.

2.3 Income Tax Reforms and Assistance Programs

The design of optimal taxes on trade and technology depends on the availability of alternative policy instruments. Propositions 3 and 4 characterize optimal taxes when there are no other tools for assisting disrupted workers. At the other extreme, if *island-specific* transfers or wage subsidies are available, there is no rationale for distorting technology—a consequence of the Second Welfare Theorem and Diamond and Mirrlees (1971). In practice, these alternative (and more desirable) instruments might be limited since identifying and targeting losers from trade and technology is challenging.[14]

This section focuses on a realistic intermediate case where governments can change the progressivity of the tax system or enact temporary assistance programs to ease transitions

[13] Alternatively, policy might subsidize reallocation efforts directly via, e.g., retraining subsidies. In Appendix Proposition 3, we recover the same formula for optimal taxes derived for the case with exogenous effort in when optimal reallocation subsidies are in place. In that case, concerns about a more gradual transition dampening reallocation efforts do not factor in the decision of how to tax new technologies.

[14] For example, Hyman (2018) finds substantial differences in the approval rate of similar Trade Adjustment Assistance applications, highlighting the high difficulty of identifying losers from trade in practice.

for disrupted households. To capture the limitations of these tools, we assume that they can only be conditioned on income, and we allow households to affect their taxable income by choosing their work effort, $n_{x,t}$, so that their income becomes $n_{x,t} \cdot w_{x,t}$. These unobserved actions limit the use of income taxes and assistance programs since they cannot distinguish between workers with a low income because their job was disrupted by technology or workers with a low income due to their lack of work effort.[15]

We focus on a tractable case where the government has access to linear income taxes and assistance programs that replace a constant fraction of income losses relative to a baseline level. With these tools in place, after-tax income becomes

$$y^h_{x,t} = (1 - \mathcal{R}_t) \cdot n_{x,t} \cdot w_{x,t} + T_t,$$

where net transfers are $T_t = \sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}} + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} n_{x,t} \cdot w_{x,t}$. The term $\mathcal{R}_t$ captures the marginal tax rate on income accounting for assistance programs. More broadly, $\mathcal{R}_t$ summarizes the insurance provided by the tax system and the safety net.

Effort is costly, and reduces households flow utility to $u(c_{x,t} - \psi(n_{x,t}))$, where ψ is a convex power function. These quasi-linear preferences imply that effort is not affected by income effects and responds to wages with a constant elasticity $\varepsilon_\ell \geq 0$.

The next proposition characterizes optimal policy when governments optimally choose paths for taxes on new technologies and the generosity of the safety net $\mathcal{R}_t$.

Proposition 5 *When reallocation effort is exogenous, optimal taxes on new technologies $\{\tau_{x,t}\}$ and marginal income tax rates $\{\mathcal{R}_t\}$ satisfy the necessary conditions*

$$\begin{aligned}(6)\qquad \tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot (1 - \mathcal{R}_t) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}\right) \\ - \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}},\end{aligned}$$

$$\begin{aligned}(7)\qquad \frac{\mathcal{R}_t}{1 - \mathcal{R}_t} = \frac{1}{\varepsilon_\ell} \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{\ell,t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot \left[(1 - \mathcal{R}_t) \cdot \frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t} - 1\right] \\ + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot n_{x,t} \cdot w_{x,t}}{m_{\ell,t}} \cdot \frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t},\end{aligned}$$

[15] One can also think of $n_{x,t}$ as costly actions that reduce the probability of loosing your job or experiencing a large income drop. For example, households might reduce their work effort and get fired because of shirking and not because of technological disruptions. Or households may decide to stop investing in their skills, causing their income to drop, not because of technology but due to their choices.

where $m_{\ell,t}$ denotes total labor income in the economy.

The degree of insurance provided by the tax system and the safety net affect optimal taxes on new technologies in three ways. First, a higher $\mathcal{R}_t$ leads to less dispersion in marginal utilities of consumption between disrupted and undisrupted households. Second, with a safety net and income taxes in the back, distorting technology to manipulate wages becomes a less powerful tool. For every dollar of higher wages at disrupted islands, households receive $1-\mathcal{R}_t$ dollars of after-tax income. Third, taxing technology creates a fiscal externality (captured in the last line of the formula), since it affects the level of work effort.

The proposition also provides a formula for the optimal level of insurance provided by the tax system and the safety net in response to technological disruptions. The formula in equation (7) shows that the optimal level of insurance trades off disincentives for work effort (the left hand side) with the direct benefits from redistribution and the pecuniary and fiscal externalities induced by general equilibrium effects.[16]

The proposition shows that, so long as $\varepsilon_\ell > 0$, the optimal marginal income tax $\mathcal{R}_t$ is in $(0,1)$ and it will be optimal to tax automation technologies or trade in the short run. Taxing technology in the short run is desirable because of its tagging value: reducing its use assists workers affected by disruptions and not those who reduced their work effort to exploit the generosity of the tax system and assistance programs.[17] A complementary intuition is that distorting technology is helpful because it directly manipulates the wage distribution in favor of disrupted workers, whereas income taxes or assistance programs are a blunt tool since they can only manipulate after tax income of all households independently of their circumstances.

The formula in equation (7) characterizes the optimal marginal income tax in an economy where the only role of income taxes is to assist disrupted households. In practice, the design of optimal income taxes also depends on the degree of inequality in permanent income. Likewise, the optimal generosity of the safety net also depends on the prevalence of idiosyncratic shocks unrelated to technological change. Both considerations are absent from our model. For this reason, we interpret the formula in equation (7) as characteriz-

[16]Shutting down general equilibrium effects by setting $\frac{\partial \ln w_{x,t}}{\partial \mathcal{R}_t} = 0$ yields the usual formula for an optimal linear income tax $\frac{\mathcal{R}_t}{1-\mathcal{R}_t} = \frac{1}{\varepsilon_\ell} \cdot \sum_{x\in\mathcal{X}} \frac{\ell_{x,t}\cdot n_{x,t}\cdot w_{x,t}}{m_{\ell,t}} \cdot \left(1 - \frac{\chi_{x,t}}{\bar{\chi}_t}\right)$.

[17]This is the same rationale for why it is optimal to distort technology in Naito (1999) and Costinot and Werning (2022). For example, in Costinot and Werning (2022), distorting technology is desirable because it redistributes towards workers whose wages are low because of technological reasons, and not towards those whose income is low due to lack of work effort. Costinot and Werning (2022) also show that this result extends to a case with general non-linear income taxes, so long as they cannot be targeted to specific islands, work effort is unobserved, and $\varepsilon_\ell > 0$.

ing the *additional* provision of insurance via income taxes and assistance programs that is justified in response to a technological disruption.

2.4 Optimal Reforms and Trade Liberalizations

We can apply the formulas in Propositions 3, 4 and 5 to the question of how to conduct economic reforms or trade liberalizations.

Consider a variant of our model where the new technology already exists at time 0 and has constant productivity A_x, but is not in use because of a distortionary tax $\bar{\tau}_x > 0$ in place preventing this, so that $\frac{1+\bar{\tau}_x}{A_{x,0}} \geq \bar{w}$ for some protected islands $x \in \mathcal{D}$. One could think of these as industries shielded from competition via trade tariffs or barriers to entry, or as industries and firms that have been subsidized at the expense of others.

Our formulas characterize the optimal path for a reform that removes these distortions. Depending on parameters, the optimal reform path could involve an instantaneous jump to $\tau_{x,0} \in (0, \bar{\tau}_x)$ and a gradual decline to $\tau_{x,t} = 0$. Or it might involve keeping the tax at $\bar{\tau}_x$ for some time and phasing it out gradually in the future. This second scenario is equivalent to announcing the reform in advance, allowing people to save and reallocate in preparation.

2.5 The Gains from Technological Gradualism

The previous section showed that it is optimal to tax technology in the short run to assist disrupted households. That does not mean that the economy benefits from moving to a counterfactual world where technological progress advances more gradually. This is a different thought experiment. Reducing the use of a new technology via taxes generates a more gradual wage path for disrupted workers *and* raises tax revenue to be distributed among households. Instead, moving to a counterfactual world where technological progress happens more gradually might benefit disrupted workers, but does not bring any additional tax revenue and ultimately reduces the frontier of what an economy can achieve.

To answer this different question, we compute *the gains from technological gradualism.*[18]

[18] An active literature in trade quantifies the gains from trade—how much welfare would decrease if a country remained in autarky instead of engaging in the observed level of trade. See for example Costinot and Rodríguez-Clare (2014) for a review of the literature quantifying the gains from trade, and Antràs et al. (2017) and Galle et al. (2022) for an approach to extend these welfare gains formulas to an environment with inequality. See also Caliendo et al. (2019) for work quantifying the gains from trade accounting for sluggish transitional dynamics. We can think of the gains from trade or technology in our context as the change in welfare resulting from the arrival of the new technology. The gains from gradualism, on the other hand, capture how the gains from trade vary with the graduality of the shock.

Suppose that the observed path for technology is differentiable with respect to time and given by $\{A_{x,t}\}_{x \in \mathcal{D}}$. The gains from technological gradualism capture the welfare gains of facing a counterfactual path of technology given by $\{A_{x,(1-\Gamma)\cdot t}\}_{x \in \mathcal{D}}$ at time t with Γ determining the level of gradualism. In this path, technology progresses more gradually ($\Gamma > 0$) or more rapidly ($\Gamma < 0$) but converges to the same steady state level.

Proposition 6 *Suppose that the government taxes new technologies optimally. The welfare gains from technological gradualism are always negative and more rapid technological change raises welfare. On the other hand, if the government does not tax new technologies nor sets assistance programs, the welfare gains from gradualism around $\Gamma \approx 0$ have an ambiguous sign and are*

$$\mathcal{W}_\Gamma = \frac{\partial W_0}{\partial \Gamma} = \int_0^\infty \left[\sum_{x \in \mathcal{D}} \left(\ell_{x,t} \cdot \chi_{x,t} \cdot w_{x,t} - \ell_t \cdot \chi_t \cdot w_t \cdot \frac{s_{x,t}}{s_t} \right) \cdot \frac{\dot{A}_{x,t}}{A_{x,t}} \cdot t \right] \cdot dt, \tag{8}$$

with $s_{x,t}$ the share of island x and s_t the share of non-disrupted islands in GDP, and χ_t the common value of $\chi_{x,t}$ in non-disrupted islands.

The gains from technological gradualism depend on tax policy. When optimal taxes on new technologies are in place, more rapid technological progress is always welcomed. This holds even if the government has no other tools for redistribution or assistance programs. This result follows from a simple envelop logic: a government can always mimic the less gradual path for technology by taxing its use.

When the government takes no action, the gains from technological gradualism can be positive or negative. More rapid technological progress increases welfare if: i. differences in consumption between disrupted and non-disrupted households are small; ii. workers reallocate rapidly; and iii. technological advances are back-loaded, so that technology increases slowly initially. When these conditions are not met, and governments do not intervene, the gains from technological gradualism are positive and society benefits from slower technological progress in automation technologies and among trade competitors.[19]

[19] The formula for the gains from gradualism in equation (8) holds for endogenous effort. Changes in reallocation effort in response to more gradual technological advances "envelope out." This is why the formulas only depend on marginal social values of consumption, $\chi_{x,t}$ and χ_t, and not on the change in reallocation effort in response to more rapid technological progress in automation or trade.

3 Application I: The Automation of Routine Jobs

3.1 Description, Empirical Evidence, and Calibration

Description: There are 5 islands. Islands 2–5 are in $\mathcal{D}$ and represent segments of routine occupations $o(x)$ (where $o(x)$ denotes the occupation associated with island x) disrupted by technological progress: i. clerical and administrative occupations (10% of employment in 1985); ii. sales occupations (5% of employment); iii. production occupations (18.5% of employment); iv. transportation and material handling occupations (4% of employment). These four occupational groups are identified as routine in Acemoglu and Autor (2011). The first island represents segments of these occupations unaffected by the automation of routine jobs plus non-routine occupations.

Not all jobs that are part of an occupation are replaced by technology. A fraction $s_{o(x)}$ of all jobs in occupation $o(x)$ are disrupted and belong to island x. This implies that island x accounts for a fraction $s_{o(x)} \cdot \Omega_{o(x)}$ of initial employment, where $\Omega_{o(x)}$ is the initial share of employment in occupation $o(x)$.[20] We treat $s_{o(x)} \in [0,1]$ as an unobservable to be calibrated in order to match the scope of the technological disruption.

The evidence in Cortes (2016): This paper estimates trends in occupational wages and shows that workers employed in routine jobs experienced lower wage growth. Cortes uses the Panel Study of Income Dynamics to estimate a variant of the model:

$$\log \text{hourly wage}_{j,o,t} = \delta_t + \text{Routine}_o \cdot \theta_t + X'_{j,t} \cdot \zeta + X'_j \cdot \zeta_t + \gamma_{j,o} + u_{j,o,t}. \tag{9}$$

The model explains hourly wages for person j employed in occupation o at time t as a function of: i. time trends, δ_t; ii. a differential time-path for routine occupations, $\text{Routine}_o \cdot \theta_t$, where Routine_o takes the value of 1 for routine occupations and θ_t captures differential wage trends in these jobs; iii. time varying individual covariates $X'_{j,t} \cdot \zeta$; iv. differential time trends by individual fixed characteristics $X'_j \cdot \zeta_t$; and v. permanent differences in the productivity of individual j in occupation o, $\gamma_{j,o}$. The last term accounts for selection in persistent attributes that make some individuals more productive at some occupations.[21]

[20] One may consider a mass 1 of islands. Each island represents a differentiated job within an occupation, with island x belonging to occupation $o(x)$, and a mass $\Omega_{o(x)}$ of islands in each occupation. The automation of routine jobs corresponds to an improvement in the productivity of specialized equipment and software that substitutes for labor in a share $s_{o(x)}$ of the jobs that are part of occupation $o(x)$.

[21] Cortes (2016) also estimates the price associated with cognitive occupations, but for our purposes, the relevant object is the price of routine occupations relative to all others.

Panels A and B in Figure 1 provide estimates of equation (9). Panel A reports estimates of θ_t from Cortes' data for different specifications: 1. controlling for permanent wage differences by individuals across occupations and demographics; 2. allowing for differential trends by region of residence and whether the person resides in urban or suburban areas; 3. controlling for union membership; and 4. allowing for differential trends over time by education level. These specifications show a permanent decline of 22–30% in relative wages paid in routine jobs since 1985. The more demanding specification that controls for trends in wages by educational levels dates the start of the decline to 1986. Panel B reports separate estimates $\theta_{o(x),t}$ for routine occupational groups using the most demanding specification. All routine occupational groups exhibit a similar pattern of declining wages, though the speed and extent of the decline varies across groups.

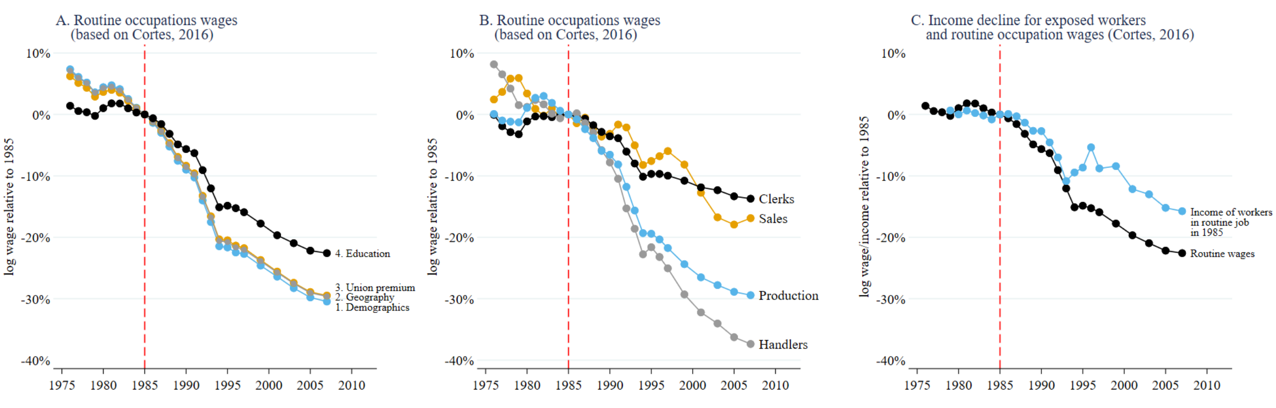

FIGURE 1: ESTIMATES OF WAGE TRENDS FOR ROUTINE OCCUPATIONS. Panel A reports estimates of θ_t in equation (9) using the data and specifications from Cortes (2016). Panel B reports separate estimates of $\theta_{o(x),t}$ for occupational groups. Panel C reports estimates of the incidence of these shocks on workers employed in routine occupations in 1985. All series smoothed using a 3-year moving average.

Cortes (2016) estimates of occupation wages over time are informative of the behavior of $w_{x,t}$ in our model, which in turn provides information on the path for technology $A_{x,t}$. For example, if all jobs within an occupation are disrupted, then $\ln(w_{o(x),t}/w_t) = \theta_{o(x),t}$, which one can use to invert for advances in technology $A_{x,t}$.

Cortes (2016) also provides estimates of future wage growth for workers employed in routine occupations at time t_0. In particular, Cortes estimates the model:

$$(10) \qquad \Delta \log \text{wage income}_{j,t} = \delta_t + \beta \cdot \text{Routine}_{j,t_0} + X'_{j,t_0} \cdot \zeta + u_{j,t}.$$

This model explains wage growth between t_0 and t as a function of individual characteristics at time t_0 and a dummy for whether the individual worked at a routine occupation at time t_0.[22] We use Cortes data an estimate this regression for $t = 1976, \ldots, 2007$, controlling for age, demographics, union membership and education in 1985. Individuals employed in routine jobs by 1985 experienced 16% less wage growth during 1985–2007 than comparable workers, which aligns with the 20-year growth estimates from Table 2 in Cortes (2016).

Panel C in Figure 1 report our preferred estimates for occupational wage changes $\theta_{o(x),t}$ from 1985 to 2007 and the incidence of this shock on workers employed in these occupations in 1985 (from equation 10). There is a large incidence of the shock, with these workers experiencing 70% of the wage decline implied by the full shock.[23] Through the lens of our model, the high incidence estimated by Cortes (2016) points to small reallocation rates α_x.

Calibration: Aggregate output y_t is given by a CES aggregator

$$y_t = \left(\nu^{\frac{1}{\sigma}} \cdot \ell_t^{\frac{\sigma-1}{\sigma}} + \sum_{x \in \mathcal{D}} \nu_x^{\frac{1}{\sigma}} \cdot y_{x,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{11}$$

We take a value of $\sigma = 0.85$ from the literature on occupational polarization (see Goos et al., 2014). We normalize status-quo wages to $\bar{w} = 1$, which implies $\nu + \sum_{x \in \mathcal{D}} \nu_x = 1$.

Technology evolves according to

$$\ln A_{x,t} = \mathcal{S}(t; \pi_x, h_x, \kappa_x) = \pi_x \cdot \frac{1 + h_x \cdot (t_f - t_0)^{-\kappa_x}}{1 + h_x \cdot (t - t_0)^{-\kappa_x}} \text{ for } t \in [t_0, t_f] \tag{12}$$

where $\mathcal{S}$ is a parametric S-curve. $A_{x,t}$ starts at 1 and converges to $\exp(\pi_x)$—the long-run level of cost-saving gains due to the technology—at time t_f. From there on, we set $A_{x,t} = \pi_x$. The parameters h_x and κ_x govern the shape of the S-curve: a higher κ_x implies an S-curve with a steeper inflection; a higher h_x implies a faster adjustment. Building on the findings in Cortes (2016), we assume that the automation of routine jobs starts in $t_0 = 1986$. Acemoglu and Restrepo (2020) estimate cost-saving gains of 30% for the automation of production jobs via industrial robots. We set $\pi_x = 30\%$ and $t_f = 2007$, which assumes similar cost-saving gains of automating other routine occupations by 2007.

[22] We focus on a variant of the specification used in Cortes (2016) that looks at total income accounting for hours worked. This is because reduced work hours might be an important margin of adjustment for workers. This strategy does not account for non-employment, though we did not find evidence of sizable employment effects in the PSID.

[23] These estimates of incidence account for non-employment and changes in hours worked. The 16% income decline is due to both lower wages and a greater likelihood of non-employment. We also estimated incidence separately for workers in each of the four routine occupational groups. The estimates range from 18% for sales, to 71–96% for the remaining groups. We use the average incidence as a target in our calibration because the group-specific estimates are noisy.

We calibrate $s_{o(x)}$ (or equivalently $\nu_x = \Omega_{o(x)} \cdot s_{o(x)}$), $A_{x,t}$, and $\alpha_{0,x} = \alpha_0$ to match the occupational wages $\theta_{o(x),t}$ over time and the incidence of the shock by 2007. These parameters are jointly calibrated, but their values are tightly linked to these moments:

- The choice of $\pi_x = 30\%$ pins the level of A_{x,t_f}.
- The occupation-level wage decline by 2007, $\theta_{o(x),t_f}$, pins $s_{o(x)}$—the exposure of each occupation to technological progress. In particular, $s_{o(x)} \to 1$ implies $\theta_{o(x),t_f} \to \ln(w_{x,t_f}/w_{t_f})$, where $w_{x,t_f}/w_{t_f}$ depends only on π_x, while $s_{o(x)} \to 0$ implies $\theta_{o(x),t_f} \to 0$.
- Occupational wages $\theta_{o(x),t}$ pin the time path for $A_{x,t}$ between $t_0 = 1986$ and $t_f = 2007$.
- The incidence of the shock pins α_0. The higher the incidence, the lower the reallocation rate. We calibrate a common $\alpha_0 = 2.7\%$ per year that matches the average incidence of 70% on exposed workers.

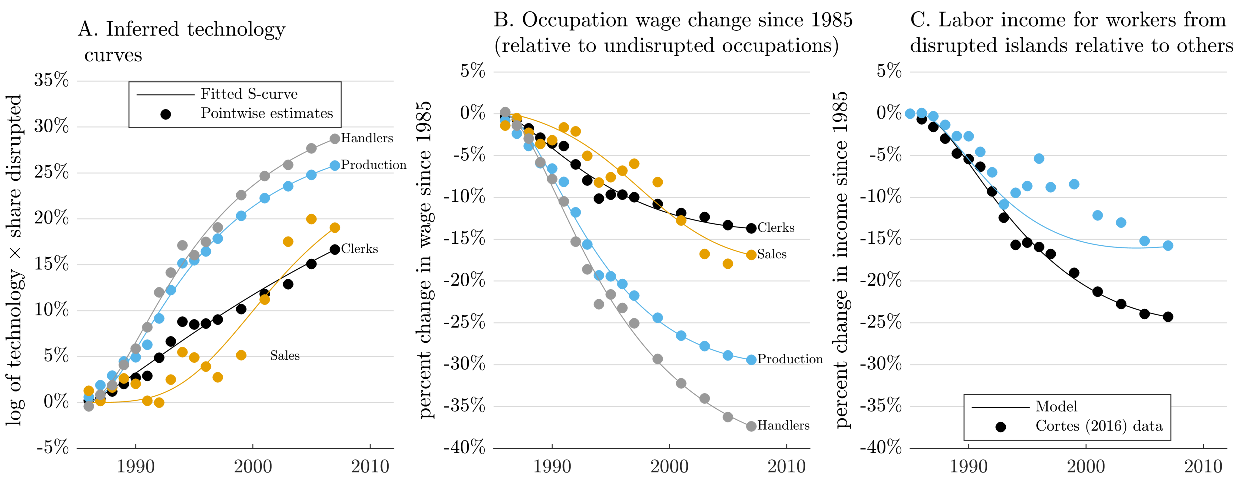

FIGURE 2: CALIBRATED PATHS FOR TECHNOLOGIES REPLACING FOR ROUTINE JOBS. Panel A reports yearly estimates of $A_{x,t}$ and their corresponding S-curve separately for each routine occupation. Panel B reports the model-implied occupational wage decline since 1985 and compares this to the estimates of $\theta_{o(x),t}$ by occupational group. Panel C reports estimates of the implied incidence of these shocks on workers employed in routine occupations by 1985.

This procedure yields yearly estimates for $\hat{A}_{x,t}$ from 1986 to 2007. Using these values, we fit the S-curve in equation (12) via non-linear least squares. Panel A in Figure 2 reports the yearly estimates of $\hat{A}_{x,t}$ and the fitted S−curves for the disrupted segments of routine occupations. To facilitate the interpretation, these estimates are scaled by $s_{o(x)}$, so that the figure is informative of the scope and the timing of the shock across routine occupations. Panels B and C show that our model matches the empirical evidence in Cortes (2016). Panel B reports the model-implied decline in relative wages by occupation since 1986 and compares it to the empirical estimates for $\theta_{o(x),t}$ from Cortes (2016). Panel C reports the model-implied incidence of the shock on workers initially employed in routine occupations. Our calibration generates a relative income decline of 16% for these workers, which matches the 70% incidence of the shock.

Table 1 reports the remaining parameters. We let $u(c) = c^{1-\gamma}/(1-\gamma)$, set the inverse elasticity of intertemporal substitution $\gamma = 2$, and set the discount and interest rate to 5% per year. For the versions of our model in which households are not hand-to-mouth, we assume zero initial assets for disrupted households. This aligns with the evidence in Kaplan et al. (2014) which points to median liquid wealth for US households of $1,714 in 2010.

3.2 Optimal policy in response to the automation of routine jobs

Using the formula in Proposition 3, we compute the optimal path for taxes on automation technologies since 1986. We first focus on the case with exogenous reallocation effort and report optimal taxes for the four scenarios described in section 2: i. hand-to-mouth; ii. no borrowing and no transition risk; iii. borrowing but transition risk; iv. ex-post complete markets. These four scenarios determine the mapping between the observed decline in income documented by Cortes (2016) and matched by our model, and the unobserved marginal utilities of consumption that are relevant for policy. We report optimal taxes obtained for an utilitarian welfare function, which implies $g_x = g = 1$.

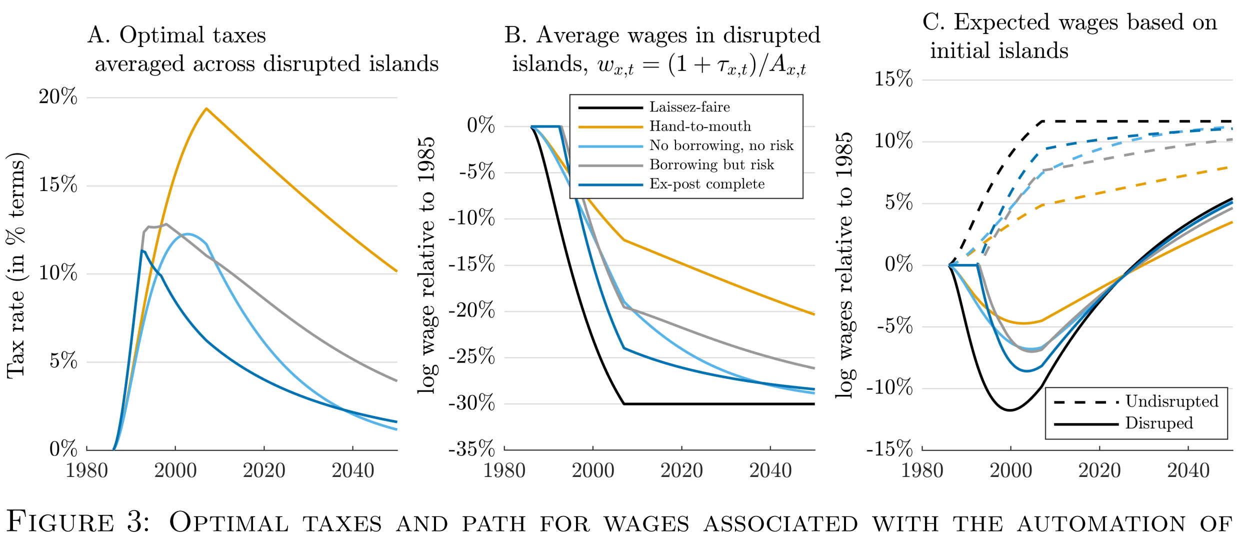

FIGURE 3: OPTIMAL TAXES AND PATH FOR WAGES ASSOCIATED WITH THE AUTOMATION OF ROUTINE JOBS. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports the average wage in disrupted islands relative to its 1985 level. This panel also includes the laissez-faire path where automation goes untaxed for comparison. Panel C reports the expected wage of workers initially employed in disrupted islands (solid lines) and non-disrupted islands (dashed lines) relative to their 1985 levels.

Panel A in Figure 3 plots optimal tax paths for these scenarios. The dark blue line provides the most conservative scenario, obtained when households can borrow and insure against the risk of transitioning late. Optimal policy calls for a short-run increase in taxes on automation technologies of 11%, phased out to a level of 4% by 2020. The comparisons across scenarios show that optimal policy calls for more aggressive and lasting taxes when households are hand-to-mouth than when they can borrow (which aligns with the findings in Beraja and Zorzi (2022)), and when households face the risk of reallocating late. This is because lasting taxes insure workers against this risk.

Panel B plots the average wage among disrupted occupational segments and compares it to a laissez-faire scenario with no taxes. Optimal policy induces a more gradual wage decline in affected islands. By 2020, wages in disrupted islands are 4–15% higher than in laissez-faire. In the two scenarios where workers can save, optimal policy fully shields workers from automation technologies for 6 years to allow them to build their savings and reallocate in advance. This is equivalent to announcing the arrival of the technology and committing to deploy it gradually 6 years from now.

Panel C plots expected wage paths for workers who were initially in disrupted islands. Relative to the previous panel, this one accounts for the role of reallocation. The solid black line shows a large income decline of 12% from 1985 to 2000, which aligns with the high estimated incidence in Cortes (2016). Optimal policy induces a more shallow and less persistent income drop of 7% for households in disrupted islands over 1985–2020. This panel also plots income for households in non-disrupted island. With no taxes on automation technologies, wages for unaffected workers grow gradually by 11%. Optimal policy makes wage growth more gradual for unaffected occupations.

Figure 4 turns to welfare implications. Panel A reports the change in welfare in consumption-equivalent terms for households initially located in unaffected islands. Panel B reports the average change in welfare for households from disrupted islands. With no taxes, the automation of routine jobs leads to a welfare gain of 7.5% for workers who are not adversely affected and a 6–8% welfare loss for workers disrupted by this technological change. In all scenarios, optimal policy leads to a sizable improvement in welfare for disrupted workers of 4–7 pp; while the cost for non-disrupted workers is small (1–2 pp).

We now consider the role of endogenous reallocation effort. We focus on scenarios i, ii, and iv, for which we have tractable formulas. Figure 5 plots optimal taxes for these scenarios. We consider a simple specification of the effort elasticities with $\varepsilon_{x'',x} = 0$ for $x'' \neq x$ and $\varepsilon_{x,x} = \varepsilon > 0$ is calibrated so that moving to the (previous) optimal plan with

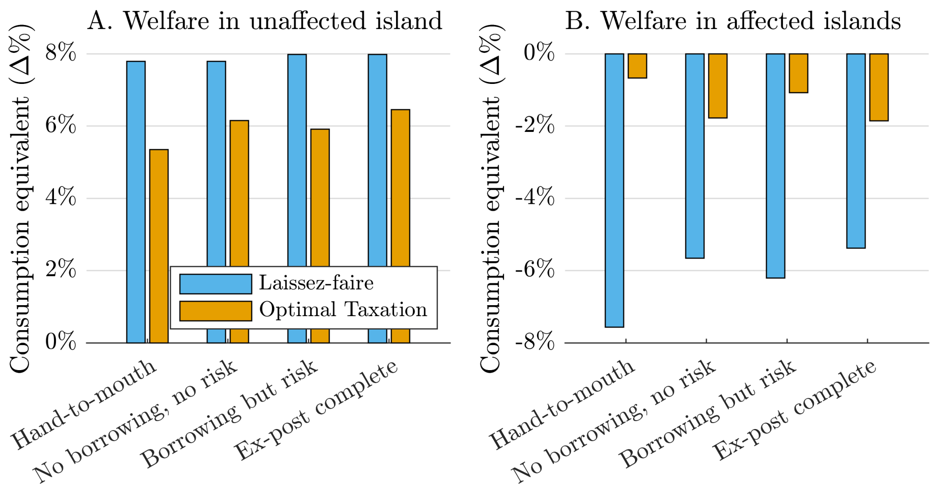

Figure 4: Welfare changes in response to the decline in routine jobs. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted occupations under laissez-faire and under optimal policy. Panel B reports average welfare changes for workers initially employed in disrupted occupations.

exogenous effort results in an offsetting reduction of reallocation effort by 10%, 20%, and 50%. A greater offset implies that moving to the (previous) optimal policy becomes more costly. The figure then provides the optimal tax in each case once we account for the induced reduction in reallocation effort using the formula in Proposition 4. Endogenous effort leads to a more rapid phase out of taxes, and in some cases, justifies a subsidy to the new technology by 2020 to provide backloaded incentives for reallocation.

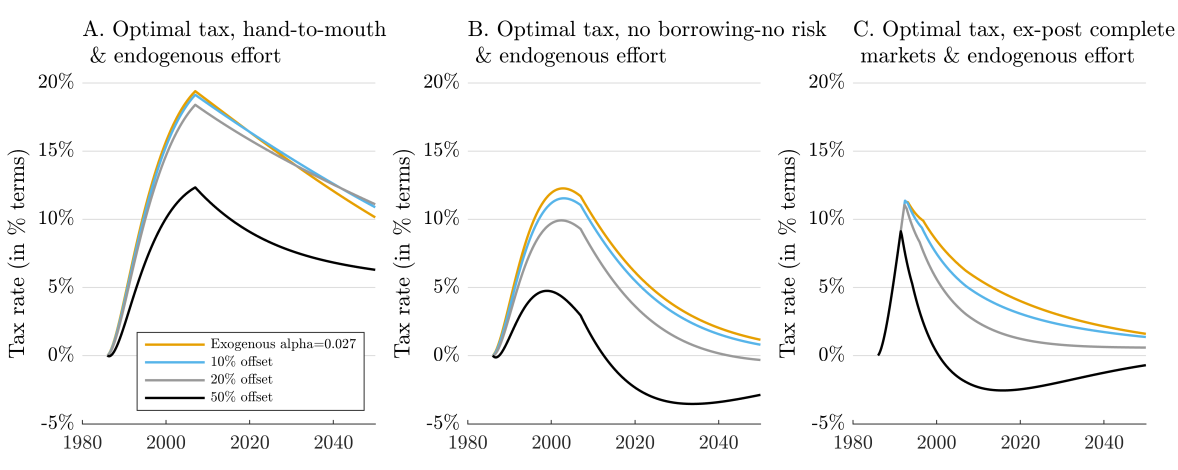

Figure 5: Optimal tax on automation with endogenous reallocation effort. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers ex-post complete markets.

We conclude by exploring the role of insurance provided by income taxes and assistance programs. Figure 6 plots optimal taxes when the government can use these alternative tools. The panels consider the same scenarios for households used above but focus on the case with exogenous reallocation effort. The blue line plots the optimal tax when work effort is endogenous and responds to wages with an elasticity $\varepsilon_\ell = 0.3$, which matches estimates of the Hicksian elasticity of labor supply (in hours) in Chetty et al. (2011), but the government does not set any assistance program or income taxes. This differs slightly from previous estimates because of endogenous changes in hours worked.

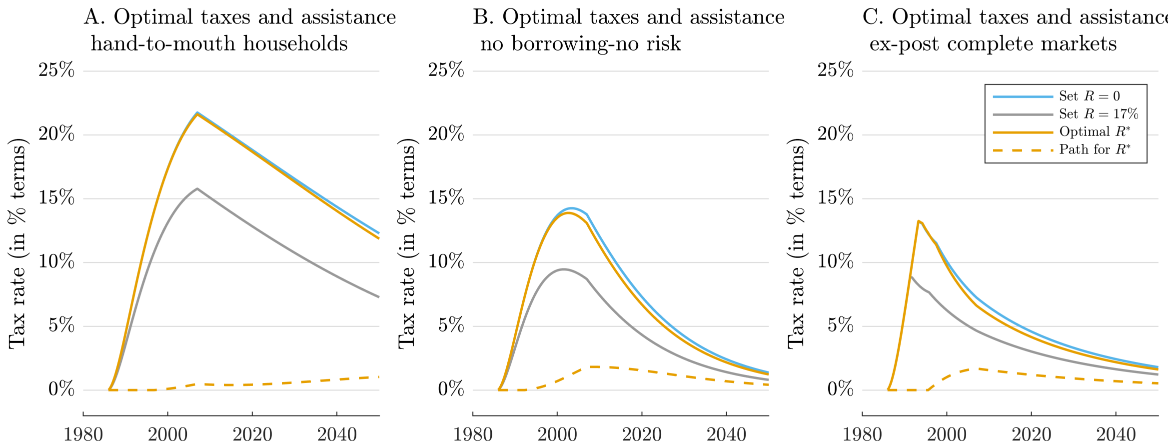

FIGURE 6: OPTIMAL TAX ON AUTOMATION TECHNOLOGIES AND ASSISTANCE PROGRAMS. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers ex-post complete markets.

The solid and dashed orange lines plot optimal paths for taxes on automation and the marginal tax rate on income $\mathcal{R}_t$ from Proposition 5. Optimal policy calls for a small increase in marginal income taxes of 1–2 pp, while optimal taxes on automation are unaffected by this additional policy lever. The reason why assistance programs are not used more intensively as part of the optimal policy bundle is that they are a blunt tool. The automation of routine jobs affects a small group of workers. Because assistance programs cannot be targeted to these households, they will be exploited by the large majority of workers who are not disrupted by technology, generating costly reductions in effort.

As explained in Section 2, our formula for the optimal level of insurance provided by taxes and assistance programs only accounts for the role of these tools in easing the transition for disrupted households. Other factors not modeled here, such as ex-ante inequality between workers or idiosyncratic income shocks might justify a more generous tax system and safety net, which provides support for disrupted workers. To account for the role of existing income taxes and assistance programs, we provide estimates (in gray) for the optimal tax on automation technologies obtained by fixing $\mathcal{R}_t = 17\%$, which matches estimates of the insurance provided by the US safety net and tax system.[24]

[24]Available estimates suggest that marginal income taxes for US workers below the median are of 7%

Our interpretation from the exercises reported in Figure 6 is that the insurance provided by the existing income tax system and safety net play lead to smaller and more short-lived optimal taxes on automation. However, our results also show that short-run taxes on automation are a better way to ease the adjustment for disrupted workers than reforming the existing safety net or raising income taxes during a transition period.

3.3 The gains from technological gradualism

Our formula for the gains from technological gradualism in Proposition 6 implies that these are negative in the case of automation technologies, even in the absence of taxes or assistance programs. A 10% increase in technological gradualism reduces the welfare gains of automation by 0.8%. From an utilitarian point of view, the US would have benefited from more rapid advances in the productivity of automation technologies. More so if these were accompanied by an optimal temporary tax on their use.

4 Application II: The China Shock

4.1 Description, Empirical Evidence, and Calibration

Description: there are 21 islands. Islands 2–21 are in $\mathcal{D}$ and represent segments of 2-digit manufacturing industries $i(x)$ disrupted by import competition from China. As before, we assume that a fraction $s_{i(x)}$ of the products or varieties in industry $i(x)$ are exposed to Chinese competition and calibrate $s_{i(x)}$ to match the scope of the disruption brought by the China Shock in each industry. This implies that the disrupted island x associated with industry $i(x)$ accounts for a fraction $\nu_x = s_{i(x)} \cdot \Omega_{i(x)}$ of initial value added, where $\Omega_{i(x)}$ is the industry share of value added.[25] Table 2 lists all 2-digit manufacturing industries, their

(see Guner et al., 2014). These are the workers that have been more exposed to technological and trade disruptions in recent years. On top of this, assistance programs, such as disability and unemployment insurance, replace an extra 6.5–10% of income losses, depending on the shock being analyzed. For example, the estimates in Tables 8 and 9 of Autor et al. (2013) and Tables A17 and A19 of Acemoglu and Restrepo (2020) show that, for every dollar of labor income loss due to trade disruptions or automation via industrial robots, government transfers increase by 10 cents.

[25] As before, one may consider a mass 1 of islands partitioned into industries. Island x belong to industry $i(x)$, and there is a mass $\Omega_{i(x)}$ of islands in each industry. Each island produces a differentiated variety. We can model the rise of Chinese imports as resulting from improvements in Chinese exporters productivity at a share $s_{i(x)}$ of the islands associated with industry $i(x)$. The remaining varieties are shielded from Chinese competition. For example, Holmes and Stevens (2014) show that import competition affected primarily large firms engaged in the production of standardized goods within exposed industries. This interpretation also aligns with the fact that there are sizable differences in Chinese import penetration across detailed industries and products, even within the 2-digit manufacturing industries used in our analysis (see Autor

SIC codes, and their shares of value added in 1991. The first island represents segments of manufacturing industries that were not exposed to Chinese competition plus all non-manufacturing industries.

The empirical evidence in Autor et al. (2013) and Autor et al. (2014): These papers provide two key moments. Autor et al. (2013) measure Chinese import penetration by industry using data from Comtrade for 1991 to 2007. They document that the increase in Chinese imports within industries is highly correlated across advanced countries, which suggest that the China Shock is driven by improvements in Chinese exporters productivity.

Panel A in Figure 7 summarizes their data and plots the *change in normalized import shares* by manufacturing industries. This is computed as

$$\text{Change in normalized import share}_i = \frac{1}{\Omega_i} \cdot \left(m_{i,t}/y_t - m_{i,t_0}/y_{t_0}\right),$$

where $m_{i,t}$ denotes the value of Chinese imports in industry i. Normalizing by Ω_i makes these estimates comparable across industries. Normalizing imports by GDP at time t ensures that this measure does not capture a mechanical increase in imports driven by US economic growth. While Chinese imports started to increase for some industries since the early 90s, there is a more pronounced and pervasive increase in 2000–2007.[26]

Panel B reports the increase in normalized import shares over 1991–2007 for 2-digit manufacturing industries. On average, Chinese imports rose by 11 pp of manufacturing value added, though there is sizable heterogeneity, with industries such as apparel, leather products, and miscellaneous manufacturing products experiencing 40–110 pp increases in import competition. The increase in normalized import shares is informative of the share of segments disrupted $s_{i(x)}$ and the path of Chinese productivity $A_{x,t}$ in these islands.

Autor et al. (2014) provide evidence that workers initially employed in industries facing more import competition from China experienced lower income growth after 1991. They use data from the Social Security Administration to estimate the model:

$$\text{cumulative earnings}_{j,t} = \beta_{\mathrm{w},t} \cdot \Delta\text{IP 91–07}_j + \theta \cdot \mathrm{X}_j + u_{j,t}, \tag{13}$$

et al., 2013).

[26]Part of the acceleration can be attributed to China's accession to the World Trade Organization (WTO) at the end of 2001, which resulted on the US granting Permanent Normal Trade Relations to China (see Pierce and Schott, 2016). This created an incentive for US firms to open plants abroad, which can be thought of as an increase in $A_{x,t}$ in our model.

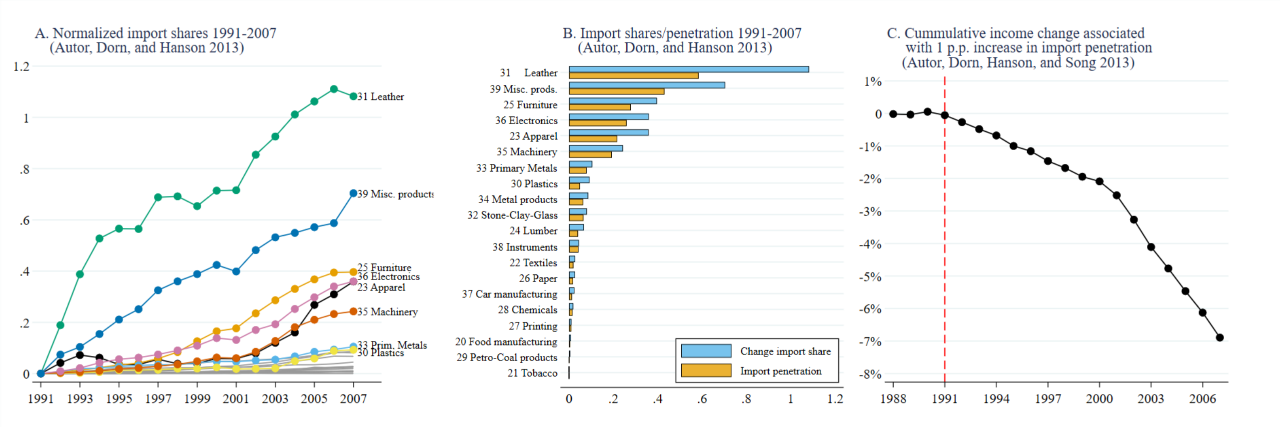

Figure 7: Measures of import competition from China and the effect of the China Shock on wage growth of exposed workers. Panel A reports estimates of normalized import shares over 1991–2007 using the data from Autor et al. (2013). Panel B reports the increase over 1991–2007 in normalized import shares and compares this to Autor et al. measure of import penetration. Panel C reports estimates from Autor et al. (2014) of the effects of a 1 pp increase in import penetration on cumulative future income growth of exposed workers.

which explains cumulative earnings for person j from 1991 to year t as a function of import penetration in their industry of employment in 1991 IP 91–07$_j$, individual characteristics X_j, and an error term.[27] Cumulative earnings are measured relative to a baseline average

$$\text{cumulative earnings}_{j,t} = \frac{\sum_{t'=1992}^{t} \text{labor income}_{j,t'}}{(1/4) \cdot \sum_{t'=1988}^{1991} \text{labor income}_{j,t'}}$$

and account for years of non-employment or zero labor income. Their import penetration measure is similar to the normalized share of imports introduced above, but differs in that it normalizes import growth by total US consumption of industry i output in 1991. For comparison, Panel B plots their import penetration measure. Both measures are highly correlated (correlation of 0.99), but normalized import shares are convenient for our model.

Panel C in Figure 7 plots the estimates of $\beta_{w,t}$ in equation (13), obtained from Figure III in Autor et al. (2014). The estimates show no pre-trends in labor income prior to 1991. Labor income then declines in relative terms, and by 2007, workers who were employed in industries with a 7.5 pp higher import penetration (the average level in manufacturing industries) saw their cumulative income from 1992 to 2007 decline by 50% of their baseline annual income (7.5× 6.8) relative to workers not exposed to Chinese competition. This siz-

[27] Autor et al. (2014) report IV estimates of equation (13). They instrument Chinese import penetration in the US using Chinese import penetration in the same industry but in other high-income countries. This strategy isolates the variation in Chinese imports coming from changes in supply. Their first-stage results suggest that 80% of variation in import penetration can be attributed to supply-side forces such as rising exporters productivity in China.

able effects suggests that the China Shock had considerable incidence on workers employed at disrupted industries, which points to limited opportunities for workers to reallocate.[28]

Calibration: Aggregate output y_t is given by the CES in (11). We set $\sigma = 2$, which matches the median elasticity of substitution across varieties at the level of aggregation in our analysis from Broda and Weinstein (2006).[29] As before, we normalize $\bar{w} = 1$.

Technology evolves according to the S-curve in (12). Building on the findings in Autor et al. (2013) and Autor et al. (2014), we assume that the China Shock starts in $t_0 = 1991$.[30] We also set a common value of $\pi_x = \pi$ across industries representing the cost-saving gains from trading with China by $t_f = 2007$. We calibrate π to match empirical estimates of price declines generated by Chinese import competition in US markets. Bai and Stumpner (2019) document that a 1% decrease in the share of goods produced domestically in an industry is associated with a 0.36–0.5% decline in consumer prices. In our model, the relationship between industry prices and domestic production shares satisfies

$$\Delta \ln P_i \approx \text{constant} + \pi \cdot \Delta \ln \text{share domestic production}_i.$$

Intuitively, substituting a variety produced domestically for a Chinese variety generates a cost-saving gain of π per variety substituted. We set $\pi = 50\%$, to match the upper end of the estimates in Table 1 of Bai and Stumpner (2019).

We calibrate $s_{i(x)}$ (or equivalently $\nu_x = \Omega_{i(x)} \cdot s_{i(x)}$), $A_{x,t}$, and $\alpha_{0,x} = \alpha_0$ to match the increase in the share of Chinese imports over 1991–2007 and the incidence of this shock on workers employed in disrupted industries. These parameters are jointly calibrated, but their values are tightly linked to these moments:

- The choice of $\pi_x = 50\%$ pins the level of A_{x,t_f}.
- Normalized import shares by 2007 are proportional to $s_{i(x)}$—the exposure of each

[28] Autor et al. (2014) also show that within a 2-digit industry, all of the incidence of the China Shock falls on workers that specialized in the detailed industries experiencing the biggest increase in import penetration. This points to limited opportunities for reallocation even within an industry.

[29] Disrupted islands correspond to specific products or varieties within each 2-digit SIC industry. The closest level of aggregation considered in Broda and Weinstein (2006) is the elasticity of substitution among products within a 3-digit SIC level. It is also important to note that $\sigma - 1$ is not equivalent to the *trade elasticity* that features in various trade models. In our model, the equivalent of a trade elasticity for island x is trade elasticity $= \sigma \cdot \frac{y_{x,t}}{k_{x,t}} - 1 > \sigma - 1$. This elasticity exceeds $\sigma - 1$, because our model features an extensive margin of trade (infinite elasticity) and an intensive margin (elasticity $\sigma - 1$). Our calibration produces an *average* trade elasticity for disrupted islands of 4.8.

[30] There was a small level of pre-existing trade with China before 1991. Appendix A.5 explains how we extend our model to capture pre-existing trade and how we deal with it in our calibration.

industry to Chinese import competition.

- The time path for normalized imports pins $A_{x,t}$ for $t = 1991, \dots, 2007$.
- The incidence of the shock pins α_0. The higher the incidence, the lower the reallocation rate. We calibrate a common $\alpha_0 = 1.8\%$ per year that matches the estimates of cumulative wage growth from Autor et al. (2014) by 2007 for affected workers.

This procedure yields estimates for $\hat{A}_{x,t}$ for all years since 1991. Using these values, we fit the S-curve in equation (12) via non-linear least squares. Panel A in Figure 8 reports yearly estimates of $\hat{A}_{x,t}$ and the fitted $S-$curves for the disrupted segments of manufacturing industries. To facilitate the interpretation, these estimates are scaled by $s_{i(x)}$, so that the figure is informative of the scope and the timing of the shock across industries. We report these for the top 8 industries with highest exposure to import competition. Panels B and C show that our model matches the empirical evidence in Autor et al. (2013) and Autor et al. (2014). Panel B reports the model-implied rise in imports since 1991 and compares it to the data. Panel C reports the incidence of the shock on workers associated with a 1 pp increase in Chinese import penetration. Our model matches the cumulative income decline for exposed workers by 2007. Though not targeted, our model matches the time path of income for exposed workers for all years.

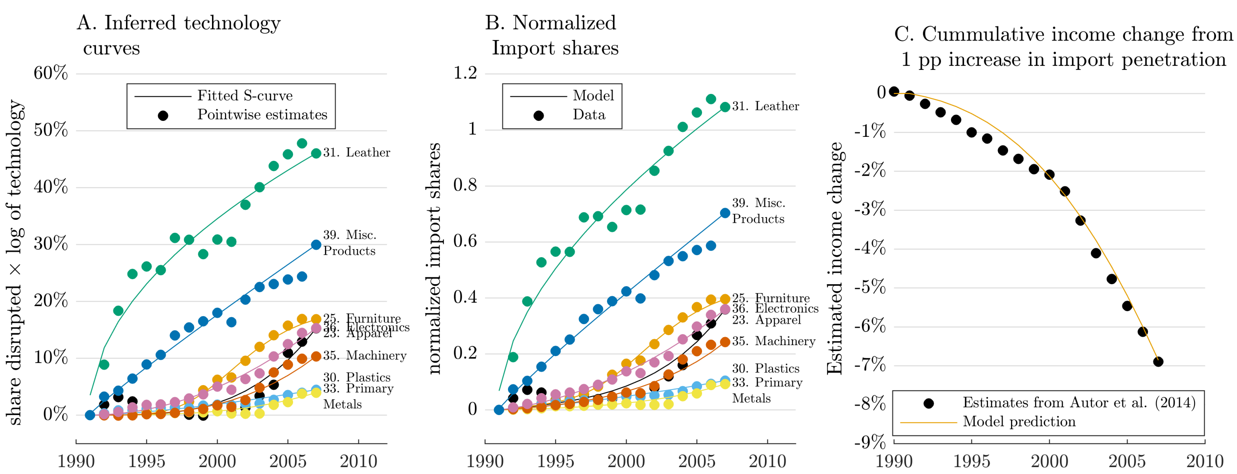

FIGURE 8: CALIBRATED PATHS FOR ADVANCES IN CHINESE EXPORTERS PRODUCTIVITY AND TARGETED MOMENTS. Panel A reports estimates for $A_{x,t}$ and their S-curve for the top 8 industries with the highest exposure to Chinese competition. Panel B reports the model-implied import shares since 1991 and compares this to the estimates in Autor et al. (2013). Panel C reports estimates of the implied incidence of these shocks on workers in industries with a 1 pp higher exposure to import penetration and compares it to the estimates in Autor et al. (2014).

Table 2 reports the remaining parameters used in our calibration, which are the same used in the application of our model to the decline in routine jobs.

The calibration for the decline in routine jobs and the China Shock vary in details but exploit similar information. In both cases, we calibrate the rate of reallocation to match empirical estimates of future wage growth for workers employed at disrupted industries or occupations. The high incidence of these shocks on these workers points to limited opportunities to reallocate. We then show that one can use trends in occupational wages or imports by industry to recover the time path for $A_{x,t}$.

4.2 Optimal policy for the China Shock

Using the formula in Proposition 3, we compute the optimal path for taxes on Chinese imports starting in 1991. We do so for the same four scenarios for households introduced before. These four scenarios determine the mapping between the observed decline in income documented by Autor et al. (2014) and matched by our model, and the unobserved marginal utility of consumption of disrupted households over time. As before, we report results for an utilitarian welfare function.

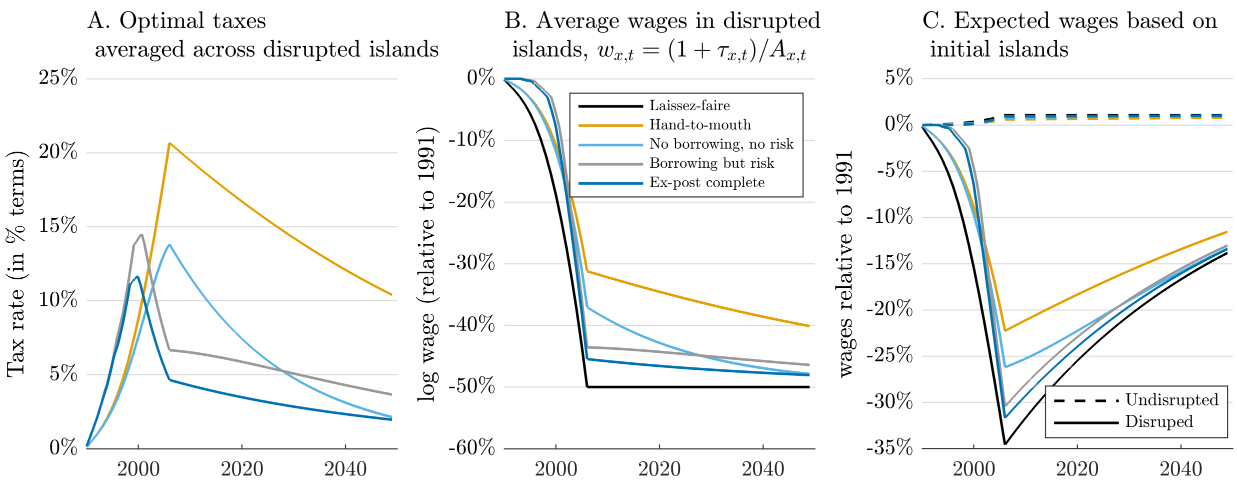

FIGURE 9: OPTIMAL TAXES AND PATH FOR WAGES ASSOCIATED WITH THE CHINA SHOCK. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports the average wage in disrupted islands relative to its 1991 level. This panel also includes the laissez-faire path where Chinese imports go untaxed for comparison. Panel C reports the expected wage of workers initially employed in disrupted islands (solid lines) and non-disrupted islands (dashed lines) relative to their 1991 levels.

Panel A in Figure 3 plots optimal tax paths for these scenarios. The dark blue line provides the most conservative scenario, obtained when households can borrow and insure against the risk of transitioning late. Optimal policy calls for a short-run increase in taxes on Chinese imports of 12%, phased out over time and reaching a level of 4% by 2020.

Panel B plots the resulting wages in disrupted islands and compares it to their level in a laissez-faire world. Optimal policy induces a more gradual reduction in wages at affected islands. By 2020, wages in disrupted islands are 5–15% higher than in a world with no taxes on Chinese imports. When workers can save, optimal policy fully delays the China Shock by 5 years, which allows workers to build their savings and reallocate in response to the impending change.

Panel C plots expected wage paths for workers initially employed in disrupted islands. Relative to the previous figure, this one accounts for the role of reallocation. The solid black line shows a large income decline of 30% from 1985 to 2000, which aligns with the high estimated incidence in Autor et al. (2014). Optimal policy induces a more modest income drop for households in disrupted islands over 1985–2040. This panel also plots income for households in non-disrupted islands over time. With no taxes on Chinese imports, wages for unaffected workers grow gradually by 1% thanks to the gains from trade.

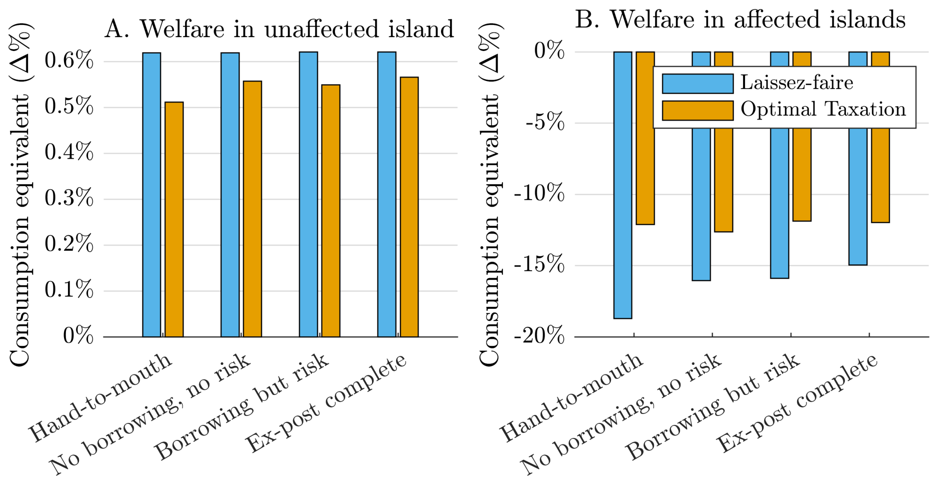

FIGURE 10: WELFARE CHANGES IN RESPONSE TO THE CHINA SHOCK. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted industries under laissez-faire and under optimal policy. Panel B reports average welfare changes for workers initially employed in disrupted industries.

Figure 10 turns to welfare. Panel A reports the change in welfare in consumption-equivalent terms for households initially employed in unaffected islands. Panel B reports the average change in welfare for households from disrupted islands. With no taxes on imports, the China Shock leads to welfare gains of 0.6% for workers who are not exposed to international competition and a 15% welfare drop for those exposed to it (though these workers represent only 1.6% of the US workforce). The small gains from trade align with the trade literature and reflect the (still) low aggregate levels of Chinese import penetration in the US. For example, Galle et al. (2022) estimate gains from trade of 0.3% from trade with China. In all scenarios, optimal policy leads to a sizable improvement in welfare for disrupted workers of 3–6 pp at a small cost for non-disrupted workers (0.05–0.1 pp).

We now consider the role of endogenous reallocation effort. As before, we focus on scenarios i, ii, iv (for which we have tractable formulas) and report estimates for optimal taxes obtained for different assumed levels of offset (the percent reduction in effort implied by moving to the optimal policy that assumes exogenous effort). Figure 11 reports our estimates. Endogenous effort leads to a more rapid phase-out of taxes on trade, and in the last scenario to a small import subsidy by 2040 to provide incentives for reallocation.

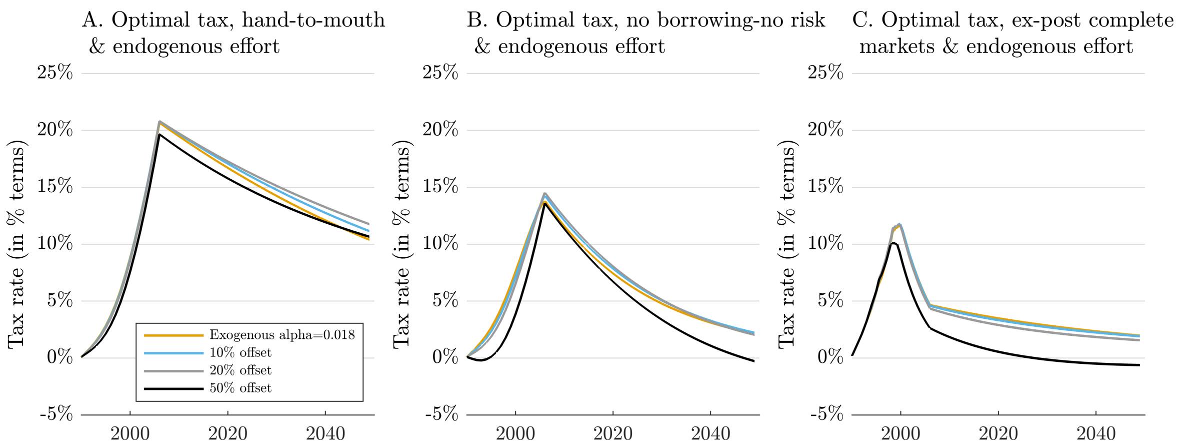

Figure 11: Optimal tax on Chinese imports when reallocation effort is endogenous. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers the case of ex-post complete markets.

Finally, we explore the role of income taxes and assistance programs in Figure 11. The panels consider the same scenarios for households used above but focus on the case with exogenous reallocation effort. The blue line plots optimal trade taxes when work effort is endogenous and responds to wages with an elasticity $\varepsilon_\ell = 0.3$ but we set $\mathcal{R}_t = 0$. The solid and dashed orange lines plot the optimal tax on automation technologies and the optimal marginal tax rate on income from Proposition 5. Optimal policy calls for a small increase in marginal income taxes of 1 pp, while optimal taxes on Chinese imports are unaffected by this additional policy lever. As before, we provide estimates (in gray) for the optimal tax on trade obtained by fixing $\mathcal{R}_t = 17\%$, which captures the existence of a safety net and income taxes that justified by considerations outside of our model. These lead to a quicker phase-out of taxes on Chinese imports.

Our conclusion from the estimates in Figure 12 is that the existing safety net and income tax system call for less pronounced taxes on trade. Still, it is optimal to set short-run taxes of the order of 7–13% on Chinese imports to ease the transition for disrupted households. Import taxes are better than reforming income taxes or expanding assistance programs

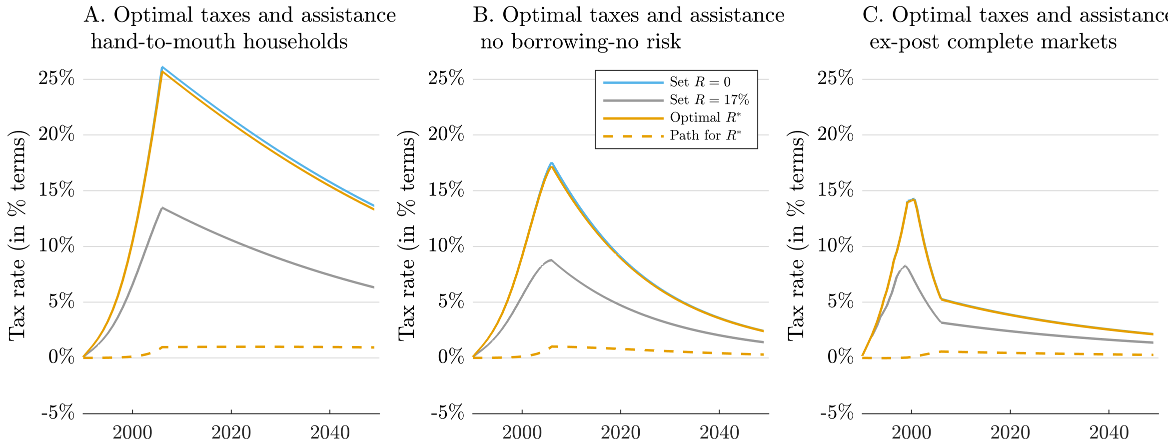

Figure 12: Optimal tax on Chinese imports and assistance programs. Panel A considers hand-to-mouth households. Panel B considers households that share reallocation risk but cannot borrow or save outside their islands. Panel C considers the case of ex-post complete markets.

because these tools cannot be targeted to the small share of workers disrupted by trade.

4.3 The gains from technological gradualism

Our formula for the gains from technological gradualism in Proposition 6 implies that these are also negative for the China Shock, even in the absence of taxes. A 10% reduction in the rate at which Chinese export productivity increased during this period would reduce the welfare gains from trade with China by 2.5%. From an utilitarian point of view, the US would have benefited from more rapid advances in Chinese export productivity. More so if these were accompanied by an optimal temporary tax on trade competition.

5 Application III: Trade Liberalization in Colombia

5.1 Description, Empirical Evidence, and Calibration

Our final application explores Colombia's trade liberalization in 1990. Before the reform, Colombia had arresting levels of trade protection, with average nominal tariffs on manufacturing imports of 40%, and effective tariffs—which account for other barriers and surcharges—reaching levels of 75% (see Goldberg and Pavcnik, 2005; Eslava et al., 2013). Pre-reform levels of trade protection also featured vast dispersion both within and across industries, with apparel and shoes enjoying effective tariffs of close to 120%, and intermediate-goods imports being subsidized or enjoying no protection. With the government of President César Gaviria in 1990, Colombia embarked in an ambitious program of economic reforms that included liberalizing labor markets and opening up to trade.[31] The initial plan was for trade liberalization to be implemented gradually. But concerns about the credibility of the reform process and the potential for the emergence of political roadblocks led to a swift implementation (see Edwards and Steiner, 2008).[32] By 1992, Colombia reduced all nominal tariffs to common international levels of close to 13% and removed almost all additional trade barriers, leading to a new trade structure with uniform effective taxes of 25% across most manufacturing industries, with the exception of imported food products.

Figure 13 depicts the large and rapid decline in effective tariffs starting in 1990 for 3-digit manufacturing industries in Colombia. Average effective tariffs declined by 45 pp from 1990 to 1992. the reform led to an immediate increase in imports shown in the middle panel, with import penetration as a share of GDP rising from 9% in 1989 to 14% in 1993, and settling at 16% by 2005. The right panel summarizes the cumulative decline in effective tariffs over 1989–2002 and the increase in normalized import shares over this period for manufacturing industries.

To map the theory to the data, we must deal with two aspects of Colombia's trade liberalization. First, tariffs were not lowered to zero—the optimal level in our model. The reform lowered tariffs to "internationally acceptable levels" of $\tau_{x,t_f} = 13\%$ for nominal tariffs and $\tau_{x,t_f} = 25\%$ for effective tariffs across most industries and products. Because we do not want to confound the gains from gradualism with issues related to the optimal long-run level of import tariffs, we assume that there is another distortion in the economy that makes the post 1990 level of protection optimal and report series for the net optimal tariff, defined as $1 + \tau^{net}_{x,t} = \frac{1+\tau_{x,t}}{1+\tau_{x,t_f}}$.[33]

Second, there were large trade volumes in some industries before the reform, especially for durable manufacturing and industries producing capital goods, many of which had been liberalized prior to 1989. We account for this feature of Colombia's tariff and trade

[31] These reforms were part of a broader regional movement away from decades of protectionism under the auspices of *import substitution programs*. Chile was at the forefront of the reform movement, and had a gradual liberalization process starting in 1975. Subsequent reformers, such as Argentina, Colombia, Costa Rica, and Nicaragua embraced a more rapid reform process. For example, Nicaragua reduced nominal tariffs from 110% to 12% from 1990 to 1992. See Edwards (1994) for more on the reform movement in Latin America during this period.

[32] The first concern was that a gradual reform would not do enough to convince Colombian firms to upgrade or restructure their operations. The second concern was that a gradual reform could be stopped before achieving the desired level of liberalization if the political climate changed or affected groups managed to organize against the reform.

[33] Formally, we assume that imported goods in island x have an exogenous subsidy of $1+\tau_{x,t_f}$, where τ_{x,t_f} denotes the effective tax rate in 2002. In this case, the optimal tax is $1+\tau_{x,t} = (1+\tau^{net}_{x,t})\cdot(1+\tau_{x,t_f})$, where $\tau^{net}_{x,t}$ is the optimal tax characterized in Proposition 3 after redefining technology to $A^{net}_{x,t} = A_{x,t}/(1+\tau_{x,t_f})$.

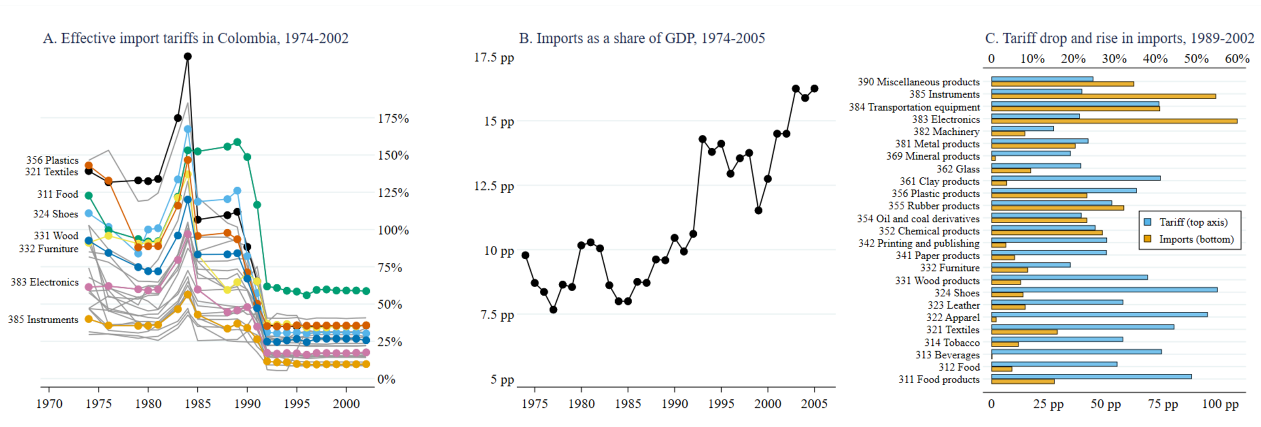

FIGURE 13: TRADE LIBERALIZATION IN COLOMBIA. Panel A reports the time series for effective tariffs for 3-digit manufacturing industries over 1974–2002. Panel B reports imports as a share of GDP. Panel C reports the percent decline in tariffs, defined as the percent change in $1 + \tau_{x,t}$ from 1989 to 2002 and the associated increase in normalized import shares for each industry. Data on tariffs comes from Colombia's *Ministerio del Comercio* and data on imports comes from the *Departamento Nacional de Planeación*, DNP.

structure by assuming that some goods and services were already produced abroad by 1989 and these goods experienced no change in tariffs during Colombia's trade liberalization. Instead, Colombia's trade liberalization worked at the extensive margin: by making it profitable to import a widening range of products that used to be produced domestically.[34]

Calibration: We consider an economy with 26 islands. Islands 2–26 represent segments of manufacturing industry $i(x)$ that survived trade competition because of the protection granted by the high tariffs in 1989, but were out-competed following the trade liberalization.[35] These segments account for a share $s_{i(x)}$ of industry i. We assume that the initial level of protection in industry $i(x)$ is set at the minimum level required to ensure that imports did not disrupt island x. This allow us to recover the pre-tax productivity of imports as $A_x = (1 + \bar{\tau}_{x,t_0}) \cdot \bar{w}$ for all disrupted islands.

As in the China-Shock application, we set $\sigma = 2$ and calibrate the shares $s_{i(x)}$ to match the observed increase in normalized import shares by industry following the trade liberaliza-

[34]A different interpretation is that trade liberalization worked at the intensive margin: by reducing tariffs for products that were already imported and not produced domestically. This alternative interpretation is at odds with the fact that industries with the highest levels of trade by 1989 had low tariffs. Likewise, there is no increase in normalized import shares for industries with the largest share of pre-existing trade, such as industrial chemicals and petroleum products (codes 351, 353) or primary metals (codes 371, 372). In fact, initial import shares explain only 10% of the increase in subsequent import penetration after 1989, which is the opposite of what one would expect if trade liberalization worked at the intensive margin.

[35]We exclude the intermediate-goods industries 351 "industrial chemicals", 353 "oil refined products", 371 "steel and iron" and 372 "primary metals" from our exercise because they were already liberalized prior to 1990 and experienced no increase in import penetration since then.

tion. Conditional on the decline in effective tariffs, industries with a larger share of exposed segments, $s_{i(x)}$, should see a more pronounced increase in normalized import shares.[36]

For this application, we do not have data on the incidence of trade liberalization on workers previously employed in exposed industries. However, Goldberg and Pavcnik (2005) provide a related piece of evidence. Exploiting variation in changes in protection over time across Colombian industries, they show that a 10 percent decrease in tariffs is associated with a decline in industry wage premiums of 1%. Their estimate of the decline in industry wage premia contains information on α. In the limit with $\alpha = \infty$, workers earn the same wage at all industries and trade does not affect industry wage premia. A value of $\alpha = 3\%$ matches Goldberg and Pavcnik (2005) estimates.

We set $t_0 = 1989$—the year before Gaviria's reforms—and feed the observed path for effective tariffs to obtain the path for wages, imports, and aggregates under the reform. Table 3 reports the remaining parameters used in our calibration, which are the same used in the application of our model to the decline in routine jobs and the China Shock.

5.2 Optimal trade liberalization

We use Proposition 3 to compute the optimal reform path for effective tariffs that would have maximized social welfare. We focus on the four scenarios described in Section 1 and adopt an utilitarian welfare function. Figure 14 reports our findings. Panel A depicts the observed path for tariffs following the 1990 trade liberalization and compares it to the optimal path implied by Proposition 3. From a welfare point of view, Colombia's trade liberalization was too rapid. Optimal policy called for an immediate drop in net tariffs to 12–15% and a gradual tariff decline reaching a level of 5–10% by 2010.

Despite the fact that some industries enjoyed more protection, optimal policy calls for a proportional reduction in tariffs across industries, retaining some of the dispersion in tariffs during the transition. This is the opposite of what one would get on pure efficiency grounds, which call for more aggressive dismantling of tariffs in heavily protected sectors (see Mussa, 1984, for a discussion of tariff dispersion in trade reforms).

Panel B shows the observed path for imports and the counterfactual path under the

[36] The assumption behind this procedure is that the observed increase in import penetration in the 90s was entirely due to the large reduction in effective tariffs. This is reasonable, especially when considering the vast drop in tariffs, and taking into account the fact that, as shown in Figure 13, the decline in tariffs was met by an immediate rise in imports. Our procedure matches the rise in imports for all industries from 1989 to 2002, except for 385 (scientific and medical instruments) and 383 (electronics). For these two industries, the restriction that $s_{i(x)} \leq 1$ binds and our model understates the increase in imports.

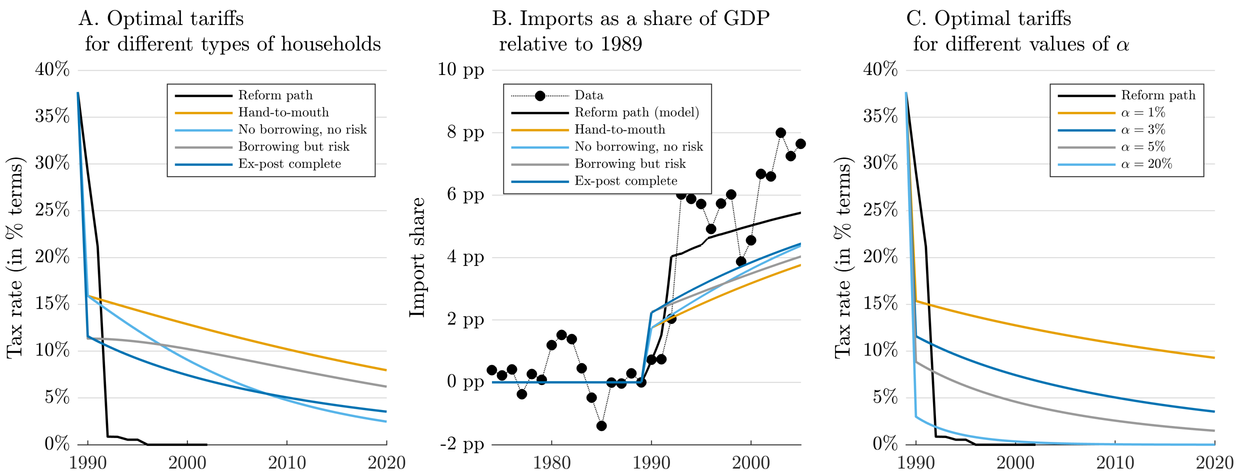

FIGURE 14: OPTIMAL TARIFFS AND OBSERVED TARIFFS FOR COLOMBIA'S TRADE LIBERALIZATION. Panel A reports optimal taxes obtained under the four scenarios introduced in Section 1. Panel B reports imports as a share of GDP relative to 1989 under the observed and the optimal policy. Panel C reports optimal taxes obtained for different values of the reallocation rate α when households face ex-post complete markets.

optimal policy. We see that both in the model and data, imports rose rapidly after the 1990 trade liberalization. Optimal policy induces a more gradual increase in imports.

Panel C reports optimal reform paths for different values of α under the conservative assumption that households can borrow and share transition risks. The swift reform conducted in Colombia (and in much of Latin America during that period) is justified for reallocation rates of 20% per year—an order of magnitude larger than our estimate.

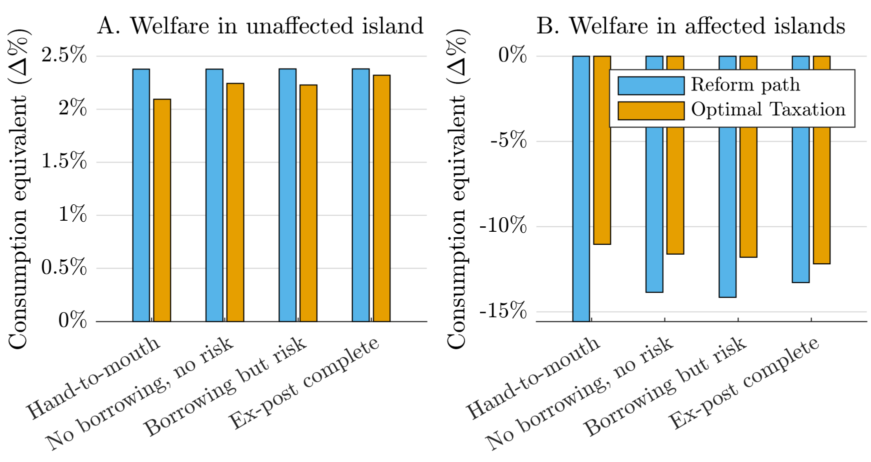

FIGURE 15: WELFARE CHANGES IN CONSUMPTION-EQUIVALENT TERMS, COLOMBIA'S TRADE LIBERALIZATION. Panel A reports consumption-equivalent welfare changes for workers initially employed in non-disrupted industries under the actual reform and under optimal policy. Panel B reports average welfare changes for workers initially employed in protected industries.

Figure 15 reports welfare gains and costs from trade liberalization under different scenarios and paths for tariffs. Colombia's trade liberalization brought welfare gains of 2.2% for unaffected workers and welfare losses of 12–14% for disrupted workers (3.4% of the workforce). A more gradual reform would mitigate losses by 1–4 pp and come at a small welfare cost for unaffected workers of 0.05-0.3 pp.

The conclusion that optimal policy calls for a more gradual trade liberalization holds when we account for endogenous reallocation effort or the availability of income taxes and assistance programs. These scenarios are summarized in Figure 16. Endogenous reallocation effort has a small effect on the optimal policy path. As before, assistance programs are a blunt tool to deal with the adverse distributional effects of rapid reforms. A marginal income tax rate of 17% (as the one induced by the US system) calls for a slightly more rapid reform, but nowhere as rapid as in practice.

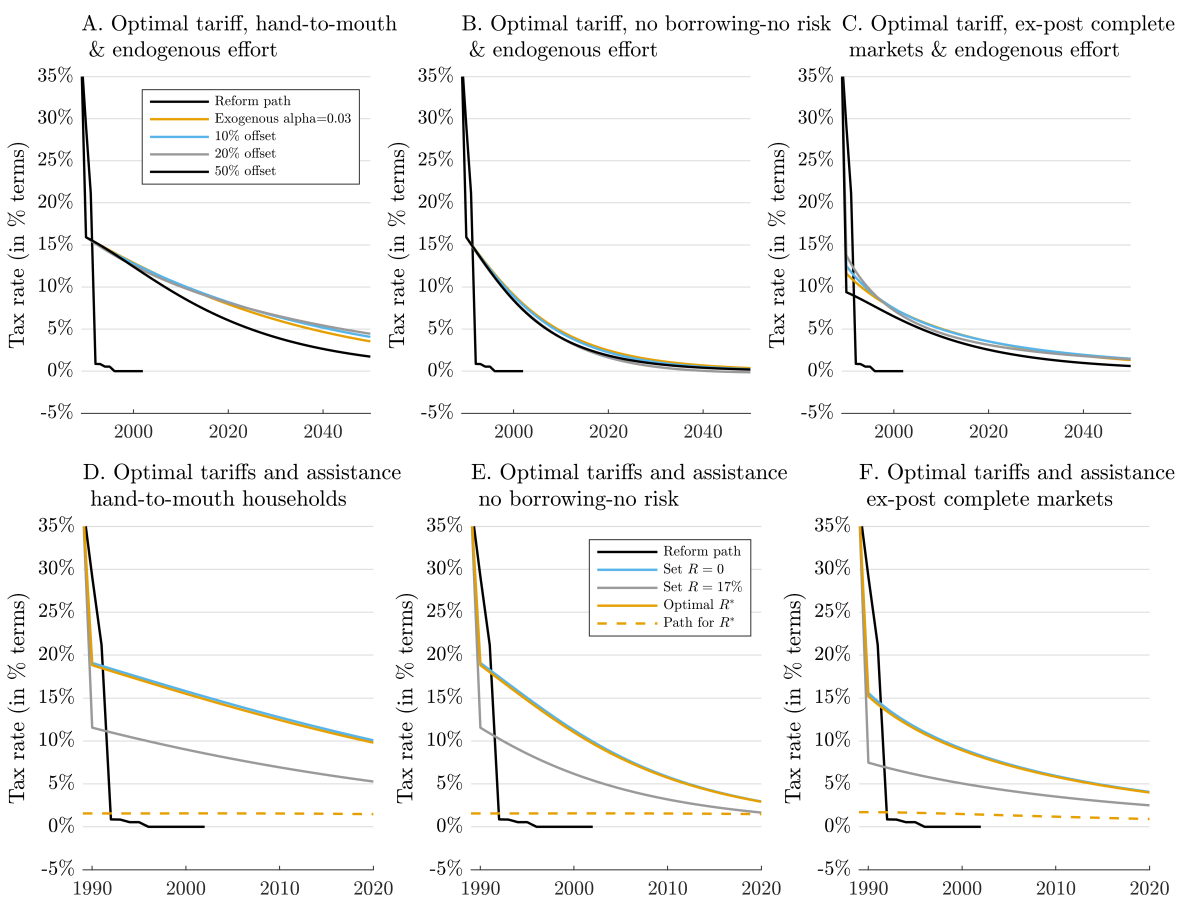

Figure 16: Optimal tariff path for Colombia's trade liberalization when reallocation effort is endogenous (top panels) or it can be complemented by assistance programs. Panels A, D consider hand-to-mouth households. Panels B, E consider households that share reallocation risk but cannot borrow or save outside their islands. Panels C, F considers the case of ex-post complete markets.

6 Concluding remarks

This paper explored how gradualism mediates the gains from trade, technological change, and reforms. We argued that gradual changes have less adverse distributional effects in the short run and justify the use of temporary taxes or gradual reforms. We provided formulas for the optimal path for taxes in response to technological change, trade, or during a reform.

We applied our theory to studying the decline in routine jobs, the rise in Chinese import competition in the US, and Colombia's trade liberalization. A version of our model calibrated to match the short run income declines experienced by some workers as a result of the automation of routine jobs or rising import competition from China suggest that optimal policy calls for temporary taxes of the order of 10%. Our formulas also suggest that the swift trade liberalization of 1990 in Colombia can only be justified in an scenario where workers can reallocate at a rate of 20% per year—an order of magnitude of what we estimate for the US.

These conclusions remained valid when we considered the possibility of dealing with disruptions by reforming the income tax system or increasing the replacement rate of assistance programs. We showed that these programs are too blunt to deal with technological disruptions that only affect some segments of the workforce, and these disruptions do not justify reforming the existing tax system or safety net. Instead, taxing new technologies or trade in the short run offers a more direct way of easing the transition for disrupted workers.

The fact that short run taxes on automation and trade are desirable does not mean that the US economy did not benefit from rapid advances in Chinese exporting productivity or the development of automation technologies. In both scenarios, we show that the gains from technological gradualism are negative, even in the absence of government policy. From a welfare point of view, rapid advances in China and in the development of automation technologies were a welcomed force. It is just that we could have made things better by taxing these technologies in the short run, easing the transition for displaced workers.

Our formulas show that the desirability of taxes and the gains from technological gradualism depend on the extent to which disrupted households cut their consumption during a period of adjustment. Most of the existing literature focuses on estimating the impact of trade and technological disruptions on income. From a policy perspective, understanding how these disruptions affect consumption seems even more important, and a natural question for future research.

One interesting extension of our theory involves a case with congestion in reallocation; for example, because retraining a vast number of people in a single period might be subject to aggregate diminishing returns. Though we believe this offers an important rationale for gradualism, we did not explore the implications of congestion in our empirical applications.

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Table 1: Calibration for the decline in routine jobs over 1985–2007.

Panel I. Islands and technology

Occupation	Employment share 1985	Wage decline 85–07	Incidence	Size disrupted islands, $\nu_{o(x)}$	Share disrupted, $s_{o(x)}$	S−curve parameters $\{\pi_x, \kappa_x\}$
	Data & moments, Cortes (2016)			*Estimated objects*		
Clerical jobs	10%	-14.4%	86.1%	5.6%	56.3%	$\{0.3, 1.0615\}$
Production jobs	18.5%	-30.1%	71.1%	16.0%	86.6%	$\{0.3, 1.7104\}$
Sales jobs	5%	-11.9%	18%	2.46%	49.1%	$\{0.3, 11.325\}$
Handling jobs	4%	-40.8%	96.4%	3.98%	99.4%	$\{0.3, 1.3335\}$

Panel II. Elasticities, reallocation rate, and households

Elasticity of substitution	$\sigma = 0.85$	From literature on polarization (see Goos et al., 2014)
Reallocation rate per year	$\alpha_0 = 2.7\%$	Calibrated to match average incidence of 71%
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Low median liquid assets in US Survey of Consumer Finances

Notes: The table summarizes the data used to calibrate the model to match the wage decline in routine jobs and the resulting parameters. The employment shares of routine occupations come from Acemoglu and Autor (2011); their wage decline from 1985–2007 from Cortes (2016); and the incidence of the wage decline also from Cortes (2016). The scale parameter of the S-curve h_x in equation (12) is not reported because it has no clear interpretation. Section 3 describes the calibration approach and data in detail.

Table 2: Calibration for the China Shock 1991–2007.

Panel I. Islands and technology

SIC code and industry	Data & moments, Autor et al. (2013), Autor et al. (2014)			Estimated objects		
	Value-added share 1991	Normalized import share 91–07 (pp)	Import Penetration 91–07 (pp)	Size disrupted islands, $\nu_{i(x)}$	Share disrupted, $s_{i(x)}$	S−curve parameters $\{\pi_x, \kappa_x\}$
20 Food & Kindred Products	1.77%	0.87	0.48	0.01%	0.74%	{0.5,3.9768}
21 Tobacco Products	0.29%	0.02	0.02	0.00%	0.01%	{0.5,14.217}
22 Textile Mill Products	0.32%	2.8	1.99	0.01%	2.38%	{0.5,4.3931}
23 Apparel	0.43%	35.97	21.76	0.13%	30.57%	{0.5,3.4398}
24 Lumber & Wood Products	0.36%	6.74	4.05	0.02%	5.72%	{0.5,2.3346}
25 Furniture & Fixtures	0.28%	39.69	27.88	0.09%	33.73%	{0.5,4.5851}
26 Paper & Allied Products	0.76%	2.75	1.83	0.02%	2.34%	{0.5,2.1375}
27 Printing & Publishing	1.31%	1.07	1.03	0.01%	0.91%	{0.5,1.6949}
28 Chemical & Allied Products	1.95%	1.94	1.58	0.03%	1.65%	{0.5,2.8831}
29 Petroleum & Coal Products	0.35%	0.54	0.13	0.00%	0.46%	{0.5,3.4844}
30 Rubber & Plastics Products	0.63%	10.53	7.95	0.06%	8.95%	{0.5,1.4856}
31 Leather & Leather Products	0.06%	108.38	58.44	0.05%	92.11%	{0.5,0.5088}
32 Stone, Clay, & Glass Products	0.43%	8.06	6.53	0.03%	6.85%	{0.5,1.0715}
33 Primary Metal Industries	0.66%	9.29	4.95	0.05%	7.90%	{0.5,3.0595}
34 Fabricated Metal Products	1.03%	8.69	6.37	0.07%	7.38%	{0.5,2.5377}
35 Industrial Machinery	1.67%	24.32	19.33	0.34%	20.67%	{0.5,2.9514}
36 Electronic Equipment	1.36%	36.04	25.96	0.41%	30.63%	{0.5,1.9978}
37 Transportation Equipment	1.88%	2.44	1.32	0.04%	2.07%	{0.5,2.5975}
38 Instruments & Related	1.04%	4.46	4.26	0.04%	3.79%	{0.5,0.9968}
39 Miscellaneous Manufacturing	0.26%	70.49	43.05	0.15%	59.91%	{0.5,0.9271}

Panel II. Elasticities, reallocation rate, and households

Elasticity of substitution	$\sigma = 2$	From Broda and Weinstein (2006)
Reallocation rate per year	$\alpha_0 = 1.8\%$	Calibrated to match incidence regressions in Autor et al. (2014)
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Low median liquid assets in US Survey of Consumer Finances

Notes: The table summarizes the data used to calibrate the model to match the China Shock and the resulting parameters. Industry value added shares come from the NBER-CES, and are adjusted using aggregate data from the BEA-BLS integrated industry accounts to recognize the fact that the NBER-CES does not remove intermediate services from value added. Normalized import shares and import penetration measures come from Autor et al. (2013) and Autor et al. (2014). The scale parameter of the S-curve h_x in equation (12) is not reported because it has no clear interpretation. Section 4 describes the calibration approach and data in detail.

Table 3: Calibration for Colombia's trade liberalization.

Panel I. Islands and technology

SIC code and industry	Data & moments, Eslava et al. (2013), DNP				Estimated objects	
	Value-added share 1989	Effective tariff 1989	Percent decline in effective tariff	Change normalized import shares 89–02	Size disrupted islands, $\nu_{i(x)}$	Share disrupted, $s_{i(x)}$
311 Food products	1.59%	48.86%	158.71%	27.6 pp	0.40%	22.19%
312 Food	1.59%	30.72%	91.34%	9.1 pp	0.18%	10.07%
313 Beverages	2.42%	41.53%	95.03%	0.4 pp	0.01%	0.37%
314 Tobacco	0.43%	32.10%	80.42%	11.9 pp	0.06%	12.92%
321 Textiles	2.01%	44.61%	111.89%	28.9 pp	0.58%	24.92%
322 Apparel	0.58%	52.73%	116.54%	2.2 pp	0.01%	1.66%
323 Leather products	0.14%	32.13%	70.53%	14.9 pp	0.03%	16.11%
324 Shoes	0.23%	55.14%	126.10%	13.9 pp	0.03%	10.15%
331 Wood products	0.15%	38.15%	84.11%	12.9 pp	0.02%	12.44%
332 Furniture	0.10%	19.30%	64.67%	16.0 pp	0.03%	22.93%
341 Paper products	0.72%	28.10%	61.72%	10.2 pp	0.10%	12.03%
342 Printing and publishing	0.59%	28.19%	73.49%	6.4 pp	0.05%	7.53%
352 Chemical products	1.40%	25.37%	48.22%	48.6 pp	0.98%	60.58%
354 Oil and coal derivatives	0.11%	21.97%	43.35%	41.8 pp	0.07%	56.32%
355 Rubber	0.31%	29.41%	65.44%	58.0 pp	0.24%	66.29%
356 Plastic products	0.55%	35.43%	93.49%	41.9 pp	0.27%	42.49%
361 Clay products	0.15%	41.27%	93.38%	6.8 pp	0.01%	6.16%
362 Glass	0.25%	21.86%	45.24%	17.3 pp	0.07%	23.34%
369 Mineral products	0.89%	19.32%	48.11%	1.8 pp	0.03%	2.63%
381 Metal products	0.65%	23.63%	59.76%	36.7 pp	0.35%	47.61%
382 Machinery (exc. electric)	0.34%	15.25%	32.95%	14.7 pp	0.09%	23.48%
383 Electronics	0.73%	21.56%	45.87%	107.6 pp	0.84%	100.00%
384 Transportation equipment	1.09%	40.92%	101.65%	73.7 pp	0.84%	67.67%
385 Instruments	0.17%	22.11%	36.90%	98.2 pp	0.20%	100.00%
390 Miscellaneous products	0.22%	24.84%	63.68%	62.4 pp	0.20%	78.62%

Panel II. Elasticities, reallocation rate, and households

Elasticity of substitution	$\sigma = 2$	Imputed from China-Shock application and Broda and Weinstein (2006)
Reallocation rate per year	$\alpha_0 = 3\%$	Matches decline in industry premium in Goldberg and Pavcnik (2005)
Inverse elasticity of intertemporal substitution	$\gamma = 2$	Standard macro calibration.
Discount rate and interest rate	$r = \rho = 5\%$	Standard macro calibration.
Initial assets	0	Imputed from China-Shock application

Notes: The table summarizes the data used to calibrate the model to match Colombia's trade liberalization and the resulting parameters. Industry value added shares come from the *Departamento Nacional De Planeacion*, DNP. Effective tariffs come from the *Ministerio de Comercio*, and are described in detail in Eslava et al. (2013). The change in import shares before and after the reform come from the *Departamento Nacional De Planeacion*, DNP. We compute the 1989 level of imports as an average over 1985–1989 and the post reform level as an average over 1998–2002. We exclude industries 351 "industrial chemicals", 353 "oil refined products", 371 "steel and iron" and 372 "primary metals" from the analysis because they was already liberalized prior to 1990 and experienced no increase in import penetration since then. Section 5 describes the calibration approach and data in detail.

Online Appendix to "Optimal Gradualism"

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A.1 Proofs for Section 1

Proof of Proposition 1:. Suppose that $k_{x,t} > 0$. We verify this is the case at the end of the proof. Firms in island x must be indifferent between producing with workers or producing using the new technology. This implies $w_{x,t} = (1 + \tau_{x,t})/A_{x,t}$.

We now show that wages across non-disrupted islands are equalized. Assumption 1 ensures that this holds at time 0. Each period, a flow $\alpha \cdot (1 - \ell_t)$ of workers joins these islands. Directed search implies that this flow of workers is allocated in a way that preserves the equality of wages. In particular, this is the case so long as new workers are allocated in proportion to island populations at the time of their arrival. This shows that, under Assumption 1, directed search is not strictly necessary. Random search as in Alvarez and Shimer (2011) suffices.

The expression pinning down the common wage w_t comes from the fact that we have normalized the price of the final good to 1, which implies that island wages lie along the iso-cost curve $1 = c^f\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)$. Note that this equation implies that w_t is implicitly a function of the vector of after tax productivities $\{(1 + \tau_{x,t})/A_{x,t}\}$.

To derive the expression for output and new technology utilization we use Shepard's lemma, which implies

$$y_{x,t} = y_t \cdot c^f_x.$$

Adding across non-disrupted islands, yields

$$\ell_t = y_t \cdot \sum_{x\notin\mathcal{D}} c^f_x = y_t \cdot c^f_w\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right),$$

which after rearrangement yields the expression for output. On the other hand, for disrupted islands we have

$$\ell_{x,t} + k_{x,t} = y_t \cdot c^f_x\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right).$$

Substituting the expression for output and rearranging yields the expression for $k_{x,t}$ in the proposition.

To conclude, we show that $k_{x,t} > 0$ as claimed in the proposition (and initially in the proof). Note that this is needed to ensure that capital is actually used and that firms are therefore indifferent between hiring workers or capital. Assumption 2 implies

$$\frac{c_x^f\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)}{c_w^f\left(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t\right)} > \frac{c_x^f\left(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w}\right)}{c_w^f\left(\{\bar{w}\}_{x\in\mathcal{D}}, \bar{w}\right)} = \frac{\ell_{x,0}}{\ell_0} \geq \frac{\ell_{x,t}}{\ell_t}.$$

Rearranging this inequality yields $k_{x,t} > 0$. ■

Proof of Proposition 2:. The first part follows from the definition of a complete market. The second and third parts follow from an envelope logic. Workers in non-disrupted island can set $\alpha_x = 0$ and achieve the exact consumption path of workers in disrupted islands, since their wages are higher (or equal) at all points in time. They can then consume the excess savings generated with this policy, which implies that they can always achieve a strictly higher utility than disrupted households. Because they are making optimal decisions, we must have $U_0 > U_{x,0}$, as wanted. ■

A.2 Proofs for Section 2

This section provides proofs for Lemma 1 and Propositions 3, 4, and A2.

Proof of Lemma 1. We first derive the expression for the change in tax revenue in (1). We prove this in a slightly more general case. For simplicity, we ignore time subscripts. Suppose there are different types of labor indexed by ℓ_j, different (untaxed) intermediate goods y_i, and different types of (taxed) capital k_m. Tax revenue is given by

$$T = \sum_m \tau_m \cdot \frac{k_m}{A_m},$$

which implies

$$dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} + \sum_m \frac{k_m}{A_m} \cdot d\tau_m.$$

Each producer f operates some CRS production function. Thus, we have

$$0 = \max_{y^f, y_i^f, k_m^f, \ell_j^f} y^f - \sum_i y_i^f \cdot p_i - \sum_m \frac{k_m^f}{A_m} \cdot (1 + \tau_m) - \sum_j \ell_j^f \cdot w_j.$$

The envelope theorem then implies

$$0 = -\sum_i y_i^f \cdot dp_i - \sum_m \frac{k_m^f}{A_m} \cdot d\tau_m - \sum_j \ell_j^f \cdot dw_j.$$

Adding this across all producers, we obtain

$$\sum_m \frac{k_m}{A_m} \cdot d\tau_m = -\sum_j \ell_j \cdot dw_j.$$

Plugging into the expression for dT yields a general version of the formula in the lemma:

$$dT = \sum_m \tau_m \cdot \frac{dk_m}{A_m} - \sum_j \ell_j \cdot dw_j.$$

We now turn to the change in welfare. The welfare function is given by

$$W_0 = \sum_{x \in \mathcal{D}} \ell_{x,0} \cdot \mathcal{W}\left(\max_{\alpha} \mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha)\right) + \ell_0 \cdot \mathcal{W}\left(\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty})\right).$$

Recall that $\mathcal{U}_x(\{w_{x,t}+T_t, w_t+T_t\}_{t=0}^{\infty}; \alpha)$ gives the maximum utility that a displaced household can achieve. The envelope theorem implies that, so long as $c_{x,t} > 0$, an infinitesimal change in $w_{x,t} + T_t$ changes $\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha)$ by

$$d\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha) = P_{x,t} \cdot e^{-\rho t} \cdot u'(c_{x,t}) \cdot (dw_{x,t} + dT_t).$$

This is because optimizing households weakly prefer consuming the additional income to any other use. After all, households could always reduce their consumption but they choose not to do that. The right hand side gives the expected marginal utility of consuming this extra income.

Likewise, so long as $c_{x,t_n,t} > 0$, an infinitesimal change in $w_t + T_t$ changes $\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha)$ by

$$\begin{aligned} d\mathcal{U}_x(\{w_{x,t} + T_t, w_t + T_t\}_{t=0}^{\infty}; \alpha) = & \int_0^t e^{-\rho t} \cdot u'(c_{x,t_n,t}) \cdot (dw_t + dT_t) \cdot \cdot \alpha_x \cdot e^{-\alpha_x t_n} \cdot dt_n \\ = & (1 - P_{x,t}) \cdot e^{-\rho t} \cdot \mathbb{E}[u'(c_{x,t_n,t}) | t_n \leq t] \cdot (dw_t + dT_t), \end{aligned}$$

where this expression integrates over all potential histories in which the household is at the non-disrupted island at time t and can consume this extra income.

Finally, so long as $c_t > 0$, an infinitesimal change in $w_t + T_t$ changes $\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty})$ by

$$d\mathcal{U}(\{w_t + T_t\}_{t=0}^{\infty}) = e^{-\rho t} \cdot u'(c_t) \cdot (dw_t + dT_t).$$

Combining these results yields the formula in equation (2).

Note that the lemma also applies when there is endogenous reallocation effort. This is because changes in reallocation effort have a second order effect on $U_{x,0}$ (households are optimizing with respect to α_x). ■

Proof of Proposition 3. The proof follows Costinot and Werning (2022). Using the definition of the χ's, we can write the change in welfare following a variation as

$$dW_0 = \int_0^{\infty} \left[\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot dw_{x,t} + \bar{\chi}_t \cdot dT_t \right] dt. \tag{A1}$$

Consider a change in taxes that changes $k_{x',t}$ by $dk_{x',t}$ but keeps the utilization of all other types of capital unchanged. At a social optimum, this variation cannot affect welfare. Thus:

$$\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot dw_{x,t} + \bar{\chi}_t \cdot dT_t = 0.$$

Using the fact that $dT_t = -\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot dw_{x,t} + \tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}}$, we obtain

$$\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot (\chi_{x,t} - \bar{\chi}_t) \cdot dw_{x,t} + \bar{\chi}_t \cdot \tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} = 0,$$

which can be re-written as

$$\tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} = \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot (\frac{\chi_{x,t}}{\bar{\chi}_t} - 1) \cdot (-dw_{x,t}).$$

Dividing by $\frac{dk_{x',t}}{A_{x',t}}$ and rearranging terms yields the formula in (3). The derivation clarifies that $\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}$ refers to a partial derivative (i.e., the percent change in wages across islands resulting from a change in $k_{x',t}$ holding all other $k_{x,t}$ as well as the allocation of workers across islands constant). ■

Proof of Proposition 4. We use Lemma 1, which continues to be valid when reallocation effort is endogenous. Suppose we are at an optimum. Consider a reform that changes $k_{x',t}$ by $dk_{x',t}$ but leaves all other $k_{x,s}$ unchanged. This reform changes α_x by $d\alpha_x$ and, because the reform kept $k_{x,s}$ fixed for all other x, s, it also changes wages and tax revenue at all points in time and islands.

Define the direct effect of the reform as the effect on welfare through wages and tax revenue holding all α_x constant. This then triggers an indirect effect via changes in α_x which affect wages and revenue at all other time periods.

For an outcome $a_{x,s}$, denote by $d_k a_{x,s}$ the direct effect of the reform—i.e., the change induced by $k_{x',t}$—and by $d_\alpha a_{x,s}$ the indirect effect—i.e., the change induced by changes in α_x.

Let's first consider the indirect effects and the determination of $d\alpha_x$. The first-order condition for α_x is $\kappa_x'(\alpha_x) = \mathcal{U}_{x,\alpha}$. Totally differentiating this equation we get

(A2)

$$\kappa_x''(\alpha_x) \cdot d\alpha_x = \sum_{x'' \in \mathcal{D}} \frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_{x''}} \cdot d\alpha_{x''} + \mathcal{U}_{x,\alpha,d,t} \cdot (d_k w_{x,t} + d_k T_t) \cdot dt + \mathcal{U}_{x,\alpha,n,t} \cdot (d_k w_t + d_k T_t) \cdot dt.$$

In this equation, $\frac{\partial \mathcal{U}_{x,\alpha}}{\partial \alpha_{x''}}$ gives the effect of changing $d\alpha_{x''}$ on $\mathcal{U}_{x,\alpha}$ via changes in wages and tax revenue over time (this object also has to be computed holding $k_{x,s}$ constant for all x, s). The equation also shows that the direct effects of the reform also alter $\mathcal{U}_{x,\alpha}$, but these effects are "of the order of" dt, since the direct effect only changes wages and tax revenue at a point in time t.

Equation (A2) is a system of equations that can be solved as

$$d\alpha_x'' = \sum_{x \in \mathcal{D}} \varepsilon_{x'',x} \cdot (\mathcal{U}_{x,\alpha,d,t} \cdot (d_k w_{x,t} + d_k T_t) + \mathcal{U}_{x,\alpha,n,t} \cdot (d_k w_t + d_k T_t)) \cdot dt \tag{A3}$$

The $\varepsilon_{x'',x}$ tell us how changes in the direct incentives to reallocate in island x affect reallocation rates from island x''.

Let $\ell_{x,0} \cdot \mu_x$ denote the welfare gains from increasing the reallocation rate out of island x. It captures all the indirect changes in wages and tax revenue generated by changes in reallocation rates (holding technology utilization constant at all points in time and islands).

We can compute the welfare gains from the reform that changes $k_{x',t}$ by $dk_{x,t}$ and leaves all other $k_{x,s}$ unchanged as

$$dW_0 = \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot (d_k w_{x,t} + d_k T_t) \cdot dt + \sum_{x'' \in \mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot d\alpha_{x''}.$$

Note that there is a dt multiplying the welfare effects of the direct effect of the reform via wages and tax revenue, since this only accrue in an instant of time. This implies that both the direct and indirect effects are "of the order of" dt.

Using the formula for $d\alpha_{x''}$ in equation (A3) we get

$$\begin{aligned}dW_0 =& \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot (d_k w_{x,t} + d_k T_t) \cdot dt \\ &+ \sum_{x\in\mathcal{D}} \sum_{x''\in\mathcal{D}} \ell_{x'',0} \cdot \mu_{x''} \cdot \varepsilon_{x'',x} \cdot (\mathcal{U}_{x,\alpha,d,t} \cdot (d_k w_{x,t} + d_k T_t) + \mathcal{U}_{x,\alpha,n,t} \cdot (d_k w_t + d_k T_t)) \cdot dt,\end{aligned}$$

which can be rewritten as

$$dW_0 = \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot \chi^{\text{end}}_{x,t} \cdot (d_k w_{x,t} + d_k T_t) \cdot dt.$$

At an optimum, we must have that this variation cannot increase welfare. Following the same steps as in the proof of Proposition 3, we get

$$\tau_{x',t} = \sum_{x\in\mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi^{\text{end}}_{x,t}}{\bar{\chi}_t} - 1\right) \cdot \left(-\frac{d_k \ln w_{x,t}}{d_k \ln k_{x',t}}\right) + \frac{\ell_t \cdot w_t}{m_{x',t}} \cdot \left(\frac{\chi^{\text{end}}_t}{\bar{\chi}_t} - 1\right) \cdot \left(-\frac{d_k \ln w_t}{d_k \ln k_{x',t}}\right)$$

This derivation also clarifies that $\frac{d_k \ln w_{x,t}}{d_k \ln k_{x',t}}$ and $\frac{d_k \ln w_t}{d_k \ln k_{x',t}}$ are partial derivatives: they correspond to the change in wages given a change in capital holding reallocation rates constant.

To complete the proof of the proposition, we compute the change in welfare driven by a change in $d\alpha_x$ holding $k_{x,s}$ constant at all periods. Equation (A1) implies that these welfare gains are given by

$$\mu_x \cdot \ell_{x,0} \cdot d\alpha_x = \int_0^\infty \left[\sum_{x''\in\mathcal{X}} \ell_{x'',s} \cdot \chi_{x'',s} \cdot d_\alpha w_{x'',s} + \bar{\chi}_s \cdot d_\alpha T_s)\right] ds.$$

The variation we are considering keeps all quantities constant (except for α_x). This implies

$$d_\alpha T_s = \sum_{x\in\mathcal{D}} d\tau_{x,s} \cdot \frac{k_{x,s}}{A_{x,s}} = -\sum_{x''\in\mathcal{X}} \ell_{x'',s} \cdot d_\alpha w_{x'',s},$$

which we can use to write the change in welfare due to changes in reallocation rates as

$$\mu_x \cdot \ell_{x,0} \cdot d\alpha_x = \int_0^\infty \left[\sum_{x''\in\mathcal{X}} \ell_{x'',s} \cdot (\chi_{x'',s} - \bar{\chi}_s) \cdot d_\alpha w_{x'',s}\right] ds.$$

The formula in (5) follows from the fact that

$$d_\alpha w_{x'',s} = \frac{\partial w_{x'',s}}{\partial \ell_{x,s}} \cdot (-s \cdot e^{-\alpha_x s}) \cdot \ell_{x,0} \cdot d\alpha_x.$$

Note that these are partial derivatives since we are interested on the effect of $\alpha_{x'}$ on wages and tax revenues holding all other α_x and $k_{x,s}$ constant. ■

A.2.1 Deriving Formulas for $\mathcal{U}_{x,\alpha}$, $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$.

Hand-to-mouth: In this case, we have

$$\mathcal{U}_x = \int_0^\infty e^{-\rho t} \cdot \left[P_{x,t} \cdot u(w_{x,t} + T_t) + (1 - P_{x,t}) \cdot u(w_t + T_t) \right] \cdot dt.$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned} \mathcal{U}_{x,\alpha} &= \int_0^\infty e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot \left[u(w_t + T_t) - u(w_{x,t} + T_t) \right] \cdot dt, \\ \mathcal{U}_{x,\alpha,d,t} &= -(t \cdot P_{x,t}) \cdot \lambda_{x,d,t}, \\ \mathcal{U}_{x,\alpha,n,t} &= (t \cdot P_{x,t}) \cdot \lambda_{x,n,t}. \end{aligned}$$

No borrowing and no transition risk: Let

$$c_{x,t} = P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t$$

In this case, we have

$$\mathcal{U}_x = \int_0^\infty e^{-\rho t} \cdot u\left(c_{x,t}\right) \cdot dt.$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned} \mathcal{U}_{x,\alpha} &= \int_0^\infty e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot (w_t - w_{x,t}) \cdot u'\left(c_{x,t}\right) dt, \\ \mathcal{U}_{x,\alpha,d,t} &= -(t \cdot P_{x,t}) \cdot \lambda_{x,d,t} + e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot (w_t - w_{x,t}) \cdot u''\left(c_{x,t}\right) \cdot P_{x,t}, \\ \mathcal{U}_{x,\alpha,n,t} &= (t \cdot P_{x,t}) \cdot \lambda_{x,n,t} + e^{-\rho t} \cdot (t \cdot P_{x,t}) \cdot (w_t - w_{x,t}) \cdot u''\left(c_{x,t}\right) \cdot (1 - P_{x,t}). \end{aligned}$$

Borrowing with transition risk: In this case there are no simple analytical expressions for $\mathcal{U}_{x,\alpha}$, $\mathcal{U}_{x,\alpha,d,t}$, $\mathcal{U}_{x,\alpha,n,t}$, nor a simple way of computing these objects numerically. For this reason, we do not analyze this scenario with endogenous reallocation effort.

Ex-post complete markets: Assume that $u(c) = c^{1-\gamma}/(1-\gamma)$ and let

$$h_{x,0} = a_{x,0} + \int_0^\infty e^{-rt} \cdot \left[P_{x,t} \cdot w_{x,t} + (1 - P_{x,t}) \cdot w_t + T_t\right] \cdot dt$$

denote the effective wealth of households in disrupted islands at time 0. We can solve analytically for $\mathcal{U}_x$ as

$$\mathcal{U}_x = \left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot h_{x,0}^{1-\gamma}/(1-\gamma).$$

Differentiating this with respect to α, and then with respect to wages at time t, we obtain:

$$\begin{aligned}
\mathcal{U}_{x,\alpha} &= \left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot \int_0^\infty e^{-rt} \cdot (t \cdot P_{x,t}) \cdot (w_t - w_{x,t}) \cdot dt, \\
\mathcal{U}_{x,\alpha,d,t} &= -\left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot e^{-rt} \cdot (t \cdot P_{x,t}) - \gamma \cdot \frac{P_{x,t}}{h_{x,0}} \cdot \mathcal{U}_{x,\alpha}, \\
\mathcal{U}_{x,\alpha,n,t} &= \left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot h_{x,0}^{-\gamma} \cdot e^{-rt} \cdot (t \cdot P_{x,t}) - \gamma \cdot \frac{1 - P_{x,t}}{h_{x,0}} \cdot \mathcal{U}_{x,\alpha}.
\end{aligned}$$

A.2.2 Optimal Assistance Programs

We now study optimal policy design for a government that can tax technology and set temporary assistance programs. As in the main text, we let $\lambda_{x,d,t} = e^{-\rho t} \cdot u'(c_{x,t} - \psi(n_{x,t}))$ denote the marginal utility of consumption at time t for households that have not reallocated, $\lambda_{x,n,t} = \mathbb{E}[e^{-\rho t} \cdot u'(c_{x,t_n,t} - \psi(n_{x,t})) | t_n \leq t]$ denotes the average marginal utility of consumption among households that reallocated by time t, and $\lambda_t = e^{-\rho t} \cdot u'(c_t - \psi(n_t))$ denotes the marginal utility of consumption of non-disrupted households at time t. We also define $\chi_{x,t}$ and $\bar{\chi}_t$ as in Proposition 3.

Without loss of generality we normalize initial effort to 1 at all islands, so that work effort is given by

$$n_{x,t} = \left(\frac{w_{x,t} \cdot (1 - \mathcal{R}_t)}{\bar{w}}\right)^{\varepsilon_\ell}.$$

Moreover, the counterfactual work effort of non-participants in non-disrupted islands is

$$n_t^* = \left(\frac{w_t}{\bar{w}}\right)^{\varepsilon_\ell}.$$

Define $\tilde{T}_t = T_t + \mathcal{R}_t \cdot w_t \cdot n_t^*$. Households' income at time t becomes

$$(1 - \mathcal{R}_t) \cdot w_{x,t} \cdot n_{x,t} + \tilde{T}_t.$$

Moreover, we can write the condition for a balanced government budget as

$$\tilde{T}_t = \sum_{x \in \mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}} + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot w_{x,t} \cdot n_{x,t}$$

This shows that our formulation for temporary assistance programs is equivalent to having a time varying linear income tax $\mathcal{R}_t$.

We first extend Lemma 1 to this environment.

LEMMA A1 (JOINT VARIATIONS LEMMA) *Consider a variation in taxes on technology and the replacement rate that induces a change in wages $dw_t, dw_{x,t}$, technology $dk_{x,t}$, tax revenue $d\tilde{T}_t$, and reallocation effort $d\alpha_x$. This variation changes tax revenue and social welfare by*

$$\text{(A4)} \qquad d\tilde{T}_t = \sum_x \tau_{x,t} \cdot \frac{dk_{x,t}}{A_{x,t}} - (1 - \mathcal{R}_t) \cdot \left(\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} \right) + m_{\ell,t} \cdot d\mathcal{R}_t + \varepsilon_\ell \cdot \mathcal{R}_t \cdot \left(\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} - m_{\ell,t} \cdot \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t} \right)$$

$$\text{(A5)} \qquad dW_0 = \int_0^\infty \left[\sum_{x \in \mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot n_{x,t} \cdot [(1 - \mathcal{R}_t) \cdot dw_{x,t} - w_{x,t} \cdot d\mathcal{R}_t] + \bar{\chi}_t \cdot d\tilde{T}_t \right] \cdot dt.$$

PROOF. The expression for the change in welfare follows from the envelope theorem and the definition of $\chi_{x,t}$ and $\bar{\chi}_t$.

Turning to the change in revenue, we have

$$d\tilde{T}_t = \sum_{x \in \mathcal{D}} \frac{dk_{x,t}}{A_{x,t}} \cdot \tau_{x,t} + \sum_{x \in \mathcal{D}} \frac{k_{x,t}}{A_{x,t}} \cdot d\tau_{x,t} + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} + m_{\ell,t} \cdot d\mathcal{R}_t + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot w_{x,t} \cdot dn_{x,t}.$$

Following the same steps from the proof of Lemma 1, we can show that $\sum_{x \in \mathcal{D}} \frac{k_{x,t}}{A_{x,t}} \cdot d\tau_{x,t} = - \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t}$. Plugging this expression into the formula for $d\tilde{T}_t$ we get

$$d\tilde{T}_t = \sum_{x \in \mathcal{D}} \frac{dk_{x,t}}{A_{x,t}} \cdot \tau_{x,t} - (1 - \mathcal{R}_t) \cdot \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} + m_{\ell,t} \cdot d\mathcal{R}_t + \mathcal{R}_t \cdot \sum_{x \in \mathcal{X}} \ell_{x,t} \cdot w_{x,t} \cdot dn_{x,t}.$$

Equation (A4) follows from the fact that $dn_{x,t} = n_{x,t} \cdot \varepsilon_\ell \cdot \left(d \ln w_{x,t} - \frac{d\mathcal{R}_t}{1 - \mathcal{R}_t} \right)$. Equation (A5) follows from the proof for Lemma 1 and the envelope condition for optimal effort. ■

We now use Lemma A1 to prove Proposition 5.

Proof of Proposition 5. The proof follows Costinot and Werning (2022). Consider a variation that changes $k_{x',t}$ by $dk_{x',t}$ but keeps the utilization of all other types of capital and the replacement rate of assistance programs unchanged. At a social optimum, this variation cannot affect welfare. Thus:

$$\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot \chi_{x,t} \cdot n_{x,t} \cdot (1-\mathcal{R}_t) \cdot dw_{x,t} + \bar{\chi}_t \cdot d\tilde{T}_t = 0.$$

For this specific variation, we have

$$d\tilde{T}_t = \tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} - (1-\mathcal{R}_t) \cdot \left(\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t}\right) + \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t},$$

which implies the necessary condition for an optimum

$$\begin{aligned}
&\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot \frac{\chi_{x,t}}{\bar{\chi}_t} \cdot (1-\mathcal{R}_t) \cdot dw_{x,t} \\
&+ \tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} - (1-\mathcal{R}_t) \cdot \left(\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t}\right) + \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} = 0.
\end{aligned}$$

Regrouping terms, we can write this as

$$\tau_{x',t} \cdot \frac{dk_{x',t}}{A_{x',t}} = \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot (1-\mathcal{R}_t) \cdot (-dw_{x,t}) - \varepsilon_\ell \cdot \mathcal{R}_t \cdot \sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t}$$

Dividing by $dk_{x',t}$ and using the fact that $m_{x,t} = k_{x,t}/A_{x,t}$ yields (6).

We now turn to the formula for optimal replacement rates. Consider a variation that changes $d\mathcal{R}_t$ but keeps technology utilization constant at all times and islands. The change in revenue from this variation is

$$d\tilde{T}_t = -(1-\mathcal{R}_t) \cdot \left(\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t}\right) + m_{\ell,t} \cdot d\mathcal{R}_t + \varepsilon_\ell \cdot \mathcal{R}_t \cdot \left(\sum_{x\in\mathcal{X}} \ell_{x,t} \cdot n_{x,t} \cdot dw_{x,t} - m_{\ell,t} \cdot \frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)$$

The welfare change from this variation is given by equation (A5). At a social optimum, this variation cannot affect welfare, which yields the necessary condition for an optimum we obtain the necessary condition for an optimum

$$\sum_{x\in\mathcal{X}} \ell_{x,t}\cdot n_{x,t}\cdot \frac{\chi_{x,t}}{\bar{\chi}_t}\cdot\left[(1-\mathcal{R}_t)\cdot dw_{x,t}-w_{x,t}\cdot d\mathcal{R}_t\right]$$
$$-(1-\mathcal{R}_t)\cdot\left(\sum_{x\in\mathcal{X}}\ell_{x,t}\cdot n_{x,t}\cdot dw_{x,t}\right)+m_{\ell,t}\cdot d\mathcal{R}_t+\varepsilon_\ell\cdot\mathcal{R}_t\cdot\left(\sum_{x\in\mathcal{X}}\ell_{x,t}\cdot n_{x,t}\cdot dw_{x,t}-m_{\ell,t}\cdot\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)=0$$

Regrouping terms, we can write this as

$$\varepsilon_\ell\cdot\mathcal{R}_t\cdot\left(m_{\ell,t}\cdot\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}-\sum_{x\in\mathcal{X}}\ell_{x,t}\cdot n_{x,t}\cdot dw_{x,t}\right)=\sum_{x\in\mathcal{X}}\ell_{x,t}\cdot n_{x,t}\cdot\left(\frac{\chi_{x,t}}{\bar{\chi}_t}-1\right)\cdot\left[(1-\mathcal{R}_t)\cdot dw_{x,t}-w_{x,t}\cdot d\mathcal{R}_t\right],$$

which dividing through by $m_{\ell,t}\cdot d\mathcal{R}_t$ yields

$$\varepsilon_\ell\cdot\mathcal{R}_t\cdot\left(\frac{1}{1-\mathcal{R}_t}-\sum_{x\in\mathcal{X}}\frac{\ell_{x,t}\cdot n_{x,t}\cdot w_{x,t}}{m_{\ell,t}}\frac{\partial\ln w_{x,t}}{\partial\mathcal{R}_t}\right)=\sum_{x\in\mathcal{X}}\frac{\ell_{x,t}\cdot n_{x,t}\cdot w_{x,t}}{m_{\ell,t}}\cdot\left(\frac{\chi_{x,t}}{\bar{\chi}_t}-1\right)\cdot\left[(1-\mathcal{R}_t)\cdot\frac{\partial\ln w_{x,t}}{\partial\mathcal{R}_t}-1\right],$$

which can be rearranged as in the formula in the Proposition.

When implementing the formula in equation (7), we compute the general equilibrium derivatives $\frac{\partial\ln w_t}{\partial\mathcal{R}_t}$ and $\frac{\partial\ln w_{x,t}}{\partial\mathcal{R}_t}$ as $(1/w_{x,t})\cdot dw_{x,t}/d\mathcal{R}_t$ and $(1/w_t)\cdot dw_t/d\mathcal{R}_t$, where these objects are given by the solution to the system of equations

$$\begin{aligned}dw_{x,t}=&\varepsilon_\ell\cdot\sum_{x'\in\mathcal{D}}f_{y_x,y_{x'}}\cdot\ell_{x',t}\cdot n_{x',t}\left(\frac{dw_{x',t}}{w_{x',t}}-\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)+\varepsilon_\ell\cdot f_{y_x,y}\cdot\ell_t\cdot n_t\cdot\left(\frac{dw_t}{w_t}-\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)\\ dw_t=&\varepsilon_\ell\cdot\sum_{x'\in\mathcal{D}}f_{y,y_{x'}}\cdot\ell_{x',t}\cdot n_{x',t}\left(\frac{dw_{x',t}}{w_{x',t}}-\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)+\varepsilon_\ell\cdot f_{y,y}\cdot\ell_t\cdot n_t\cdot\left(\frac{dw_t}{w_t}-\frac{d\mathcal{R}_t}{1-\mathcal{R}_t}\right)\end{aligned}$$

■

A.3 Proofs for Section 2.5

Proof of Proposition 6. Suppose the government sets no taxes nor assistance programs. Consider a new path for technology that changes wages by $dw_{x,t}$ and dw_t. The resulting change in welfare is

$$dW_0=\int_0^\infty\left(\sum_{x\in\mathcal{D}}\ell_{x,t}\cdot\chi_{x,t}\cdot dw_{x,t}+\ell_t\cdot\chi_t\cdot dw_t\right)\cdot dt.$$

Note that changes in reallocation effort are second order. This is because in this case we are varying technology and letting $k_{x,t}$ adjust, which implies that wages are entirely pinned down by technology and independent of reallocation rates.

Differentiating the ideal-price index condition $c^f(\{w_{x,t}\}_{x\in\mathcal{D}}, w_t) = 1$, we obtain

$$s_t \cdot d\ln w_t + \sum_{x\in\mathcal{D}} s_{x,t} \cdot d\ln w_{x,t} = 0.$$

Substituting in the formula for the change in welfare and rearranging terms yields

$$dW_0 = \int_0^\infty \left(\sum_{x\in\mathcal{D}} \left(\ell_{x,t} \cdot \chi_{x,t} \cdot w_{x,t} - \ell_t \cdot \chi_t \cdot w_t \cdot \frac{s_{x,t}}{s_t}\right) \cdot d\ln w_{x,t}\right) \cdot dt.$$

Finally, from $w_{x,t} = 1/A_{x,(1-\Gamma)\cdot t}$, we get $d\ln w_{x,t} = \frac{\dot{A}_{x,t}}{A_{x,t}} \cdot t \cdot d\Gamma$, which gives the formula in equation (8).

To prove the second part of the Proposition, we show that the government can always change taxes in response to improvements in technology in a way that leaves the economy and welfare unchanged and generates additional revenue. Suppose that $\tau_{x,t}$ is the optimal path for taxes when the productivity of new technology is $A_{x,t}$. Suppose that productivity shifts up to $\tilde{A}_{x,t} \geq A_{x,t}$. Taxes can then be adjusted up to a level $\tilde{\tau}_{x,t}$ defined as

$$\frac{\tilde{A}_{x,t}}{1+\tilde{\tau}_{x,t}} = \frac{A_{x,t}}{1+\tau_{x,t}},$$

and which satisfies $\tilde{\tau}_{x,t} \geq \tau_{x,t}$ while at the same time keeping transfers T_t unchanged. This choice of taxes and transfers ensures that prices, utilities, reallocation rates, and the equilibrium remain unchanged. To conclude the proof we need to show that this policy is feasible, or

$$T_t \leq \sum_{x\in\mathcal{D}} \tilde{\tau}_{x,t} \cdot \frac{k_{x,t}}{\tilde{A}_{x,t}}.$$

This follows from the fact that

$$T_t = \sum_{x\in\mathcal{D}} \tau_{x,t} \cdot \frac{k_{x,t}}{A_{x,t}} = \sum_{x\in\mathcal{D}} \frac{\tau_{x,t}}{1+\tau_{x,t}} \cdot \frac{1+\tilde{\tau_{x,t}}}{\tilde{\tau_{x,t}}} \cdot \tilde{\tau_{x,t}} \cdot \frac{k_{x,t}}{\tilde{A}_{x,t}} \leq \sum_{x\in\mathcal{D}} \tilde{\tau_{x,t}} \cdot \frac{k_{x,t}}{\tilde{A}_{x,t}},$$

as wanted. ■

A.4 Theoretical Extensions

This section provides theoretical extensions of our baseline model.

A.4.1 Inequality Between and Within Islands

This section extends our results to an economy with ex-ante differences in labor productivity within and between islands. We also discuss a rationale for ignoring pre-existing income differences across islands when deciding how to compensate the losers of globalization and technological progress.

We consider a model where workers differ in absolute advantage. Households are endowed with $\xi > 0$ units of labor. We refer to ξ as the type of the household. The distribution of ξ in island x has cdf $\Phi_x(\xi)$, and the distribution of ξ in non-disrupted islands has cdf $\Phi(\xi)$. This definition implies $\int_\xi \xi \cdot d\Phi_x(\xi) = \ell_{x,0}$ and $\int_\xi \xi \cdot d\Phi(\xi) = \ell_0$.

We make the following assumptions:

- The utility function is of the form $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$ for some $\gamma > 0$.
- A household of type ξ receives labor income $\xi \cdot w_{x,t}$ in island x and a proportional lump-sum rebate of $\xi \cdot T_t$. It also faces a cost of reallocation $\xi^{1-\gamma} \cdot \kappa_x(\alpha)$ and is endowed with initial assets $\xi \cdot a_{0,x}$.
- The budget restrictions faced by households imply that a consumption plan $\xi \cdot c$ is feasible for a household of type ξ from island x if and only if $\xi' \cdot c$ is feasible for a household of type ξ' from island x.

These assumptions imply that all households in island x choose paths for consumption and savings that are proportional to each other. In particular, let $\{c_{x,t_n,t}, c_{x,t}, \alpha_x\}$ denote the optimal consumption plan for a household with $\xi = 1$ from island x, and let $\{c^\xi_{x,t_n,t}, c^\xi_{x,t}, \alpha^\xi_x\}$ denote the optimal consumption plan for a household of type ξ from island x. The assumptions imply $c^\xi_{x,t_n,t} = \xi \cdot c_{x,t_n,t}$, $c^\xi_{x,t} = \xi \cdot c_{x,t}$, and $\alpha^\xi_x = \alpha_x$. In addition, households' utility is $U^\xi_{x,0} = U_{x,0} \cdot \xi^{1-\gamma}$, and their marginal utilities of consumption are $\lambda^\xi_{x,d,t} = \lambda_{x,d,t} \cdot \xi^{-\gamma}$ and $\lambda^\xi_{x,n,t} = \lambda_{x,n,t} \cdot \xi^{-\gamma}$. The same applies to households from non-disrupted islands.

Consider a welfare function of the form

$$W_0 = \sum_{x \in \mathcal{D}} \int_\xi \mathcal{W}\left(((1-\gamma) \cdot U^\xi_{x,0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi_x(\xi) + \int_\xi \mathcal{W}\left(((1-\gamma) \cdot U^\xi_0)^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi(\xi).$$

This welfare function accounts for heterogeneity in ξ. We do not require the welfare function to be symmetric, and in particular, we let $\mathcal{W}$ depend on ξ to capture societal preferences for redistribution across households with different types. We also wrote the welfare function in terms of consumption equivalent terms $((1-\gamma)\cdot U^{\xi}_{x,0})^{\frac{1}{1-\gamma}}$ and $((1-\gamma)\cdot U^{\xi}_{0})^{\frac{1}{1-\gamma}}$, but this is done for tractability only.

PROPOSITION A1 *The results in Propositions 3, 4 and 5 apply to this general economy with Pareto weights re-defined as*

$$g_x = \int_{\xi} \mathcal{W}'\left(\xi \cdot ((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot ((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}} \cdot \frac{\xi \cdot d\Phi_x(\xi)}{\ell_{x,0}}$$
$$g = \int_{\xi} \mathcal{W}'\left(\xi \cdot ((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot ((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}} \cdot \frac{\xi \cdot d\Phi(\xi)}{\ell_{0}}.$$

Proof:. Using the fact that $U^{\xi}_{x,0} = U_{x,0}\cdot \xi^{1-\gamma}$ and $U^{\xi}_{0} = U_{0}\cdot \xi^{1-\gamma}$, we can rewrite the welfare function as

$$W_0 = \sum_{x\in\mathcal{D}} \int_{\xi} \mathcal{W}\left(\xi \cdot ((1-\gamma)\cdot U_{x,0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi_x(\xi) + \int_{\xi} \mathcal{W}\left(\xi \cdot ((1-\gamma)\cdot U_{0})^{\frac{1}{1-\gamma}}; \xi\right) \cdot d\Phi(\xi).$$

Changes in welfare are then given by

$$dW_0 = \ell_{x,0}\cdot g_x \cdot dU_{x,0} + \ell_0 \cdot g \cdot dU_0,$$

which coincides with the change in welfare in the main text. As explained in the main text (see footnote 10) all the results in the paper hold for arbitrary Pareto weights, g_x, g, and so in particular, they also hold after redefining g_x and g. ■

The proposition illustrates how ex-ante inequality affects optimal policy. Suppose that $\mathcal{W}'(c;n) = c^{-\eta}$ for $\eta \geq \gamma$, so that the welfare function is scale free, symmetric, and concave in individual utilities. Then:

$$g_x = \left(\int_{\xi} \xi^{-\eta} \cdot \frac{\xi \cdot d\Phi_x(\xi)}{\ell_{x,0}}\right) \cdot \left(((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{1}{1-\gamma}}\right)^{-\eta} \cdot ((1-\gamma)\cdot \mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}}$$
$$g = \left(\int_{\xi} \xi^{-\eta} \cdot \frac{\xi \cdot d\Phi(\xi)}{\ell_{0}}\right) \cdot \left(((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{1}{1-\gamma}}\right)^{-\eta} \cdot ((1-\gamma)\cdot \mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}}.$$

Ex-ante inequality across households only matters via the terms $\int_{\xi} \xi^{-\eta} \cdot \frac{\xi \cdot d\Phi_x(\xi)}{\ell_{x,0}}$ and $\int_{\xi} \xi^{-\eta} \cdot \frac{\xi \cdot d\Phi(\xi)}{\ell_{0}}$. These terms are larger for islands x with households that have fewer units of labor on average, introducing a motive for taxing $k_{x,t}$ more aggressively due to its tagging value. On the other hand, within island inequality does not affect optimal taxes conditional on these tagging terms. Optimal taxes are also zero in the long run, since distorting $k_{x,t}$ loses its tagging value as people reallocate away from island x.

The proposition also identifies conditions under which ex-ante inequalities do not interact with the problem of protecting losers. Suppose that $\mathcal{W}$ satisfies

$$\mathcal{W}'(\xi \cdot c; \xi) = \mathcal{W}'(c).$$

This captures a situation where the public considers it fair for households of type ξ to enjoy a higher consumption, proportional to their higher labor endowment. In this case

$$\begin{aligned} g_x =& \mathcal{W}'\left(((1-\gamma) \cdot \mathcal{U}_{x,0})^{\frac{1}{1-\gamma}}\right) \cdot ((1-\gamma) \cdot \mathcal{U}_{x,0})^{\frac{\gamma}{1-\gamma}} \\ g =& \mathcal{W}'\left(((1-\gamma) \cdot \mathcal{U}_{0})^{\frac{1}{1-\gamma}}\right) \cdot ((1-\gamma) \cdot \mathcal{U}_{0})^{\frac{\gamma}{1-\gamma}} \end{aligned}$$

and inequality of labor endowments between and within islands is irrelevant for the problem of compensating winners and losers. This offers a rationale for ignoring ex-ante inequalities across (and within) islands when selecting optimal taxes on technologies or trade motivated exclusively by compensating the losers. In particular, we can ignore ex-ante inequalities across islands if they are considered fair.

A.4.2 Retraining Subsidies and Active Labor Market Policies

As our second extension, we consider the problem of taxing technologies when the government has access to other tools that allow it to select the socially optimal level of reallocation. These tools might include retraining subsidies or active labor market policies.

Proposition A2 *Suppose the planner has other policy tools that implement the optimal social level of reallocation. A necessary condition for an optimal tax sequence is that:*

$$\tau_{x',t} = \sum_{x \in \mathcal{X}} \frac{\ell_{x,t} \cdot w_{x,t}}{m_{x',t}} \cdot \left(\frac{\chi_{x,t}}{\bar{\chi}_t} - 1\right) \cdot \left(-\frac{\partial \ln w_{x,t}}{\partial \ln k_{x',t}}\right), \tag{A6}$$

where the multipliers on the right-hand side are now evaluated along an equilibrium with the socially optimal level of α_x.

Proof:. Optimal reallocation effort maximizes social welfare. The envelope theorem implies that the effect of any reform on welfare is equal to the direct effect holding α_x constant, which leads to the same optimality condition as in Proposition 3. ■

A.5 Calibration and Details of Numerical Algorithms

A.5.1 Calibration Details, China Shock

Calibrating π: Industry prices are initially given by $P_i = 1$. Following the disruption, we get a price index

$$P_{i,t_f} = c_i(W_t, \exp(-\pi)),$$

for some cost function c_i with $c_i(1,1) = 1$. Assuming that π is small, we can log-linearize this equation around $(1,1)$ as

$$\ln P_{i,t_f} \approx \text{share domestic production}_{i,t_f} \cdot \ln W_t - \text{share Chinese production}_{i,t_f} \cdot \ln \pi.$$

This implies

$$\ln P_{i,t_f} =\approx \text{share domestic production}_{i,t_f} \cdot (\ln W_t + \ln \pi) - \ln \pi.$$

Let $s = \max\{\text{share Chinese production}_{i,t_f}\}$ and suppose that s is small, as is the case in the data. Then

$$\ln P_{i,t_f} =\approx \ln \text{share domestic production}_{i,t_f} \cdot \ln \pi - \ln \pi.$$

This shows that the regression in Bai and Stumpner (2019) across industries identifies $\ln \pi$.

Pre-existing trade: In the applications of our framework to the China Shock and Colombia's trade liberalization, we have to deal with the fact that there was some pre-existing trade.

For the China Shock, we handle pre-existing trade by assuming that there is a mass $\nu_{p(i)}$ of islands associated with industry i that were already replaced by Chinese imports and hosted no workers by 1991. We normalize the cost of Chinese imports in these islands to 1, which implies that the cost function associated with (11) becomes

$$c_u^f(\{w_x\}, w) = \left(\nu_p + \nu \cdot w^{1-\sigma} + \sum_{x \in \mathcal{D}} \nu_x \cdot w_x^{1-\sigma}\right)^{\frac{1}{1-\sigma}},$$

where $\nu_p = \sum_i \nu_{p(i)}$. The normalization $\bar{w} = 1$ in status quo then requires $\nu_p + \nu + \sum_{x \in \mathcal{D}} \nu_x = 1$. In our calibration, we set $\nu_p = 2.5\%$—the share of imports in GDP before the China Shock.

We assume that the China Shock is driven by advances in the productivity of Chinese imports at other islands, and not by cost reductions of established products. These assumptions imply that the status-quo level of imports in industry i is

$$\frac{m_{i,t_0}}{y_{i,t_0}} = \nu_{p(i)};$$

while imports in industry i at time t after the China Shock are given by

$$\frac{m_{i,t}}{y_{i,t}} = \nu_{p(i)} + \frac{1}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right),$$

where x is defined as the island associated with industry i (i.e., the one for which $i(x) = i$). In this expression, the first term accounts for imports at islands with pre-existing trade and the second term accounting for imports in new islands. The change in normalized import shares at time t is then equal to

$$\text{(A7)} \qquad \text{Change in normalized import share}_{i(x),t} = \frac{1}{\Omega_i} \cdot \frac{1}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right) \text{ for } x \in \mathcal{D}.$$

Equation (A7) provides a system of equation across industries that we use to calibrate $\nu_{i(x)}$ and $s_{i(x)}$ in a first step to match the change in normalized imports by 2007 (recall that $A_{x,t_f} = \exp(\pi)$ at this point), and then to calibrate a path for $A_{x,t}$ in a second step, as described in the main text.

For Colombia's trade liberalization, we assume that a mass $\nu_{p(i)}$ of segments were already produced internationally and hosted no workers by 1989. In addition, we assume this segments were not protected by 1989, and experienced no subsequent decline in tariffs after the 1990 trade liberalization. Under these assumptions, we have that the status-quo level of imports in industry i is

$$\frac{m_{i,t_0}}{y_{i,t_0}} = \nu_{p(i)};$$

while imports in industry i at time t after the liberalization are

$$\frac{m_{i,t}}{y_{i,t}} = \nu_{p(i)} + \frac{1 + \tau_{x,t}}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right),$$

where x is defined as the island associated with industry i (i.e., the one for which $i(x) = i$).

The change in normalized import shares at time t is then equal to

$$\text{(A8)} \quad \text{Change in normalized import share}_{i(x),t} = \frac{1}{\Omega_i} \cdot \frac{1+\tau_{x,t}}{y_t \cdot A_{x,t}} \cdot \left(\ell_t \cdot \frac{c_x^f}{c_w^f} - \ell_{x,t} \right) \text{ for } x \in \mathcal{D}.$$

Equation (A8) provides a system of equation across industries that we use to calibrate $\nu_{i(x)}$ and $s_{i(x)}$ to match the increase in normalized import shares between 1989 and 2002.

A.5.2 Implementing the Formulas in Equations (3) and (4):

This subsection describes the numerical procedure used to compute optimal taxes.

Exogenous reallocation effort. We compute optimal taxes with exogenous reallocation effort as follows:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire).
2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$, identified with the superscript (n) below.
3. Use equation (3) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x\in\mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{(n)}}{\bar{\chi}_t^{(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{(n)}}{\bar{\chi}_t^{(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

4. Repeat steps 2–3 until convergence.

Endogenous reallocation effort. We compute optimal taxes with exogenous reallocation effort as follows:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire) and the observed rate of reallocation $\alpha_x^{(0)}$.
2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$ and $\alpha_x^{(0)}$, identified with the superscript (n) below.
3. Use equation (3) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x\in\mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(- \frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

4. Update the reallocation rate using

$$\alpha_x^{(n+1)} = \alpha_x^{(n)} + \Delta\alpha_x^{(n)},$$

where $\Delta\alpha_x^{(n)}$ is given by

$$\Delta\alpha_x^{(n)} = \sum_{x''} \varepsilon_{x,x''} \cdot \int_0^\infty \left(\mathcal{U}_{x'',\alpha,d,t}^{(n)} \cdot (\Delta^{(n)} w_{x'',t} + \Delta^{(n)} T_t) + \mathcal{U}_{x'',\alpha,n,t}^{(n)} \cdot (\Delta^{(N)} w_t + \Delta^{(n)} T_t) \right) \cdot dt,$$

and $\Delta^{(n)} w_{x'',t}$, $\Delta^{(n)} w_t$, $\Delta^{(n)} T_t$ is the change in wages and tax revenue generated by the update in taxes from iteration n to $n+1$.

5. Repeat steps 2–4 until convergence.

This procedure only requires us to specify values for $\varepsilon_{x,x''}$ and solve for the optimal tax and the equilibrium path without having to specify the κ function. This comes at the cost of assuming that the elasticities $\varepsilon_{x,x''}$ remain roughly unchanged for the variations in taxes considered. It also ignores the effect of changes in household utility on the multipliers g_x, which is second order due to the envelope theorem, but could be non-negligible for large changes in reallocation effort α_x.

As an alternative, we experimented with the following procedure, which requires parametrizing the κ_x function:

1. Start with $\tau_{x,t}^{(0)} = 0$ (laissez-faire) and the observed rate of reallocation $\alpha_x^{(0)}$.

2. Compute equilibrium objects for $\tau_{x,t}^{(n)}$ and $\alpha_x^{(0)}$, identified with the superscript (n) below.

3. Compute $\varepsilon_{x,x''}^{(n)}$ by solving the system of equations in (A2).

4. Use equation (3) to update optimal taxes as

$$\tau_{x',t}^{(n+1)} = \sum_{x \in \mathcal{D}} \frac{\ell_{x,t} \cdot w_{x,t}^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_{x,t}^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_{x,t}^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right) + \frac{\ell_t \cdot w_t^{(n)}}{m_{x',t}^{(n)}} \cdot \left(\frac{\chi_t^{\text{end},(n)}}{\bar{\chi}_t^{\text{end},(n)}} - 1 \right) \cdot \left(-\frac{\partial \ln w_t^{(n)}}{\partial \ln k_{x',t}^{(n)}} \right),$$

using the values of $\varepsilon_{x,x''}^{(n)}$ to compute the χ's.

5. Update the reallocation rate using

$$\kappa'(\alpha_x^{(n+1)}) = \mathcal{U}_{x,\alpha}^{(n)}.$$

6. Repeat steps 2–5 until convergence.

A.5.3 Details for the Savings Problem with no Risk Sharing:

As explained in the text, households problem can be summarized by the following system of HJB equations

$$\rho v_x(a,t) - \dot{v}_x(a,t) = \max_c u(c) + \frac{\partial v_x(a,t)}{\partial a} \cdot (ra + w_{x,t} - c) + \alpha_x \cdot (v(a,t) - v_x(a,t)),$$
$$\rho v(a,t) - \dot{v}(a,t) = \max_c u(c) + \frac{\partial v(a,t)}{\partial a} \cdot (ra + w_t - c).$$

Here, $v_x(a,t)$ is the value function of households in disrupted islands at time t with assets a when they exert reallocation effort α_x (kept as an implicit argument to simplify notation), and $v(a,t)$ is the value function of households in non-disrupted islands with assets a.

Let $h_{x,t} = \int_t^\infty e^{-(s-t)r} \cdot w_{x,s} ds$ and $h_t = a_t + \int_t^\infty e^{-(s-t)r} \cdot w_s ds$. We can rewrite these HJB equations using $z = a + h$—effective wealth—as the state variable:

$$\rho v_x(z,t) - \dot{v}_x(z,t) = \max_c u(c) + \frac{\partial v_x(z,t)}{\partial z} \cdot (rz - c) + \alpha_x \cdot (v(z + h_t - h_{x,t}) - v_x(z,t)),$$
$$\rho v(z) = \max_c u(c) + \frac{\partial v(z)}{\partial z} \cdot (rz - c).$$

Note that the HJB equation for $v(z)$ is now stationary, since interest rates are constant. For $u(c) = c^{1-\gamma}/(1-\gamma)$, we can solve analytically for $v(z)$ as

$$v(z) = \left[r - \frac{1}{\gamma}(r-\rho)\right]^{-\gamma} \cdot \frac{z^{1-\gamma}}{1-\gamma}.$$

Moreover, policy functions in the non-disrupted island are given by

$$c_t = \left[r - \frac{1}{\gamma}(r-\rho)\right] \cdot z_t, \qquad \dot{z}_t = \frac{1}{\gamma}(r-\rho) \cdot z_t.$$

This implies

$$\lambda_{x,n,t} = \frac{1}{1-P_{x,t}} \cdot \int_0^t e^{-\rho t} \cdot \alpha_x \cdot e^{-\alpha_x t_n} \cdot \left(\left[r - \frac{1}{\gamma}(r-\rho)\right] \cdot (z_{x,t_n} + h_{t_n} - h_{x,t_n}) \cdot e^{\frac{1}{\gamma}(r-\rho)(t-t_n)}\right)^{-\gamma} \cdot dt_n,$$

where $z_{x,t}$ denotes the effective wealth held by households in disrupted islands at time t.

This expression uses the fact that

$$c_{x,t_n,t} = \left[r - \frac{1}{\gamma}(r-\rho)\right] \cdot (z_{x,t_n} + h_{t_n} - h_{x,t_n}) \cdot e^{\frac{1}{\gamma}(r-\rho)(t-t_n)}.$$

To characterize $z_{x,t}$ we solve the HJB equation for $v_x(z,t)$ numerically using the finite-differences method described in Achdou et al. (2021). This method characterizes the common path of consumption $c_{x,t}$ and assets $z_{x,t}$ for households in disrupted islands starting from $z_{x,0} = a_{x,0} + h_{x,0}$. From this method, we also obtain

$$\lambda_{x,d,t} = e^{-\rho t} \cdot c_{x,t}^{-\gamma}.$$

Figure A1 plots typical path for consumption $c_{x,t}$ and assets $z_{x,t}$ starting from $z_{x,0} = h_{x,0} = 1$ in an economy where $h_t - h_{x,t}$ is positive and rises from 0.3 to 0.5 over time. For this examples, we consider a baseline scenario with $\alpha_x = 5\%$, $r = \rho = 5\%$, and $\gamma = 2$ and report variants.

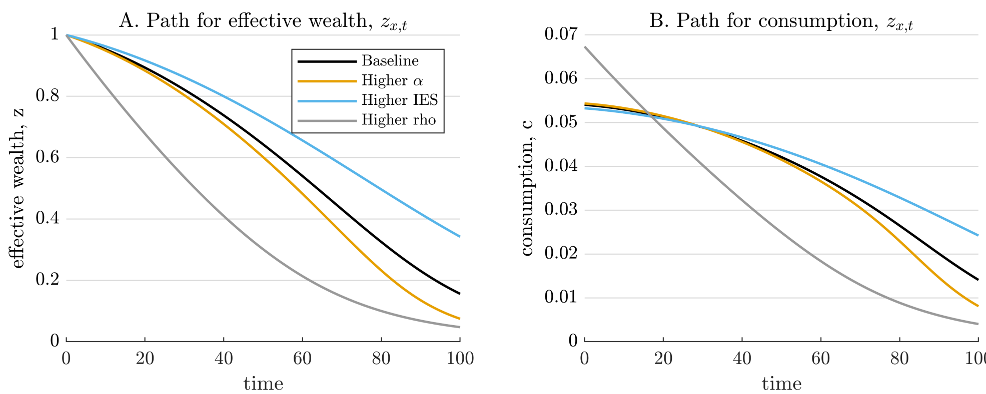

Figure A1: Consumption and wealth path in disrupted islands. The figure reports examples of the optimal path for effective wealth and consumption in disrupted islands when households can borrow but face uncertainty regarding when they will reallocate. These paths are obtained numerically using the finite-differences method described in Achdou et al. (2021).