Is the Electricity Sector a Weak Link in Development?

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Abstract

This paper asks whether increasing productivity in the electricity sector can yield larger long-run GDP gains than suggested by electricity's small share of aggregate economic activity. We answer this question using a dynamic model in which electricity is a strong complement to other inputs in production. We parameterize the model using our own new measures of electricity-sector TFP across countries. The model predicts modest long-run GDP gains from improving electricity-sector TFP, contrary to the notion that electricity is a weak link. Parameterizations that make electricity a weak link mostly require the electricity sector to be counterfactually large or unproductive.

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1. Introduction

Should some sectors of the economy be considered "weak links" in the development process, whereby modest increases in sectoral productivity have large aggregate effects? A short list of candidates would have to include the electricity sector, which provides an essential input for nearly all the goods and services that characterize advanced economies. It is hard to point to a modern manufacturing process that can thrive without electric power, and the service sector just wouldn't be the same without lighting, refrigeration, and a place to plug in a computer. Gordon (2017) argues that electricity was the foundation for much of U.S. productivity growth throughout the 20th century. Jones (2011) uses electricity as a motivating example in his seminal study of intermediate goods and weak links in development, and Kremer (1993), in his famous O-ring paper, points to electricity as a potential source of productivity-reducing bottlenecks in low-income countries.

This paper quantitatively assesses the case that the electricity sector is a weak link in development. The basic idea is that if developing countries are ineffective at making electricity, and there are few substitutes for electricity in the production process, this could impede capital accumulation and hinder long-run growth. To formalize this idea, we build a dynamic multi-sector model deliberately composed of elements familiar to macroeconomists. The primary element is a final-goods production function that features a low substitution elasticity between electricity and other productive inputs (Atkeson and Kehoe, 1999; Hassler, Krusell and Olovsson, 2021; Casey, 2023). The second element involves sectoral linkages, as in Jones (2011), where final goods are partially reused as intermediates. The third element assumes that less developed countries exhibit particular inefficiencies in producing capital goods, following Hsieh and Klenow (2007). Lastly, the model introduces tax-like distortions in electricity production, as in Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), resulting in elevated electricity costs for users. This inclusion is motivated by substantial evidence indicating that electricity markets are distorted in developing countries, prompting firms to generate their own power at higher costs (see e.g., McRae, 2015; Allcott, Collard-Wexler and O'Connell, 2016; Burgess, Greenstone, Ryan and Sudarshan, 2020).

We first consider an analytical special case of the model in which final goods are produced using a Leontief aggregate of electricity and other inputs, intermediate linkages are shut down, and electricity is produced using only capital. We show that the long-run aggregate effects of an improvement in electricity TFP are higher than suggested by Hulten's (1978) approximation when initial electricity TFP levels are low enough or distortions are high enough. These long-run GDP gains arise as the economy reallocates capital out of the electricity sector and accumulates new capital, and are higher in economies with initially less productive or more distorted electricity sectors.

This prediction highlights the need for measures of electricity-sector TFP in developing and advanced economies. We construct a new database consisting of real inputs and outputs in the electricity production, transmission, and distribution sector covering countries of all income levels. Electricity outputs have the advantage of being measured in physically homogeneous units – megawatt hours – that sidestep some of the challenges faced in measuring other goods and services at international prices (see e.g. Feenstra, Inklaar and Timmer, 2015). We measure capital stocks using a database of large-scale electric power plants plus our own estimates of small-scale electricity generation. We measure labor and primary-energy inputs (such as coal), plus labor's share in electricity production, using a mix of industrial censuses and firm-level annual reports.

To measure TFP in the electricity sector at the country level, we propose and calibrate an aggregate production function for the electricity sector in which electricity output is a constant-returns function of capital, labor, and primary energy inputs. We show that labor's share of revenue in electricity is low and roughly constant across countries. This finding supports our assumption that the electricity sector production function is Cobb-Douglas in labor and a capital-fuel composite input. Using this production function and our cross-country database, we measure electricity-sector TFP in 80 countries ranging from Ethiopia to Norway.

We document very little variation in electricity-sector TFP across countries. In our preferred specification, the poorest quartile of countries has around 86 percent the TFP level in electricity as the richest ones. As a frame of reference, the poorest countries have aggregate TFP in the Penn World Tables (PWT) that are around 36 percent as high as the richest. We show that our results are robust to several alternative measurement and modeling assumptions and when considering country differences in energy mixes.

Just how large of an aggregate effect arises from a positive shock to electricity TFP is a quantitative question. We parameterize the model to match our cross-country electricity data as well as aggregate data from the PWT. We solve the model for a wide range of GDP per capita levels and show that it is consistent with salient cross-country statistics, such as labor- and capital-productivity in electricity, employment shares in electricity, and capital-output ratios.

We use the estimated model to compute the effects of a large TFP increase in the electricity sector. We then ask whether the resulting increase in aggregate productivity is larger than suggested by the electricity sector's Domar weight, measured as the ratio of gross electricity output to GDP. We do this both for the short run, where the capital stock is held fixed and factors are not allowed to reallocate across sectors, and the long run, where capital is accumulated until a new steady state is reached. We find that raising electricity TFP by fifty percent would lead to average long-run GDP per capita gains of about 1.5 percent. By comparison, the simple approximation suggested by Hulten (1978) predicts a GDP gain of 1.7. This is inconsistent with electricity being a weak link. We find similarly modest gains from removing distortions in the electricity sector, which we estimate would raise GDP by 0.6 percent.

To guide intuition for why electricity is not a weak link in our quantitative model, we illustrate several alternative scenarios in which it *is* a weak link. When initial electricity TFP is counterfactually lower, the long-run GDP gains from increasing electricity TFP can be potentially much larger than suggested by Hulten. However, in this case, the electricity sector's initial employment share is also much larger than in the data. We find a similar result for counterfactually lower initial levels of TFP in the capital goods sector, and substantially larger initial distortions in the electricity sector. An alternative formulation for distortions, where electricity is lost due to unpredictable outages, makes the electricity sector better resemble a weak link. Yet it requires that firms do not or cannot insure against these losses at any price. We conclude that richer mechanisms than the intuitive ones present in our model are required for electricity to be a weak-link sector.

2. Model

This section describes our macroeconomic model of the electricity sector. We first specify the environment and then focus on a special case that can be characterized analytically.

2.1. Environment

Final goods are produced with a CES production function:

$$Y=\left[\alpha\left(A_X Z^{\gamma}(K_X^{\theta_X}L_X^{\theta_X})^{1-\gamma}\right)^{\frac{\mu-1}{\mu}}+(1-\alpha)E^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{1}$$

where K_X and L_X are capital and labor inputs, E is the electricity input, and and Z denotes final goods put back into the production process as intermediates. A_X denotes TFP in final-goods production; θ_X captures the importance of capital; $\alpha\in(0,1)$ denotes the importance of non-electricity inputs; γ represents the importance of the reused final goods; and $\mu\in[0,\infty)$ is the elasticity of substitution between electricity and non-electricity inputs. This specification is adapted from that of Hassler et al. (2021), who argue that there is very limited ability to substitute other inputs for energy in the aggregate production function, suggesting a value of μ close to zero.

The production of electricity requires capital, K_E, labor L_E, and a primary energy input, denoted by F, and representing fuel inputs. Since there is no standard choice for an electricity-sector aggregate production function, for now we write the production function as:

$$E=A_E G(K_E,L_E,F) \tag{2}$$

where A_E is the TFP of the electricity sector and we assume that $G(\cdot)$ exhibits constant returns to scale (based on a replication argument) and diminishing marginal products for each factor of production. We will later consider alternative assumptions about sector returns to scale and, after examining the data more carefully, specify a specific functional form for the production function.

As in Greenwood, Hercowitz and Krusell (1997), we posit an investment-goods sector that can convert one unit of the final good into A_I units of capital. In the cross-country setting, Hsieh and Klenow (2007) find that the relative price of investment goods is higher in poor countries, which they interpret (as we do) as lower relative productivity in producing capital goods in poor countries. With this assumption, the law of motion for capital accumulation becomes

$$K_{t+1} = (1-\delta)K_t + A_I I_t \tag{3}$$

where K_t is the capital stock, δ is the depreciation rate of capital, and I_t is the amount of investment.

There is a representative household in the economy with preferences:

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{4}$$

where $\beta \in (0,1)$ is the discount factor, C_t denotes consumption, and $U(\cdot)$ is an increasing and concave utility function continuously differentiable on $(0,\infty)$. The household owns all capital in the economy, rents the capital to firms, and supplies one unit of labor inelastically.

All sectors in the economy are competitive and the representative producers earn zero profits in equilibrium. The electricity sector, in addition, faces a distortion as in Hsieh and Klenow (2009), where a fraction $\tau_E \in (0,1)$ of its revenue gets taxed away. This is a simple way to accommodate the empirical findings that electricity is often distorted in developing countries (see e.g. Burgess et al., 2020). In response, producers resort to self-generated power (Allcott et al., 2016), which costs substantially more than grid power (World Bank, 2007).

We assume that the final good is the numeraire and that primary energy inputs are imported from abroad at an exogenous international price P_F. Countries differ in their TFP terms – A_X, A_E, and A_I – and the distortion in their electricity sector – τ_E. We are interested in how the economy responds in the long run to an exogenous change in these productivity and distortions, particularly A_E.

2.2. Analytical Case

To guide intuition, we consider a particular case of the model that can be solved analytically. Here we describe the properties of the model; the derivations are presented in the Online Appendix.

We make three simplifying assumptions. First, capital is the only input into electricity production, so that $E = A_E K_E$, and all labor is supplied to the final-goods sector. Second, final goods are not used as intermediate inputs, so $\gamma = 0$. Third, the elasticity of substitution between the non-electricity inputs and electricity is zero, making the final goods production function Leontief:

$$Y = \min\left[\nu A_X K_X^{\theta_X}, E\right]. \tag{5}$$

These assumptions are not as restrictive as they might seem. In the next section, we provide evidence that the labor share in electricity production is quite low, and argue that the same is true for the elasticity of substitution between electricity and other inputs. The intermediate share is certainly not zero, but we show later that its value does not meaningfully alter our quantitative conclusions.

We are interested in how aggregate output responds in the long run to an increase in electricity-sector TFP, A_E. To build intuition, it is useful to first consider how aggregate output responds in the short run, where factors cannot reallocate. In this analytic case of the model, the answer is *not at all* (see Baqaee and Farhi, 2019; Baqaee and Rubbo, 2023). As factors cannot reallocate, the extra electricity does not increase output since it cannot substitute for other inputs at all.

In the long run, the economy may reallocate capital out of the electricity sector and accumulate new capital. We show that the long-run output increase arising from a change in A_E is:

$$\frac{d\ln Y}{d\ln A_E} = \frac{\theta_X}{1-\theta_X}\frac{1/\beta - 1 + \delta}{(1-\tau_E)A_E A_I - (1/\beta - 1 + \delta)}. \tag{6}$$

An increase in A_E leads to gains in GDP that are increasing in θ_X and τ_E and decreasing in A_E and A_I. That the gains increase in θ_X is a familiar result of neoclassical models. The effect of A_E, A_I, τ_E are not, to our knowledge, present in prior work, and capture the possibility that more capital may be reallocated or accumulated in response to an electricity productivity increase when A_E or A_I are low to begin with, or τ_E is high.

The intuition for this result is that when A_E is low, the economy has a large fraction of its capital stock allocated to the electricity sector. If A_E increases, the economy can reallocate some of this large capital stock into the non-electricity sector and productively accumulate more capital. When A_E is already high, further increases in A_E do not lead to much capital reallocation or accumulation and the aggregate gains are relatively lower.

Under what conditions does a positive shock to A_E increase GDP in this economy by more than Hulten's prediction, using the electricity sector's Domar weight?

We can write the Domar weight of the electricity sector in this economy as

$$\lambda_E \equiv \frac{P_E E}{Y} = \frac{P_E E}{E} = P_E = \frac{r}{(1-\tau_E)A_E}, \tag{7}$$

where r is the steady state return to capital: $r = (1/\beta - 1 + \delta)/A_I$. The elasticity in (6) is greater than the Domar weight of electricity when

$$(1-\tau_E)A_E < r\frac{1-\theta_X}{1-2\theta_X}. \tag{8}$$

This expression shows that the electricity sector can be a weak link if the electricity sector is sufficiently unproductive or distorted to begin with. Motivated by this result, we turn in the next section to the empirical question of how electricity-sector TFP varies between poor and rich economies.

3. Data and Cross-Country Facts

In this section, we summarize our data and findings on how TFP in the electricity sector varies across countries.

3.1. Data Sources

We compile data from various public and proprietary sources to construct a new cross-country data set on the electricity sector. We take a broad definition of the electricity sector that includes production, transmission, and distribution. Our data cover the year 2012, which was the most recent year that all data were available.

Electricity Capital Stocks. We collected data on electricity capital stocks from the World Electric Power Plants (WEPP) database, provided by S&P Global Platts. The database provides a global inventory of more than 200,000 power plants. We calculate aggregate capacity, measured in MW, and the average age of power plants that produce electricity in each country year.

One limitation of the WEPP data is that it does not cover small-scale power producers, which may be more common in lower-income countries. We therefore supplement the information from WEPP with data we constructed on the capacity of imported electricity generators. This exercise is inspired by Caselli and Wilson (2004) and is based on the assumption that the majority of small-scale electricity capital is imported rather than domestically produced.

We collect data on the quantity of AC generator imports for 147 countries from the UN Comtrade database – the largest repository of international trade data, containing over 3 billion records. The data subdivide AC generators by electrical capacity: less than 75kVa, 75-375kVa, 375-750kVa, and more than 750 kVa. To be consistent with our other capacity data we convert this data into MW of capacity using a standard conversion factor.

We calculate that, on average, generator capacity accounts for 7.6 percent of overall capacity. In the bottom quartile of the income distribution, generator capacity accounts for 6.7 percent of overall capacity on average. In the top quartile, generator capacity accounts for 5.2 percent.

In addition to generation capital, we also consider transmission and distribution capital. Data on the capacity of transmission transformers, distribution transformers, and generator step-up transformers was provided by Kalt et al. (2021). In practice, we find that transmission and distribution capital is almost perfectly correlated with generation capital (Figure A1). This suggests a Leontief production function between generating capacity and transmission capital:

$$K_E = \min[K_E^G, \psi K_E^{T\&D}],$$

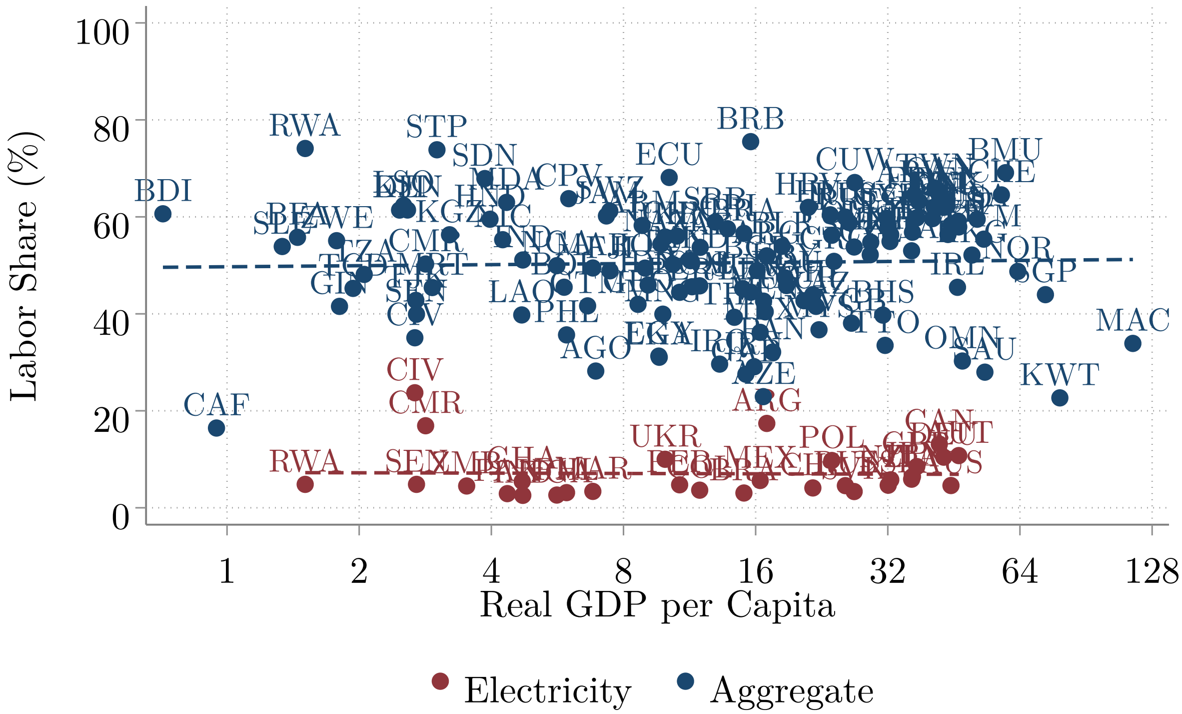
where we allow the parameter ψ to vary across countries in order to equate the two inputs. In practice, this assumption makes little difference for any of our conclusions, and we get nearly identical results assuming a constant value of ψ across countries.

Electricity Output and Fuel Inputs. We obtained data on physical electricity output and primary energy (fuel) inputs from the International Energy Agency (IEA). We collected data on electricity and combined heat and power plants from the energy balance tables of each country. We measure electricity output in megawatt hours (MWh), and primary energy inputs in millions of British Thermal Units.

In addition to the electricity output and inputs from power plants, we estimate the output and inputs related to self-produced electricity from generators. To do this, we calculate electricity output per MW of power plant capacity and fuel inputs per MW of power plant capacity for each country and scale this by self-generation capacity. Finally, we adjust the amount of electricity from power plants by subtracting transmission and distribution losses. Data on transmission and distribution losses is provided by the IEA and measured as a percent of total electricity output that is lost before reaching end users.

Electricity Employment. We compiled data on employment in the electricity sector from national statistics offices, and most often from statistical yearbooks or industrial censuses. In certain cases, we were only able to find employment in aggregated utilities, which include gas and water. This results in an overestimate of aggregate employment in the electricity sector for some countries. However, we find that our findings are qualitatively and quantitatively robust to restricting our sample to countries that explicitly report electricity employment, or to scaling utilities employment by the average electricity-utilities employment ratio (∼80 percent). We also scale employment to

Figure 1: Labor's Share of Revenue in the Electricity Sector



account for the labor required for self-generation. We calculate the number of workers per MW of power plant capacity in each country and scale this by self-generation capacity.

Labor's Share of Revenue. In addition to collecting data on output and different factors of production, we also measure the labor share of revenue (and value-added) in the electricity sector. To do this we collected and harmonized data from privately-owned electricity generation firms that we extracted from annual reports, financial statements, and SEC filings. This primary data collection effort resulted in usable information for a subset of 69 electricity companies in 31 countries. For each company, we compute total payments to labor, total sales, and total purchases of intermediates (fuel inputs). We compute labor's share of revenue as total payments to labor divided by total electricity sales. All monetary data was converted into thousands of USD. When values were not presented in US dollars, we converted the data using the average monthly exchange rate for the year of operation. To calculate national labor shares, we first collapsed the data across firm-years within a country, and then by country.

Figure 1 plots labor's share of revenue in electricity for the countries we were able to collect data for in red dots, and the aggregate labor shares from the PWT in blue dots. The average labor share in electricity is about 7 percent, with little variation throughout the income distribution. A bivariate regression of labor's share in electricity on real GDP per capita yields a slope coefficient that is very small and statistically insignificant from zero.

3.2. Electricity-Sector TFP

Measuring electricity-sector TFP at the country level requires taking a stand on the form of the production function in equation (2). We first assume that the production function features constant returns to scale based on the standard replication argument. Note that this is *not* the same as assuming that a particular electricity plant can scale up indefinitely without raising its costs. Constant returns at the industry level simply mean that as long as new producers can enter the electricity sector, growth in the size of an economy (and hence overall electricity needs) can be accommodated by the entry of new producers using the same technologies as the incumbents.

The roughly constant labor share of revenue in the electricity sector suggests a Cobb-Douglas specification between labor and other inputs. The other inputs are capital and fuel, which we assume are not substitutable (Christensen and Greene, 1976). Putting these assumptions together yields our aggregate production function for electricity:

$$E = A_E \left(\min\left[K_E, \chi F\right]\right)^{\theta_E} L_E^{1-\theta_E} \tag{9}$$

where χ represents the weight of fuel in the production function and θ_E denotes the importance of the capital-fuel bundle. We allow χ to vary across countries to capture the different intensities in using renewable energy to produce electricity. Intuitively, a country that relies more on renewable energy requires less fuel to generate electricity, which leads to a higher capital-fuel ratio for its electricity sector, hence a higher value of χ.

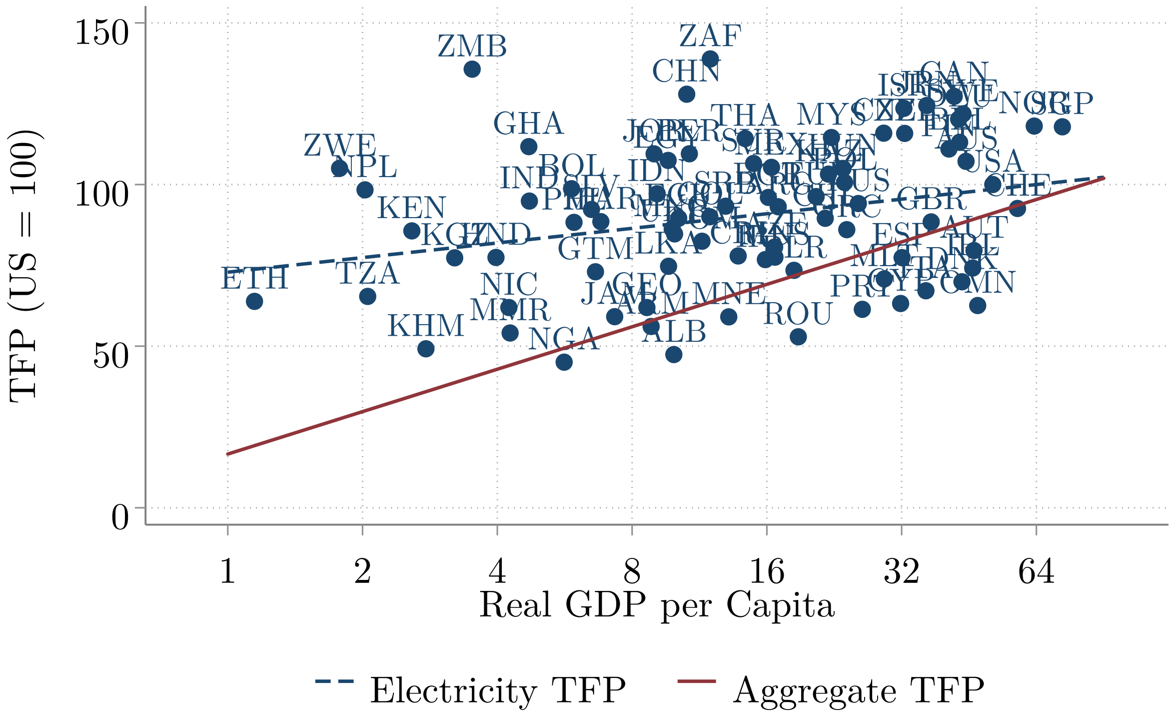
Using equation (9), we can measure TFP in the electricity sector in each country. As we take the electricity sector to be competitive, the exponent over labor is equal to the ratio of expenditures on labor to total revenue. The data in Figure 1 above suggest a value of 0.93 for θ_E.

Figure 2 shows that there is a positive, but very small, association between electricity TFP (where the U.S. is normalized to 100) and GDP per capita.[1] The poorest quartile of countries has 86 percent the TFP level in electricity as the richest one, putting them close to the world productivity frontier (Table A1) The electricity sector is thus a far cry from the manufacturing or agriculture sectors, for which there is at least as much, if not more, variation in productivity as GDP per capita (Restuccia, Yang and Zhu, 2008; Herrendorf, Rogerson and Valentinyi, 2022). The solid line in Figure 2 plots aggregate TFP according to the PWT as a frame of reference. The poorest quartile has 36 percent the aggregate TFP level as the richest quartile (Table A1).

Our TFP results are robust to various controls and alternative measurement assumptions (Table

[1] Figure A2 plots electricity labor productivity and capital productivity separately. Electricity labor productivity varies substantially with GDP per capita whereas electricity capital productivity is only slightly lower in less developed countries.

Figure 2: Electricity-Sector TFP Across Countries



A2). The TFP gaps in electricity are similar when we exclude distribution losses. Excluding own production of electricity yields a similarly small slope of electricity TFP on GDP per capita. Cross-country differences in the share of electricity coming from renewables do not drive our results. Finally, there needn't be constant returns to scale at the sector level (see Basu and Fernald, 1997), though plausible alternative assumptions do not change our conclusion that electricity TFP is similar in rich and poor countries. With aggregate decreasing returns to scale (of degree 0.9), the slope coefficient of electricity TFP is only slightly larger than in Figure 2. With aggregate increasing returns to scale (of degree 1.1) the slope coefficient becomes even smaller, and statistically insignificant. In Table A3 we document that the most meaningful predictor of cross-country variation in TFP is the share of transmission and distribution losses. Other factors, such as the age of the capital stock, renewable share, and ruggedness of the topography contribute little to explaining the variation in TFP that exists. After controlling for transmission and distribution losses the association between electricity TFP and real GDP per capita is no longer statistically significant.

4. Quantitative Analysis

In this section, we parameterize the model to match our cross-country evidence and simulate the quantitative effects of increasing TFP in the electricity sector.

4.1. Parameterization and Model Fit

We begin by assigning some parameters common values from the literature. We set the discount factor β to be 0.96 since each period represents one year. The depreciation rate is chosen to be 0.05. We assume a capital's share in the non-electricity stage of production, θ_X, of 0.4. For the intermediate goods share, we follow Jones (2011) and set $\gamma = 0.5$, which yields an empirically plausible ratio of gross output to GDP of two.

There is no standard value for the elasticity of substitution between electricity and other inputs in the final goods production function, μ. The arguments made by Hassler et al. (2021) and Casey (2023) suggest a low value, and Atalay (2017) points to low substitution elasticities for purchased intermediates more generally. In our benchmark calibration we choose a value of 0.2. We explore sensitivity to other choices later.

We use data from the PWT to calculate the productivity of the capital goods sector, A_I. Specifically, we assume that A_I^i equals the relative price of consumption goods to investment goods in country i in the PWT.

We take electricity TFP in country i, A_E^i, directly from our cross-country data. The exogenous international price for primary energy, P_F, is calibrated to 0.0216 to match the world average revenue share of fuel in electricity production.

We assume that τ_E decreases linearly with the log of GDP per capita across countries and that $\tau_E = 0$ for the country with the highest GDP per capita in our sample. Specifically, the distortion follows

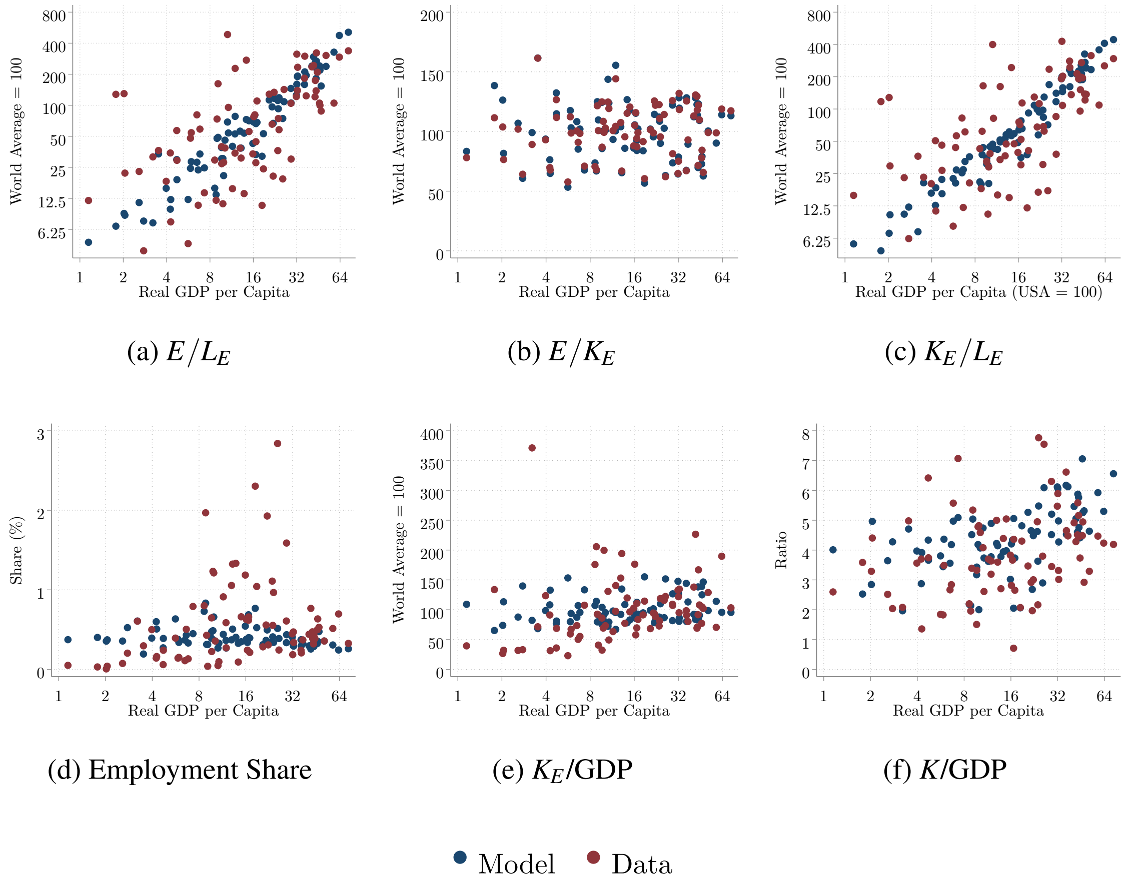
$$\tau_E^i = 0.25 - 0.25 \frac{\ln GDPpc^i}{\ln \overline{GDPpc}} \tag{10}$$

where $\overline{GDPpc}$ is the GDP per capita of the richest country in our sample. The intercept of 0.25 is consistent with a cost of self-generation that is four times as high as grid electricity in the world's poorest countries, with final goods producers using self-generated electricity around one-third of the time (World Bank, 2007). We then calibrate α, the weight on non-electricity input, to the U.S. employment share in the electricity sector so that $\alpha = 1 - 4 \times 10^{-7}$.

Finally, we calibrate A_X for each country to match GDP per capita in constant international prices from the PWT.

Figure 3 plots the model's predictions for the full cross-section of countries and the corresponding values in the data. Panels (a) and (b) show that, in both the model and data, labor productivity (E/L_E) is sharply increasing in GDP per capita, whereas electricity capital productivity (E/K_E) increases only slightly. The increase in labor productivity is due in large part to the significant

Figure 3: Model Fit to Cross-Country Data



(a) E/L_E (b) E/K_E (c) K_E/L_E

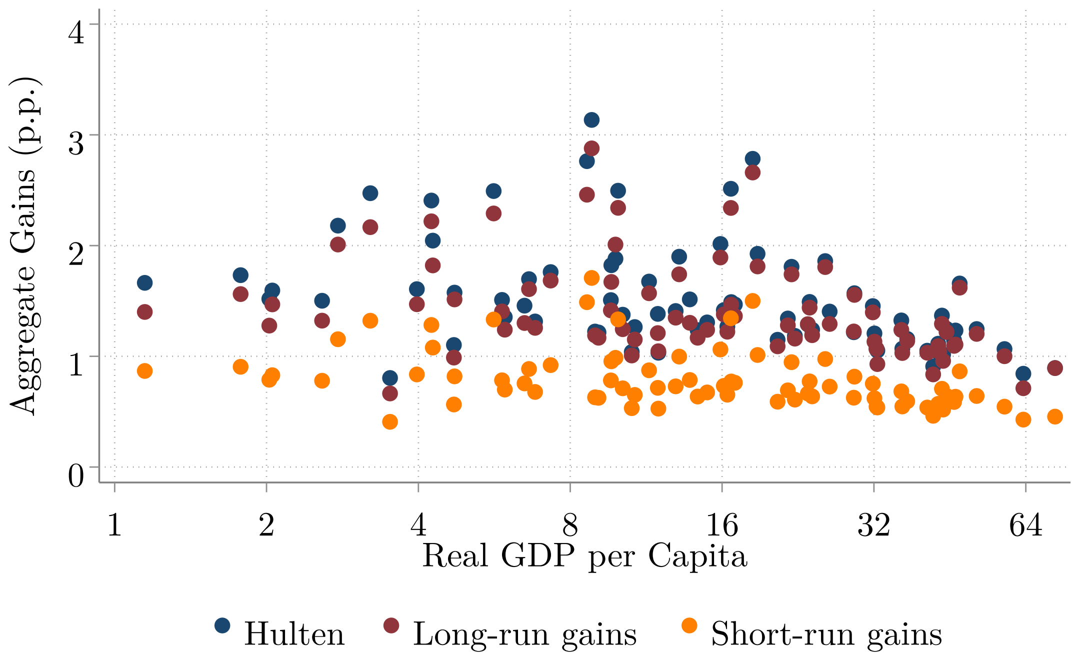
(d) Employment Share (e) K_E/GDP (f) K/GDP

increase in electricity capital per worker with GDP per capita (shown in panel (c)). The modest increase in capital productivity is a result of the modest increase in electricity TFP. Given that capital's share is so close to one in electricity, electricity TFP and capital productivity mechanically have similar behavior in the model.

Panel (d) shows that the model's predictions for electricity employment share are largely consistent with the data. Both are low overall and increasing modestly in income per capita. The model does not replicate the very high electricity employment shares for five former Soviet nations (though data for these countries cover the utilities sector, rather than just the electricity sector). Panels (e) and (f) show that the model does a reasonable job of matching the electricity capital to GDP ratios and aggregate capital to GDP ratios. Both are increasing somewhat in income per capita, and aggregate capital stocks range from an average of around three times GDP in the poorest countries to around six times GDP in the richest.

Another statistic of interest is the price of electricity across countries. Comparisons are more challenging here since there are so few electricity price observations for the poorest countries in

Figure 4: GDP Gains from Raising Electricity TFP by 50 Percent



the International Comparison Program's (ICP) disaggregated price data. Our model predicts that countries in the lowest quartile of the income distribution have electricity prices that are 1.7 times as high as countries in the richest quartile. The ratio is 2.4 in the data from the ICP's 2011 round (see Figure A3), with a wide confidence interval, and we cannot reject that the slopes are the same in the model and data.

Overall, despite its simplicity, the model does a reasonable job of matching the relevant cross-country statistics. This lends some confidence that its predictions for the aggregate impacts of significant increases in electricity TFP may also be reasonable.

4.2. Is Electricity a Weak-Link Sector?

We use the calibrated model to explore the effects of a large increase in electricity-sector TFP. For each country in our sample, we increase the TFP of the electricity sector by 50 percent and compute the new steady-state of the economy.

We plot three series in Figure 4. The dark blue series denotes for each country what the aggregate gains are as predicted by Hulten's theorem. Strictly speaking, Hulten's approximation is for short-run changes, though it is still a useful frame of reference in our setting. We observe that Hulten's predictions have a modestly decreasing trend in GDP per capita, indicating that poorer countries have larger Domar weights for the electricity sector and should thus see larger aggregate gains.

The orange series represents the "short-run" gains from the shocks. These are the immediate response of the transitional dynamics when the electricity TFP shock takes place unexpectedly after all factors of production are allocated and only the reused final goods, Z, are allowed to adjust. It is evident from the figure that the short-run aggregate gains in all countries are smaller than Hulten's prediction, consistent with the work of Baqaee and Farhi (2019).

The aggregate gains in the long run, denoted by the red dots, are substantially larger than those in the short run. However, the long-run gains from the 50 percent increase in electricity TFP still fall below Hulten's predictions, as reflected in Figure 4. This means that the electricity sector is not a weak link even with the extra channel of capital accumulation; the aggregate capital stock is only 0.5 percent higher on average in the new steady state.

The finding that electricity is not a weak link is robust to different values of γ and μ. Donovan (2021) documents that the intermediate share is as high as 0.7 in some countries, and using $\gamma = 0.7$ hardly alters our conclusions. Lower values of μ, which are used in Hassler et al. (2021) and Casey (2023), make electricity even less of a weak link.

To illustrate why electricity is not a weak link sector in our main exercise, it is useful to consider some cases in which it *is* a weak link. Table 1 summarizes these cases, which focus on countries in the lowest income quartile in our sample. We find that the electricity sector may be a weak link when it is initially much less productive, when the productivity of turning final goods into capital goods is much lower, or when the electricity sector is much more severely distorted. See Panel A of Table 1. For instance, when A_E is initially set to just 1 percent of its original value, then a 50 percent increase in A_E raises GDP by a robust 268.1 percent (compared to 40.5 percent suggested by Hulten's approximation). Yet the initial Domar weight is implausibly large, at 83.7 percent. Counterfactually high initial Domar weights are also present when initial A_I is set to be only 1 percent of its initial value and τ_E is set to be 90 percent in the poorest countries.

Another way to improve the electricity sector is to remove the distortions it faces. It is possible that these collective distortions, rather than electricity productivity per se, are the problem. We show in Panel B of Table 1 that our model also predicts small long-run gains from reducing distortions in electricity. The first row under Panel B shows that removing all distortions in our model only leads to an average of 0.6 percent long-run gains for the poorest countries. The bottom row in Panel B shows that if τ_E were 0.9 to begin with, then removing the distortion would raise GDP by 25.8 percent. The Domar weight would again be unrealistically high, however.

In Panel C of Table 1, we present a case where the electricity sector becomes a weak link without a counterfactually high Domar weight. In this exercise, we assume that producers plan and hire factors of production with the benchmark values of the parameters. The electricity sector is still

Table 1: How *Could* the Electricity Sector Be a Weak Link?

	Model Gains (%)	Hulten Gains (%)	Model / Hulten (%)	Domar Weight (%)
A. Gains from 50% rise in A_E				
Benchmark	1.5	1.7	91	4.1
A_E 1% of original value	268.1	40.5	622	83.7
A_I 1% of original value	76.6	27.5	263	59.7
$\tau_E = 0.9$	8.4	8.4	98	20.0
B. Gains from removing frictions				
Changing baseline τ_E's to 0	0.6	/	/	4.1
Changing τ_E from 0.9 to 0	25.8	/	/	20.0
C. Gains from fixing "unpredictable outages"				
Restoring 33%-loss due to outages	3.5	/	/	2.5
Restoring 50%-loss due to outages	12.9	/	/	2.0

distorted, but not in the sense that there is a wedge between the user price and the seller price. Instead, we model distortions such that a fraction of produced electricity is lost unexpectedly, never delivered, and cannot be recovered with any means. This exercise has a similar spirit to the model of Fried and Lagakos (2023), where electricity gets rationed in equilibrium, and producers use other inputs less efficiently as a result.

We then restore the loss in electricity production and compute the gains in aggregate output. The first row under Panel C shows the case where a 33 percent loss is restored. This scenario is comparable in some ways to the 50 percent gains in electricity TFP in our main quantitative exercise, since both entail 50 percent more electricity capacity than before. We see that the gains in the steady state generated by the model are more than twice as large as what Hulten's theorem predicts for an increase in the TFP of electricity in our benchmark exercise. The gains become even greater (especially relative to the Domar weight) when we fix an electricity sector that loses half of its output. In the long run, GDP can rise by 12.9 percent with an initial Domar weight of 2 percent.

The electricity sector becomes a weak link in this case because the loss of power makes the input mix inefficient in the production of final goods. Since final-goods producers hire factors of production based on competitive prices and cannot insure against the loss of power, they always hire too much labor and capital, which are then idled. The contrast between Panel C of Table 1 and our benchmark results shown in Figure 4 shows one possibility through which the electricity sector could be a weak link in development. Yet it remains unanswered, just as in the model of Fried and Lagakos (2023), why producers would idle resources rather than generating their own power.

5. Conclusion

This paper asks whether the electricity sector is a weak link in the development process. We answer this question in a dynamic multi-sector model in which electricity is a strong complement to capital and labor inputs in production. The model captures the idea that almost every type of capital equipment relies on electricity to operate, and that it is hard to find good substitutes for electricity in the production process. As such, if poor countries are not efficient at producing electricity, this could limit capital accumulation and stall development.

Using new cross-country evidence, we show that electricity-sector TFP in low-income countries is not too far behind that of advanced countries. When calibrated to match this evidence, our model implies that electricity is not a weak link. Substantial increases in electricity TFP do not translate to larger long-run GDP gains than suggested by Hulten's approximation.

The lesson of this paper is not that the electricity sector cannot be a weak link. Instead, the implication is that richer mechanisms are needed for it to be a weak link. Unreliable supply that leads to idle resources is one promising channel, but it is unclear why producers do not anticipate the under-supply of electricity and hold back on other inputs, or self-insure against outages using generators. Another possibility is that improper electricity supply damages capital equipment or output directly during the production process. Future research could fruitfully pursue these and other channels.

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Online Appendix

Contents

A. Additional Tables and Figures

Table A1: Electricity Productivity Summary Statistics

	Average			
	Labor Productivity (KWh / worker)			Real GDP Per Worker
	(1)	(2)	(3)	(4)
Bottom Quartile	1,354	1,038	1,065	6,270
Second Quartile	2,048	1,868	1,884	20,426
Third Quartile	1,850	1,655	1,665	44,000
Top Quartile	5,026	4,715	4,731	95,586
Top/Bottom	3.71	4.54	4.44	15.24
	Capital Productivity (Hours Per Year)			Real GDP / K
	(1)	(2)	(3)	(4)
Bottom Quartile	3,813	3,422	3,433	0.17
Second Quartile	4,208	3,797	3,823	0.24
Third Quartile	3,998	3,463	3,512	0.31
Top Quartile	3,784	3,236	3,265	0.53
Top/Bottom	0.99	0.95	0.95	3.12
	Electricity TFP (US = 100)			Aggregate TFP
	(1)	(2)	(3)	(4)
Bottom Quartile	96.92	86.62	87.65	36.03
Second Quartile	99.66	95.35	96.23	56.89
Third Quartile	85.51	83.73	83.95	72.35
Top Quartile	102.77	101.85	102.32	98.54
Top/Bottom	1.06	1.17	1.16	2.73
Losses	No	Yes	Yes	–
Self-Generation	No	No	Yes	–

Notes: This table reports summary statistics of productivity in the electricity sectors of the countries in our data. The first panel covers labor productivity, measured in megawatt hours per worker per year. The second panel covers capital productivity, measured in hours per year. The third panel covers TFP, where the value for the US is normalized to be 100. Column 1 reports measures that don't account for transmission & distribution losses or small-scale capital. Column 2 reports measures that adjust for transmission & distribution losses. Column 3 reports measures that adjust for both transmission & distribution losses and small-scale capital. Column 4 reports aggregate economy measures for comparison.

Table A2: Alternate Regressions and Measures of Electricity TFP

	Electricity TFP (US = 100)				
	(1)	(2)	(3)	(4)	(5)
$\log_2$ GDP per capita	4.52**	6.63***	-2.07	4.83***	1.37
	(1.80)	(1.39)	(3.18)	(1.79)	(1.85)
Bottom Quartile	79.44	42.47	148.92	77.51	92.44
Top Quartile	98.34	70.27	140.60	98.12	97.49
Top/Bottom	1.24	1.65	0.94	1.265	1.05
Electricity Sector Returns to Scale	$\nu = 1.0$	$\nu = 0.9$	$\nu = 1.1$	$\nu = 1.0$	$\nu = 1.0$
Includes Small-scale Capital	Yes	Yes	Yes	No	Yes
Adjusted for Transmission Losses	Yes	Yes	Yes	Yes	No
Observations	80	80	80	80	80

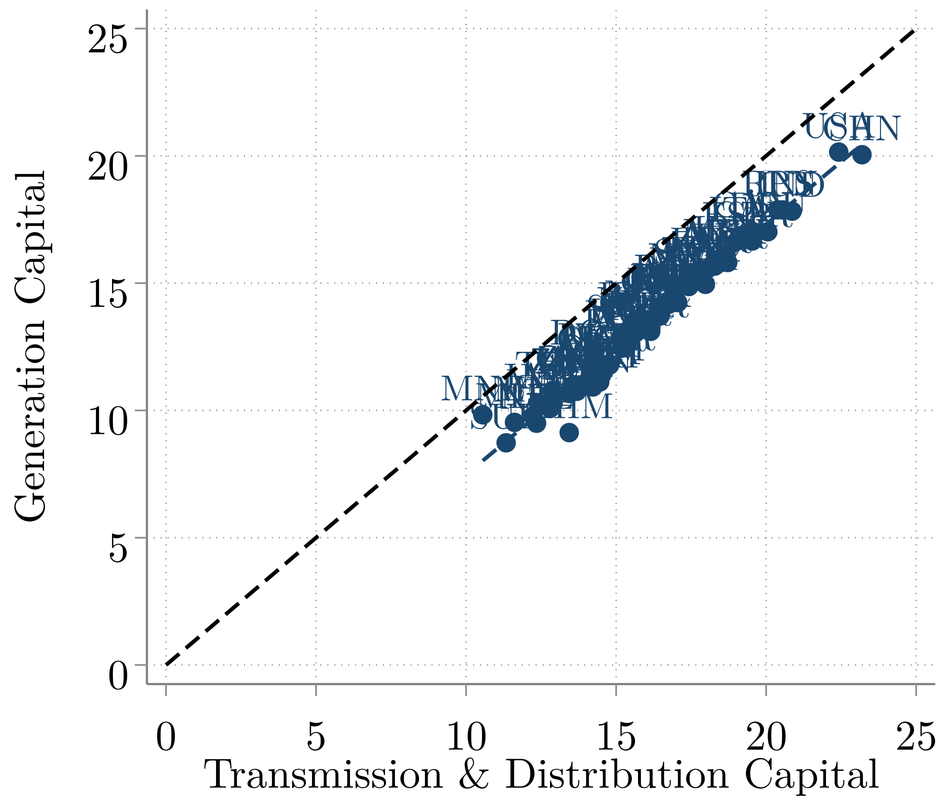
Notes: This table reports how electricity sector TFP varies with GDP per capita under alternative measures of electricity TFP. The first column reproduces the coefficient reported in the main body of the paper, from a bivariate regression of electricity sector TFP on GDP per capita in log base 2. The second and third columns re-compute TFP assuming that the electricity sector exhibits decreasing returns to scale ($\nu = 0.9$) or increasing returns to scale ($\nu = 1.1$). The fourth column removes the adjustment we made to include small-scale electricity capital, e.g., generators. Column 5 removes the adjustment we made to remove transmission losses from output. Significance levels are indicated as * 0.10 ** 0.05 *** 0.01.

Table A3: Correlates of Electricity TFP

	Electricity TFP (US = 100)						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\log_2$ GDP per capita	4.50**	-3.67	-3.39	-2.41	-2.52	-2.38	-2.38
	(1.80)	(2.35)	(2.41)	(2.37)	(2.40)	(2.43)	(2.46)
T&D Losses (%)		-2.34***	-2.21***	-2.18***	-2.12***	-2.11***	-2.04***
		(0.58)	(0.53)	(0.58)	(0.58)	(0.57)	(0.57)
Ruggedness (%)			-0.12	-0.22	-0.23	-0.20	-0.20
			(0.12)	(0.16)	(0.16)	(0.16)	(0.16)
Hydro Share (%)				0.16	0.12	0.08	0.07
				(0.11)	(0.12)	(0.13)	(0.13)
Capital Age					0.26	0.20	0.16
					(0.38)	(0.37)	(0.37)
Small Island						-11.70	-12.92*
						(7.50)	(7.60)
OPEC							-22.93
							(15.63)
Observations	78	78	78	78	78	78	78
Adjusted R^2	0.07	0.26	0.26	0.27	0.27	0.28	0.29

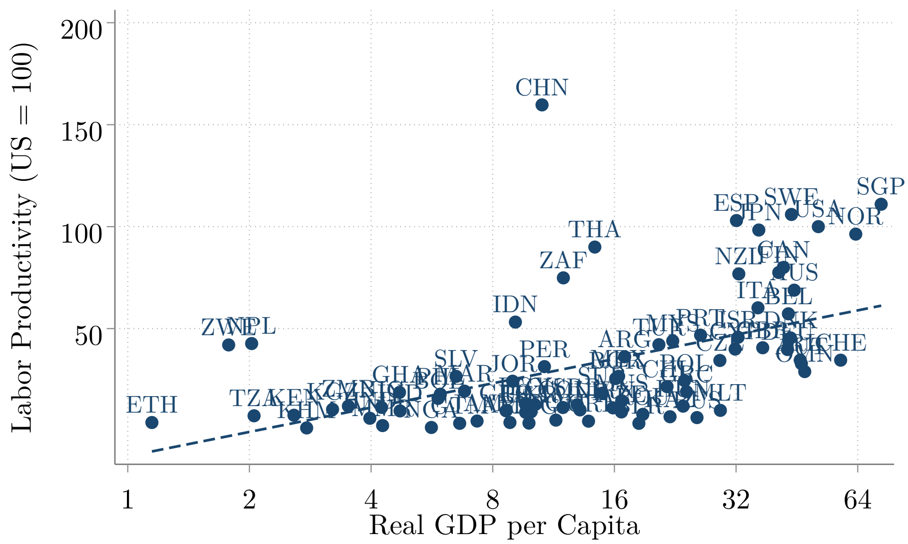
Notes: This table presents the correlates of electricity sector TFP. The first column presents the bivariate correlation between electricity sector TFP and GDP per capita in log base 2, restricting the sample to the countries that have no missing values for any of our covariates. Column 2 adds the additional covariate that we estimate is associated with electricity sector TFP – the share of electricity output that is lost during transmission and distribution. We see that the relationship between electricity sector TFP and GDP per capita is completely mediated when we control for transmission and distribution losses. Electricity sector TFP and transmission and distribution losses are negatively correlated and transmission and distribution losses are negatively correlated with GDP per capita. These inferences are not affected by the inclusion of additional controls. Additional controls are added sequentially and are: the ruggedness of the land, the share of electricity capacity that is hydro, the average age of the electricity capital stock, whether the country is a small island, or OPEC member. These inferences hold when including continent fixed effects (column 5). Robust standard errors are reported in parentheses. Significance levels are indicated as * 0.10 ** 0.05 *** 0.01.

Figure A1: Electricity Generation Capital vs. Electricity Transmission and Distribution Capital

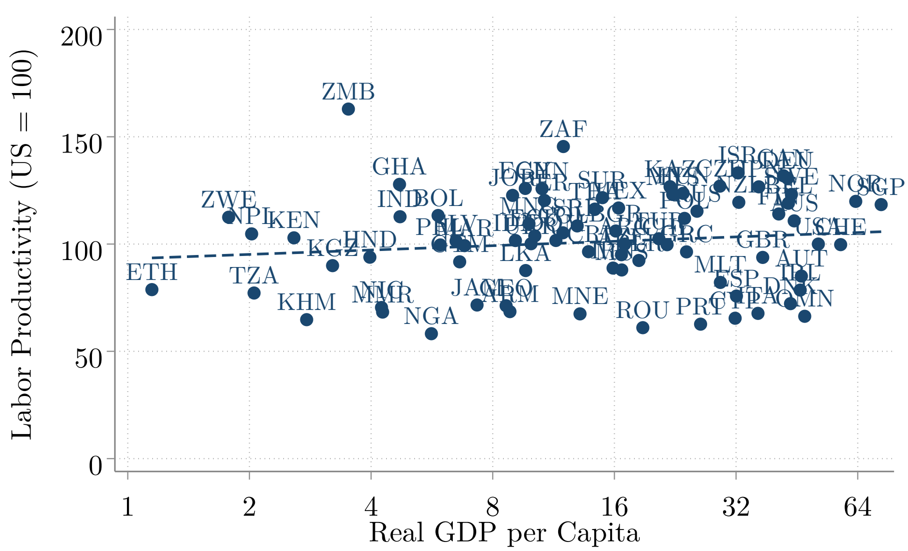


Notes: This figure reports the bivariate relationship between the capacity (in MW) of generation capital and the capacity (in MW) of transmission & distribution capital in the electricity sector. Both variables are transformed on a $\log_2$ scale. We observe a very strong relationship between the two measures of capital. In all cases countries have more transmission & distribution capacity than generation capacity. This is reasonable as electricity output is constrained by the ability to transmit and distribute what is being generated.

Figure A2: Electricity-Sector Labor and Capital Productivity Across Countries



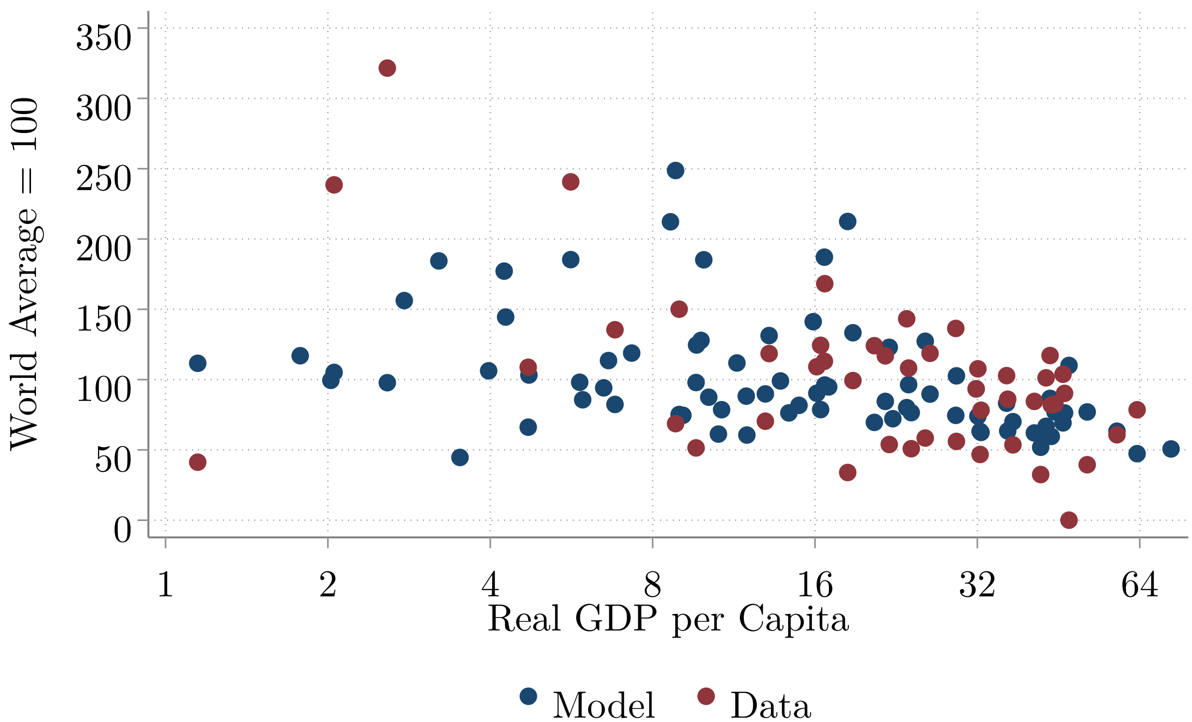
(a) Labor Productivity



(b) Capital Productivity

Notes: These figures document the relationship between electricity sector labor productivity and real GDP per capita (Panel (a)) and electricity sector capital productivity and real GDP per capita (Panel (b)). Data on real GDP per capita are taken from the PWT version 9.1. Electricity sector labor productivity and capital productivity are calculated from the data described in section 3.1. We estimate a strong negative association between electricity sector labor productivity and real GDP per capita. By contrast, we estimate a much weaker relationship between capital productivity and real GDP per capita.

Figure A3: Prices of Electricity



Notes: This figure shows how the model's predictions for prices vary with GDP per capita. Data on real GDP per capita is taken from the PWT version 9.1. Data on electricity prices is from the International Comparison Program's 2011 round. The model's prediction is for after-tax prices.

B. Model Appendix

In this section, we lay out the full model in detail and the optimality conditions and solve for the steady state. We take A_X, A_E, A_I, and τ_E as constants.

2.1. Environment

Final goods are produced with the technology in (1):

$$Y = \left[\alpha \left(A_X Z^{\gamma} (K_X^{\theta_X} L_X^{1-\theta_X})^{1-\gamma} \right)^{\frac{\mu-1}{\mu}} + (1-\alpha) E^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$

as described in section 2. In the production function, K_X, L_X, Z, and E denote the amount of capital, labor, reused final goods, and electricity put into the production process. The economy has an exogenous productivity A_X in producing the final goods. The elasticity of substitution between electricity and other inputs is denoted by μ. The parameter α governs how important non-electricity inputs are relative to electricity. The parameter γ determines the extent to which final goods are reused in production and θ_X is the importance of capital in value-added relative to labor.

Electricity is produced with capital K_E, labor L_E, and imported fuel F with the technology (9):

$$E = A_E \left(\min[K_E, \chi F] \right)^{\theta_E} L_E^{1-\theta_E}$$

as we infer from our data. The TFP of the sector is denoted by A_E and χ governs how intensively the electricity sector uses fuel to couple with each unit of capital. The parameter θ_E pins down the relative importance of the capital-fuel bundle to labor.

A final sector in this economy turns final goods into capital with productivity A_I. This means that capital accumulation in this model follows (3):

$$K_{t+1} = (1-\delta)K_t + A_I I_t.$$

We now turn to characterize the producer and household problems. The final goods producer faces the problem

$$\max Y - Z - rK_X - wL_X - P_E E \tag{11}$$

each period. We take the final goods as numeraire so the prices of both Y and Z are 1.

The electricity sector is distorted in the way that a portion of its revenue gets taxed away. Specifically, the problem is

$$\max (1-\tau_E) P_E E - rK_E - wL_E - P_F F \tag{12}$$

where τ_E is the tax rate and P_F is the exogenous international price of fuel. We assume that a fraction of the distortion may be rebated to the household in a lump-sum fashion. This transfer is denoted by

$$T = \zeta \tau_E P_E E \tag{13}$$

where $\zeta \in [0,1]$.

The representative household solves

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{14}$$

by choosing consumption and investment for each period, subject to the budget constraint

$$r_t K_t + w_t + T_t \geq C_t + I_t \tag{15}$$

and the law of motion (3).

2.2. Equilibrium Conditions

Substituting the law of motion (3) into the budget constraint (15) of the households, the Lagrangian of the household problem is

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \left(\beta^t U(C_t) + M_t \left[w_t + r_t K_t + T_t - C_t - (K_{t+1} - (1-\delta)K_t)/A_I \right] \right) \tag{16}$$

where M_t is the Lagrangian multiplier of each period. The first-order conditions lead to

$$M_t = \beta^t U'(C_t) \tag{17}$$

and

$$\frac{1}{\beta} \frac{U'(C_t)}{U'(C_{t+1})} = A_I r_{t+1} + 1 - \delta. \tag{18}$$

The representative firm producing the final goods, Y, solves the problem

$$\max_{K_{X,t}, L_{X,t}, Z_t, E_t} \left[\alpha \left(A_X Z_t^{\gamma} \left[K_{X,t}^{\theta_X} L_{X,t}^{1-\theta_X} \right]^{1-\gamma} \right)^{\frac{\mu-1}{\mu}} + (1-\alpha) E_t^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}$$
$$r_t K_{X,t} - w_t L_{X,t} - Z_t - P_{E,t} E_t \tag{19}$$

The first-order conditions imply

$$\alpha(1-\gamma)\theta_X \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\mu}} \frac{X_t}{K_{X,t}} = r_t \tag{20}$$

$$\alpha(1-\gamma)(1-\theta_X) \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\mu}} \frac{X_t}{L_{X,t}} = w_t \tag{21}$$

$$\alpha\gamma \left(\frac{Y_t}{X_t} \right)^{\frac{1}{\mu}} \frac{X_t}{Z_t} = 1 \tag{22}$$

$$(1-\alpha)\left(\frac{Y_t}{E_t}\right)^{\frac{1}{\mu}} = P_{E,t} \tag{23}$$

where X_t is shorthand for $A_X Z_t^{\gamma}(K_{X,t}^{\theta_X} L_{X,t}^{1-\theta_X})^{1-\gamma}$.

The electricity producer solves the problem

$$\max_{K_{E,t},L_{E,t},F_t} (1-\tau_E)P_{E,t}A_{E,t}\min[K_{E,t},\chi F_t]^{\theta_E} L_{E,t}^{1-\theta_E} - r_t K_{E,t} - P_F F_t - w_t L_{E,t} \tag{24}$$

and the optimality conditions require that

$$K_{E,t} = \chi F_t \tag{25}$$

$$(1-\theta_E)(r_t + P_F/\chi)K_{E,t} = \theta_E w_t L_{E,t} \tag{26}$$

$$(1-\theta_E)(1-\tau_E)P_{E,t}E_t = w_t L_{E,t} \tag{27}$$

The final goods market clears when the demand is equal to the supply. Final goods may be used for consumption, investment, intermediate goods in the production process, or exports for trade balance against the imported fuel. Mathematically,

$$Y = C + I + Z + P_F F. \tag{28}$$

2.3. Steady State

In the steady state, the household optimality conditions are reduced to

$$r^* = \frac{1/\beta - 1 + \delta}{A_I} \tag{29}$$

$$I^* = \frac{\delta K^*}{A_I} \tag{30}$$

Using the optimality conditions, we can express all quantities in terms of the amount of labor used in production:

$$Z^* = L_X^* w^* \frac{\gamma}{(1-\gamma)(1-\theta_X)} \tag{31}$$

$$K_X^* = L_X^* \frac{w^*}{r_*} \frac{\theta_X}{1-\theta_X} \tag{32}$$

$$K_E^* = L_E^* \frac{w^*}{r^* + P_F/\chi} \frac{\theta_E}{1-\theta_E}. \tag{33}$$

Therefore,

$$E^* = A_E L_E^* \left(\frac{1}{1-\tau_E} \frac{\theta_E}{1-\theta_E} \frac{w^*}{r^* + P_F/\chi}\right)^{\theta_E} \tag{34}$$

and

$$X^* = A_X L_X^* \left(w^* \frac{\gamma}{(1-\gamma)(1-\theta_X)} \right)^{\gamma} \left(\frac{w^*}{r^*} \frac{\theta_X}{1-\theta_X} \right)^{\theta_X(1-\gamma)} \tag{35}$$

In addition, the first-order condition of the non-electricity goods producer also implies

$$X^* = E^* \left(\frac{\alpha}{1-\alpha} \frac{P_E^*}{P_X^*} \right)^{\mu} \tag{36}$$

where P_X denotes the shadow price of X, which is the non-electricity goods producer's costs of non-electricity inputs.

In equilibrium, the three equations above give us the allocation of resources with the labor market clearing condition $L_E^* + L_X^* = 1$, given the steady state prices w^*, r^*, P_E^*, and P_X^*.

We now show how we solve for the wage rate and the prices quantitatively. In any period, the following conditions must be satisfied for the firm's optimality conditions and free entry conditions to hold:

$$P_{Y,t} = \left(\alpha^{\mu} P_{X,t}^{1-\mu} + (1-\alpha)^{\mu} P_{E,t}^{1-\mu} \right)^{\frac{1}{1-\mu}} \tag{37}$$

$$P_{X,t} = \frac{1}{A_X} \left(\frac{P_{Y,t}}{\gamma} \right)^{\gamma} \left(\frac{r_t}{(1-\gamma)\theta_X} \right)^{(1-\gamma)\theta_X} \left(\frac{w_t}{(1-\gamma)(1-\theta_X)} \right)^{(1-\gamma)(1-\theta_X)} \tag{38}$$

$$P_{E,t} = \frac{1}{1-\tau_E} \frac{1}{A_E} \left(\frac{r_t + P_F/\chi}{\theta_E} \right)^{\theta_E} \left(\frac{w_t}{1-\theta_E} \right)^{1-\theta_E} \tag{39}$$

We observe that $P_{X,t}$ and $P_{E,t}$ are both functions increasing in w_t and r_t. Since P_X and P_E are both strictly increasing in w, P_Y is normalized, and the steady state rental rates r^* are pinned down by exogenous parameters, there must exist a unique value of w^* so that all the above conditions hold.