ARTICLE

QUANTUM PATENTS

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ABSTRACT

Quantum Patents are patents with claims relating to quantum computing. The number of Quantum Patents granted by the USPTO is rapidly increasing each year. In addition, Quantum Patents are increasing in total market value. While the literature on technology patents is visibly scaling, literature specifically focused on Quantum Patents is non-existent. This Article draws on a growing body of quantum computing, intellectual property, and technology law scholarship to provide novel Quantum Patent analysis and critique.

This Article contributes the first empirical Quantum Patent review, including novel technology descriptions, market modeling, and legal analysis relating to Quantum Patent claims. First, this Article discusses the two main technical approaches to Quantum Computing. This discussion explores the relationship between Adiabatic Quantum Computers and Gate-Model Quantum Computers, as well as various quantum software frameworks. Second, this Article models an evolving Quantum Patent dataset, offering economic insights, claims analysis, and patent valuation strategies. The data models provide insight into an uncharted patent market alcove, shining light on a completely new economy.

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I. INTRODUCTION

As society approaches the twilight of technological tradition,[2] progress appears to be slowing. We find ourselves 2,500 years in the future, asking the same questions the ancients asked, stilled puzzled by the same phenomena, and perplexed by paradox. The nature of space and time gives rise to many paradoxes. Several early paradoxes were authored by Zeno of Elea, who wrote a book on paradox around the year 490 B.C.[3] Zeno's story starts with the ordinary notion

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[2] *See* MARK TWAIN, THE ADVENTURES OF TOM SAWYER 254 (1876) ("[T]he twilight of tradition . . . .").

[3] John Palmer, *Zeno of Elia*, STANFORD ENCYCLOPEDIA OF PHILOSOPHY (database updated Jan. 2017), https://plato.stanford.edu/entries/zeno-elea [https://perma.cc/M6K2-MQ6L] (Plato's work estimates Zeno was born around 490 BC. A pupil of Parmenides, the father of metaphysics, Zeno wrote a book of paradoxes defending Parmenides philosophy. Today, most of what is known of Zeno comes from Aristotle, who unsuccessfully attempted to refute Zeno's arguments against motion).

of motion, an arrow leaving a bow, then shows how contradiction results from motion's logical consequence.[4] Thus, Zeno's Dichotomy Paradox asserts the non-existence of motion, drawing on the principle of infinite divisibility.[5] Zeno's Dichotomy Paradox remained unsolved until Isaac Newton developed his Laws of Motion and Gravity in the year 1687.[6] The Principle of Relativity upended Newton's Laws when Albert Einstein revealed his vision of the world in 1905, forming the foundation for modern conceptions of the physical universe.[7]

Contemporaneous to Einstein, but just before the turn of the twentieth-century, Thomas Edison and Nikola Tesla were finding ways to harness electric power by manipulating the movement of electrons across conductive wire.[8] Edison and Tesla were inventing new ways to generate and transmit electrical power for what became the Edison Electric empire. Interestingly, quantum mechanical uncertainties did not seem to stifle electric generation, nor transmission in commercial settings. As a result, it was not until a half-century later that these ideas would collide with quantum physics.[9]

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[4] The arrow paradox conjures questions relating to fundamental assumptions about motion. Soaring through the air, the arrow approaches the half-way point to the target. Before the arrow can reach the half-way point, the arrow must move half the distance to that point. Applying the intuitive notion of infinite divisibility, the arrow needs to pass through an infinite number of half-way points to reach the half-way point. Passing through an infinite amount of points takes, theoretically, an infinite amount of time. Thus, contrary to conventional wisdom, it is impossible for the arrow to reach the bullseye because the arrow must traverse an infinite number of points in a finite amount of time before the arrow can reach the bullseye. So, our intuition about motion is a lie and the arrow never hits the target. *See* PAUL E. CERUZZI, COMPUTING: A CONCISE HISTORY ix (2012).

[5] Nick Huggett, *Zeno's Paradoxes*, STANFORD ENCYCLOPEDIA OF PHILOSOPHY (database updated June 2018), https://plato.stanford.edu/entries/paradox-zeno/ [https://perma.cc/S6L8-PR4Z].

[6] ROGER R. BATE, DONALD D. MUELLER, JERRY E. WHITE, FUNDAMENTALS OF ASTRODYNAMICS 51 (1971).

[7] 2 ALBERT EINSTEIN, ON THE ELECTRODYNAMICS OF MOVING BODIES (1905), *reprinted in* THE COLLECTED PAPERS OF ALBERT EINSTEIN THE SWISS YEARS: WRITINGS, 1900-1909 140 (Anna Beck trans.) (1989) ("We shall raise this conjecture (whose content will be called 'the principle of relativity' hereafter) to the status of a postulate and shall introduce, in addition, the postulate, only seemingly incompatible with the former one, that in empty space, light is always propagated with a definite velocity *V* which is independent of the state of motion of the emitting body.").

[8] Apparatus for The Transmission of Electrical Power, U.S. Patent No. 265,786 (filed Aug. 7, 1882) (assigned to Edison); *see also* Electric Lamp, U.S. Patent No. 223,898 (filed Nov. 4, 1879) (assigned to Edison); *see also* Method of Converting and Distributing Electric Currents, U.S. Patent No. 382,282 (filed Dec. 23, 1887 (assigned to Tesla); *see also* Pyromagnetic Electric Generator, U.S. Patent No. 428,057 (filed May 26, 1887) (assigned to Tesla).

[9] Memory System for a Multi-Chip Digital Computer, U.S. Patent No. 3,821,715 (filed Jan. 22, 1973) (assigned to Hoff, Jr. et al.).

Richard Feynman laid the groundwork for the study of quantum physics by analyzing the differential equations of Schrödinger and the matrix algebra of Heisenberg.[10] Conceptually, Feynman was the first to discuss the intuition behind quantum computers,[11] specifically to evolve computers from binary logic to a higher-order logic using quantum mechanical[12] properties like superposition.[13] As Stephen Hawking later argued, a weakness in Einstein's theory is that, although it furnishes field equations, it does not provide boundary conditions for them.[14] Feynman's greatest idea was to exploit this weakness to improve computational systems.[15]

It was another half-century before Feynman's idea would be realized. In 2006, D-Wave Systems Inc. (D-Wave), a Canadian quantum computing company, was awarded U.S. Patent No. 7,135,701.[16] Adiabatic quantum computation with superconducting qubits serves as the foundation of quantum computational architectures today.[17] As a field of study, Quantum Computing rests at the intersection of physics, computer science, electrical engineering, and philosophy. From an engineering perspective, a quantum computer is a machine harnessing the principles of quantum mechanics to perform transformations. In short, the quantum computer is the twenty-first century's greatest technical achievement.[18]

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[10] R.P. Feynman, *Space-Time Approach to Non-Relativistic Quantum Mechanics*, 20 REVS. MOD. PHYS. 367, 367 (1948).

[11] Richard P. Feynman, The Principle of Least Action in Quantum Mechanics, 1-2 (May 1942) (Ph.D. dissertation, Princeton University) (ProQuest) ("Planck's discovery in 1900 of the quantum properties of light led to an enormously deeper understanding of the attributes and behavior of matter, through the advent of the methods of quantum mechanics. . . . The fundamental (microscopic) phenomena in nature are symmetrical with respect to interchange of past and future.").

[12] Jenann Ismael, *Quantum Mechanics*, STANFORD ENCYCLOPEDIA OF PHILOSOPHY (database updated Sept. 10, 2020), https://plato.stanford.edu/entries/qm/ [https://perma.cc/H3P8-S3KJ] (quantum mechanics is the scientific discipline concerned with the motion and interaction of sub-atomic particles).

[13] R.P. Feynman, *Space-Time Approach to Non-Relativistic Quantum Mechanics*, 20 REVS. MOD. PHYS. 367, 368 (1948) (superposition describes an instance where a sub-atomic particle occupies two independent positions simultaneously).

[14] S.W. Hawking, Properties of Expanding Universes (Feb. 1966) (Ph.D. dissertation, University of Cambridge) (on file with University Library of Cambridge) (arguing Einstein's viewpoint does not give a unique model for the universe but allows a whole series of models. Thus, a theory providing boundary conditions, restricting the possible solutions based on fundamental principles. For example, in electrodynamics, there are equal numbers of sources of positive and negative sign, their fields can cancel each other out, and the total field can be zero apart from local irregularities).

[15] Feynman, *supra* note 13, at 2.

[16] Adiabatic Quantum Computation with Superconducting Qubits, U.S. Patent No. 7,135,701 (filed Mar. 28, 2005) (issued Nov. 14, 2006) (assigned to D-Wave Systems Inc.).

[17] *Id.*

[18] PAUL E. CERUZZI, COMPUTING: A CONCISE HISTORY 103 (MIT Press ed., 2012) ("Second only to the airplane, the microprocessor was the greatest innovation of the twentieth

This Article coins the term Quantum Patents, referring to patents with claims relating to quantum computing.[19] Further, this Article contributes the first empirical Quantum Patent review, including novel technology descriptions, market modeling, and legal analysis relating to Quantum Patent claims. Part II begins by discussing the two main technical approaches to quantum computing hardware, and then discusses various quantum software implementations. Part III introduces the Quantum Patent Dataset, discusses Quantum Patent claim drafting – providing critique to specific patents, and develops data models and metrics for Quantum patent valuation.

II. QUANTUM COMPUTERS

A quantum computer is a physical system harnessing quantum effects to perform computation.[20] Quantum computers differ from classical computers because of the way in which they process information.[21] Classical computers process information with bits, a binary representation.[22] However, quantum computers process information with qubits, which represent information in a complex vector space.[23]

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century."); *see also* Ryszard Winiarczyk, Piotr Gawron, Jarosław A. Miszczak, Łukasz Pawela, & Zbigniew Puchala, *Analysis of Patent Activity in the Field of Quantum Information Processing*, 11 INT. J. QUANTUM INFORM. 1, 1 (2013) ("In the last two decades of the twentieth century, the groundwork for quantum computing was laid.").

[19] The term quantum computing commonly refers to both hardware and software. *See* Jacob Biamonte, Peter Wittek, Nicola Pancotti, Patrick Rebentrost, Nathan Wiebe & Seth Lloyd, *Quantum Machine Learning,* 549 NATURE 195, 195 (2017).

[20] Aleksey K. Fedorov, Evgeniy O. Kiktenko & Alexander I. Lvovsky, *Quantum Computers Put Blockchain Security at Risk*, 563 NATURE INT'L J. SCI. 465, 466 (2018); *see also* Quantum Processor, U.S. Patent No. 9,779,360 (filed June 30, 2016); *see also* Multiple-Qubit Wave-Activated Controlled Gate, U.S. Patent No. 9,432,024 (filed Sept. 2, 2014).

[21] Biamonte et al., *supra* note 19, at 196; *see also* Approximate Gate and Supercontrolled Unitary Gate Decompositions for Two-Qubit Operations, U.S. Patent No. 10,474,960 (filed Aug. 14, 2019).

[22] CERUZZI, *supra* note 18, at 3.

[23] ELEANOR RIEFFEL & WOLFGANG POLAK, QUANTUM COMPUTING 14 (MIT Press ed., 2011); *see also* Sys. & Methods for Improving the Performance of a Quantum Processor via Reduced Readouts, U.S. Patent No. 10,031,887 (filed Sept. 3, 2015); Eleanor Rieffel & Wolfgang Polak, *An Introduction to Quantum Computing for Non-Physicists*, 32 ACM Computing Surveys 300, 306 (2000) ("A quantum bit, or qubit, is a unit vector in a two-dimensional complex vector space for which a particular basis, denoted by $\{|0\rangle, |1\rangle\}$, has been fixed. . . . Or $|0\rangle$ and $|1\rangle$ could correspond to the spin-up and spin-down states of an electron.").

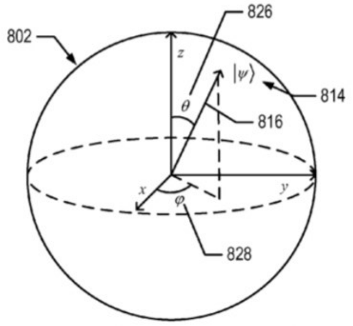

Figure 1[24]

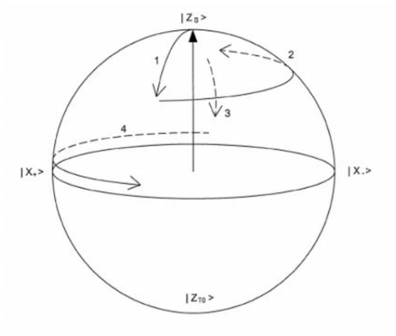

Figure 2[25]

To illustrate, *Figure 1* and *Figure 2* are patent drawings for qubits. The mathematical abstraction is intended to mirror the difference between classical and quantum states in physics.[26]

In short, the qubit is an innovation advancing the goal to improve the efficiency and power of classical computing methodologies with quantum mechanics.[27] A qubit may represent a zero, one, or zero and one simultaneously in a state of superposition.[28] The qubit allows for faster computing and less electrical power consumption compared to its classical counterpart.[29] Generally, there are

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[24] Method & Sys. for Decomposing Single-Qubit Quantum Circuits into a Discrete Basis, U.S. Patent No. 9,836,698 fig.2D (filed Jul. 19, 2012) ("Fig. 2D shows a complete representation of the Bloch sphere for an arbitrary state vector $|\psi\rangle$ 814. The unit vector 816 representing the qubit state vector $|\psi\rangle$ [814] can be specified with two angles θ [826] and ϕ [828]. Using the polar form for the complex coefficients α and β, state vector $|\psi\rangle$ can be expressed as: $|\psi\rangle = r_\alpha e^{i\phi_\alpha}|0\rangle + r_\beta e^{i\phi_\beta}|1\rangle$").

[25] Graphene Valley Singlet-Triplet Qubit Device and the Method of the Same, U.S. Patent No. 9,126,829 fig.5 (filed Jan. 13, 2012). ("Fig. 5 shows an initial qubit state manipulated in the alternating sequences consisting of $R_x(\theta_x)$ in the Bloch sphere according to the present invention.").

[26] Indeed, a qubit is a unit vector in a two-dimensional complex vector space for which a particular basis, $\{|0\rangle, |1\rangle\}$ has been fixed. For example, $|0\rangle$ and $|1\rangle$ may correspond to the spin-up and spin-down states of an electron. *See* LEONARD SUSSKIND & ART FRIEDMAN, QUANTUM MECHANICS: THE THEORETICAL MINIMUM 2 (2014); *see also* Embedding Electronic Structure in Controllable Quantum Systems, U.S. Patent No. 10,417,574 (filed Nov. 4, 2014) (assigned to President and Fellows of Harvard College).

[27] PATRICIA MALONEY FIGLIOLA, CONG. RSCH. SERV., R45409, QUANTUM INFORMATION SCIENCE: APPLICATIONS, GLOBAL RESEARCH AND DEVELOPMENT, AND POLICY CONSIDERATIONS 1 (2019).

[28] *See* RIEFFEL & POLAK, *supra* note 23, at 301 (superposition refers to electrons simultaneously occupying multiple positions in space).

[29] Brian Seamus Haney, *Blockchain: Post-Quantum Security & Legal Economics*, 24 N.C. BANKING INST. 117, 131 (2020).

two different types of quantum computers,[30] adiabatic quantum computers and gate model quantum computers.[31]

A. Adiabatic Quantum Computer

Adiabatic quantum computers (AQCs) are supercomputers harnessing natural quantum state evolution to perform computation.[32] AQCs use liquid nitrogen and liquid helium to cool a specialized quantum chip to 0.015 Kelvin,[33] a temperature 175x colder than interstellar space.[34] Instead of using Silicon like traditional computer chips,[35] the quantum chip uses a metal called Niobium.[36] The chip contains 2048 qubits in a 16x16 cell matrix, with 8 bits per cell.[37] The Niobium is looped throughout the chip, connecting the qubits and acting as a superconducting metal where each loop models a quantum spin.[38] And, when cooled to the near zero Kelvin temperature at which it is stored, the chip becomes a superconductor, a metal with properties including zero electrical resistance and

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[30] Recent research indicates a third variant is developing, ion trap quantum computers. *See* Fault Tolerant Scalable Modular Quantum Computer Architecture with an Enhanced Control of Multi-Mode Couplings Between Trapped Ion Qubits, U.S Patent No. 9,858,531 (filed Aug. 1, 2014); *see also* Ion Trap in a Semiconductor Chip, U.S Patent No. 7,411,187, (filed May 23, 2006).

[31] Ehsan Zahedinejad & Arman Zaribafiyan, Combinatorial Optimization on Gate Model Quantum Computers: A Survey 1 (Aug. 16, 2017), https://arxiv.org/pdf/1708.05294.pdf [https://perma.cc/C3PA-QDHC].

[32] '701 Patent, *supra* note 16; *see also* Quantum Computing with D-Wave Superconductors, U.S. Patent No. 6,649,929 (filed May 16, 2002).

[33] Kelvin is a standard measure of thermodynamic temperature. CONSULTATIVE COMMITTEE FOR THERMOMETRY, SI BROCHURE 2 (9th ed. 2019).

[34] *See The COBE Far Infrared Absolute Spectrophotometer (FIRAS)*, NASA https://lambda.gsfc.nasa.gov/product/cobe/about_firas.cfm [https://perma.cc/7H7M-QKYZ] (last visited Sept. 24, 2020) (calculating a definitive cosmic microwave background temperature of 2.725±0.002 K).

[35] *See* CERUZZI, *supra* note 18 at 68-69; *see also* Memory Sys. for a Multi-Chip Digital Computer, U.S. Patent No. 3,821,715 (filed Jan. 22, 1973) (assigned to Intel Corporation).

[36] Methods of Adiabatic Quantum Computation, U.S. Patent No. 8,504,497 (filed Jul. 28, 2010) (assigned to D-Wave Systems Inc.) ("One hardware approach uses integrated circuits formed of superconducting materials, such as aluminum or niobium.").

[37] Press Release, D-Wave, D-Wave Breakthrough Demonstrates First Large-Scale Quantum Simulation of Topological State of Matter (August 22, 2018), https://www.dwavesys.com/press-releases/d-wave-breakthrough-demonstrates-first-large-scale-quantum-simulation-topological [perma.cc/S8SD-LUVL]; *see also* Probabilistic Inference in Machine Learning Using a Quantum Oracle, U.S. Patent No. 10,339,466 (filed Sept. 11, 2014) (discussing AQC applications for improved machine learning techniques).

[38] Quantum Computing with D-Wave Superconductors, U.S. Patent No. 6,649,929 (filed May 16, 2002).

magnetic flux fields.[39] These properties allow the chip to exhibit quantum mechanical effects and eliminate noise during the computational process.[40] From a computational perspective, AQCs use the Adiabatic Theorem,[41] which is composed of two parts, the Ising Model and a traverse magnetic field.[42]

i. Ising Model

The Ising Model is traditionally used in statistical mechanics, where variables are binary and the relationship between variables is represented by couplings.[43] The Ising Model, uses a Hamiltonian[44] energy measurement function to describe the total amount of energy in a quantum system.[45] The input for the Hamiltonian function is the state of the system.[46] And, the output is the energy measurement

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[39] *See* Universal Adiabatic Quantum Computing with Superconducting Qubits, U.S. Patent No. 10,037,493 (filed Oct. 21, 2014); *see also* '929 Patent, *supra* note 38.

[40] *See* T.D. Ladd, F. Jelezzko, R. Laflamme, Y. Nakamura, C. Monroe & J. L. O'Brien, *Quantum Computers*, 464 NATURE 45, 49 (2010).

[41] *See* Adiabatic Quantum Computation With Superconducting Quibits, U.S. Patent No. 7,418,283 (filed Mar. 28, 2005) ("One definition of an adiabatic process is a process that occurs in a system without heat entering or leaving the system. . . . Such a system is adiabatic because the population of the various states of the quantum system has not been altered as a result of the change. Hence, if the populations have not changed, the temperature of the system has not changed, and therefore no heat has entered or left the system."); *see also* Tameem Albash & Daniel A. Lidar, *Adiabatic Quantum Computation*, REV. MOD. PHYS., Jan.-Mar. 2018, at 5.

[42] Augusto César Lobo, Rafael Antunes Ribeiro, Clyffe de Assis Ribeiro & Pedro Ruas Dieguez, *Geometry of the Adiabatic Theorem*, 33 EUR. J. PHYSICS, 1063 (2012), https://iopscience.iop.org/article/10.1088/0143-0807/33/5/1063/meta [https://perma.cc/J8TB-NRMU].

[43] Sorin Istrail, *Statistical Mechanics, Three-Dimensionality and NP-Completeness: I. Universality of Intractability for the Partition Function of the Ising Model Across Non-Planar Lattices*, 32 ACM SYMPOSIUM ON THEORY OF COMPUTING 89 (2000); *see also* Albash & Lidar, *supra* note 41, at 4.

[44] *See* '493 Patent, *supra* note 39; *see also* SUSSKIND & FRIEDMAN, *supra* note 26, at 274 ("In quantum mechanics, the Hamiltonian controls a systems evolution through the time-dependent Schrödinger equation:

$$i\hbar\frac{\partial|\Psi\rangle}{\partial t} = \mathrm{H}|\Psi\rangle.$$

The time-dependent Schrödinger equation establishes the connection between the Hamiltonian and unitary operations.

[45] *See* SUSSKIND & FRIEDMAN, *supra* note 26, at 274.

[46] '701 Patent, *supra* note 16.

of the system.[47] In other words, the Hamiltonian returns the energy measurement for any particular state.[48]

ii. Traverse Magnetic Field

In addition to the Ising Model, AQCs second essential element is a traverse magnetic field, which can be manipulated to solve optimization problems.[49] During the computational process, each qubit begins in a state of superposition encoded in a circular magnetic field.[50] Then, a barrier is raised and a magnetic field is applied to the qubits.[51] As the magnetic field is applied, each qubit moves toward a classical state, ending as a zero or one.[52] The qubits minimize their energy in the presence of the magnetic field, according to a bias.[53] Additionally, links between qubits, called couplers, allow for the resulting states of multiple qubits to affect one another.[54]

In sum, AQCs harnesses natural quantum state evolution to solve optimization and sampling problems.[55] More specifically, AQCs measure quantum state evolution with a Hamiltonian function, manipulating a magnetic field to perform computation.[56] A major advantage of the AQC model is its scalability.[57] As a result, AQCs are the first type of quantum computer capable of real world

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[47] The Ising Model is defined:

$$H_s(s) = -\frac{1}{2}\sum_i \Delta(s)\sigma_i^x + \varepsilon(s)\left(-\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right).$$

Here, $H_s(s)$ is the system's energy measurement. The initial Hamiltonian is defined:

$$-\frac{1}{2}\sum_i \Delta(s)\sigma_i^x,$$

which is the lowest energy state where all qubits are in a superposition of all states. And, the Final Hamiltonian is defined:

$$\varepsilon(s)\left(-\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right),$$

which is the lowest energy state for the system. In essence, the Hamiltonian is the sum of the Initial Hamiltonian and the Final Hamiltonian. *See* Cost function deformation in quantum approximate optimization, U.S. Patent No. 10,452,990 (filed Nov. 28, 2017) (issued Oct. 22, 2019) (assigned to International Business Machines Corporation).

[48] Mohammad H. Amin, Evgeny Andriyah, Jason Rolfe, Bohdan Kulchytskyy & Roger Melko, *Quantum Boltzmann Machine*, 8 Phys. Rev. X 021050-1 (2018).

[49] *See* Method and Sys. for Solving Integer Programming and Discrete Optimization Problems Using Analog Processors, U.S. Patent No. 7,877,333 (filed Sep. 5, 2007).

[50] Method for Adiabatic Quantum Computing Comprising of Hamiltonian Scaling, U.S. Patent No. 7,788,192 (filed Jan. 22, 2007).

[51] Biamonte et al., *supra* note 19, at 197.

[52] *See* '701 Patent, *supra* note 16.

[53] *See* '493 Patent, *supra* note 39.

[54] *See* Sys. and Methods for Real-Time Quantum Computer-Based Control of Mobile Systems, U.S. Patent No. 9,400,499 (filed Oct. 2, 2015).

[55] *See* '333 Patent, *supra* note 49.

[56] *See* '493 Patent, *supra* note 39.

[57] *See* Rieffel & Pollack, *supra* note 23; *see also* D-Wave, *supra* note 37.

application.[58] However, a potential drawback is it is arguably incapable of scaling to a universal quantum computer.[59]

B. Gate Model Quantum Computer

The second type of quantum computer is the Gate Model Quantum Computer (GMQC).[60] In contrast to AQCs, which utilize a quantum state's natural evolution, GMQCs manipulate quantum state evolution.[61] In this approach, quantum circuits are established from a combination of electrical and mechanical components and circuitry.[62] Further, "qubits are acted upon by sequences of logical gates that are the compiled representation of an algorithm."[63] The GMQC has two conceptual elements, the quantum circuit and gate transformation.[64]

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[58] *See* '499 Patent, *supra* note 54; *see also* I. Stewart, D. Ilie, A. Zamyatin, S. Werner, M. F. Torshizi & W. J. Knottenbelt, *Committing to Quantum Resistance: A Slow Defence for Bitcoin Against a Fast Quantum Computing Attack.*, 5 THE ROYAL SOC. 3 (2018), http://dx.doi.org/10.1098/rsos.180410 [https://perma.cc/CLP5-J7DZ] (discussing quantum computing applications for blockchain mining).

[59] *See* '283 Patent, *supra* note 41 (AQC is universal in that it is able to convert any input state into any output state. However, unlike the circuit model of quantum computing, there is no application of a predetermined set of one- and two-qubit unitary gates at precise times); *see also* Joel M. Gottlieb, Introduction to the Physics of D-Wave and Comparison to Gate Model, North Carolina State University (March 20, 2018), https://arcb.csc.ncsu.edu/~mueller/qc/qc18/readings/gottlieb2.pdf [https://perma.cc/7LUJ-DKHJ]; *see also* Cruz-Santos, Venegas-Andraca & Lanzagorta, A QUBO Formulation of Minimum Multicut Problem Instances in Trees for D-Wave Quantum Annealers, Scientific Reports (2019), https://www.nature.com/articles/s41598-019-53585-5 [https://perma.cc/C2KW-U9Y3] (The D-Wave hardware has some limitations. For example, the D-Wave hardware has a total 2048 qubits, which logically limits problem parameters. The current hardware is also limited in its connectivity, which uses a graph structure to describe the chain of physical qubits); *see also* Maria Schuld, Ilya Sinayskiy & Francesco Petruccione, *Prediction by linear regression on a quantum computer*, 5 (2016), https://arxiv.org/abs/1601.07823v2 [https://perma.cc/3G5P-PAHE] ("We described an algorithm for a universal quantum computer to implement a linear regression model for supervised pattern recognition.")

[60] *See* '024 Patent, s*upra* note 20.

[61] *See* Quantum Communication Link Robust Against Photon Loss, U.S. Patent No. 10,439,735 (filed Mar. 7, 2017) (issued Oct. 8, 2019) (assigned to International Business Machines Corporation).

[62] *See* '960 Patent, s*upra* note 21.

[63] '497 Patent, s*upra* note 36, col. 1 l. 48-49.

[64] *See* '499 Patent, *supra* note 54.

i. Quantum Circuit

In essence, GMQCs uses a circuit, replacing classical gates with quantum equivalents.[65] However, a quantum circuit can process information in a manner significantly different from binary digital techniques based on transistors.[66] In the circuit model, "qubits remain coherent over time periods much longer than the single-gate time."[67] A key advantage to GMQCs is their universal nature,[68] meaning GMQCs "… have qubits that can be connected to all of their neighbor qubits, and typically can run all or virtually all types of algorithms."[69] For GMQCs, the main goal is to control and manipulate quantum state evolution over time with gate transformations.[70]

ii. Gate Transformations

A quantum gate is a state transformation acting on qubits.[71] Some sequences of quantum gates are called quantum gate arrays.[72] In quantum information processing, "gates are mathematical abstractions useful for describing quantum algorithms."[73] Indeed, "quantum gates do not necessarily correspond to physical objects as they do in the classical case."[74] Instead they may represent the relationship between entangled electrons – as modeled by quantum mechanics.[75] For

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[65] *See* '024 Patent, s*upra* note 20, col. 1 l. 21-24 ("Analogous to how classical algorithms can be built from a universal logic gate, such as a NAND gate, all quantum algorithms can be constructed from a universal set of quantum gates.").

[66] *See* '960 Patent, s*upra* note 21.

[67] '497 Patent, s*upra* note 36, col. 1 l. 51.

[68] '960 Patent, *supra* note 21, col. 1 l. 27 ("[A] conventional approach to [quantum circuit design] can use a universal quantum computing circuit that can be utilized for virtually all types of algorithms.")

[69] *Id.* at col. 1 l. 29-32.

[70] *See* Artur Ekert, Patrick Hayden & Hitoshi Inamori, *Basic Concepts in Quantum Computation*, CORNELL U. ARXIV.ORG (Feb. 1, 2008), https://arxiv.org/abs/quant-ph/0011013 [https://perma.cc/7RTA-2RAY] (discussing qubit control toward a target state).

[71] *See infra* Appendix B; *see also* RIEFFEL & POLAK, *supra* note 23, at 74; *see also* Eckert et al., *supra* note 70, at 2 ("A *quantum logic gate* is a device which performs a fixed unitary operation on selected qubits in a fixed period of time and a *quantum network* is a device consisting of quantum logic gates whose computational steps are synchronized in time.").

[72] *See* Operating a Multi-Dimensional Array of Qubit Devices, U.S. Patent No. 9,892,365, at [57] col. 5 l. 37-54 (filed Feb. 27 2015) (issued Feb. 13, 2018) (assigned to Rigetti & Co., Inc.); *see also* Apparatus and Methods for Optical Neural Network, U.S. Patent No. 10,268,232, at [57] col. 11 l. 34-36 (filed June 2, 2017) (issued April 23, 2019) (assigned to Massachusetts Institute of Technoloygy) (discussing quantum gate array applications for developing neural networks).

[73] RIEFFEL & POLAK, *supra* note 23, at 74.

[74] *Id.*

[75] *See* BRIAN GREENE, FABRIC OF THE COSMOS: SPACE. TIME. AND THE TEXTURE OF REALITY 128, 117–21 (2004) (discussing the properties of quantum particles).

example, the controlled-NOT (C_{NOT}) gate[76] operates on two qubits by changing the second bit if and only if the first bit equals one.[77] Another important quantum gate representation, the Hadamard gate, produces an equal superposition of states.[78]

The GMQC's main advantage is the potential to scale to a universal quantum computer.[79] A Universal Quantum Computer is a quantum computer that can simulate certain other quantum computer.[80] However, the GMQC has drawbacks as well. For example, GMQCs, by requiring explicit excited states as computational states, are sensitive to decoherence and noise.[81] Further, GMQCs are slower to scale than AQCs; meaning, practically, GMQCs have less qubits.[82]

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[76] '735 Patent, supra note 61, col. 1 l. 43-45 ("In computing science, the controlled NOT gate (also C-NOT or CNOT) is a quantum gate that is an essential component in the construction of a quantum computer.").

[77] *See* RIEFFEL & POLAK, supra note 23, at 33, 75, 77 (The C_{not} gate's importance in quantum computing stems from its ability to change the entanglement between two qubits. The C_{not} gate is defined:

$$C_{not} = |0\rangle\langle 0|\otimes I + |1\rangle\langle 1|\otimes X.$$

Here, I is an identity transformation, and X is negation, and ⊗ is the tensor product. Interestingly, Cnot is unitary and is its own inverse.).

[78] *See id.*, at 76. The Hadamard matrix transformation is represented:

$$\mathrm{H}=\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}.$$

[79] The GMQC is universal in the sense any unitary is decomposable it into elementary gates. Thus, any unitary operation may be approximated by a finite set of gates. *See* '024 Patent, *supra* note 20, col. 1 l. 16-44 ("Superconducting qubits have made considerable progress recently in experimental controls for generating a universal set of quantum gates for quantum computing."); *see also* G. B. Lesovik, I. A. Sadovskyy, M. V. Suslov, A. V. Lebedev & V. M. Vinokur, *Arrow of Time and Its Reversal on the IBM Quantum Computer*, SCIENTIFIC REPORTS, at 1 (March 13, 2019), https://www.nature.com/articles/s41598-019-40765-6 [https://perma.cc/8ZDY-XLYH] (experimentally demonstrating backward time dynamics for an electron scattered on a two-level impurity).

[80] In other words, a quantum computer is universal in the sense that it can take any quantum state or qubits and transform them into any other quantum state or qubits. *See* '192 Patent, *supra* note 50, col. 1 l. 27-40.

[81] *See* '497 Patent, *supra* note 36, col. 1 l. 53-67 ("Much research has been focused on developing qubits with coherence sufficient to form the basic information units of circuit model quantum computers.").

[82] *See* Zahedinejad & Zaribafiyan, *supra* note 31, at 3 ("Over the past decade, there has been a great deal of progress in designing adiabatic quantum devices, with the D-Wave 2000Q quantum computing machine, with more than two thousand qubits, being the latest quantum adiabatic optimizer."); *see also* Gerhard W. Dueck, Anirban Pathak, Md Mazder Rahman, Abhishek Shukla & Anindita Banerjee, *Optimization of Circuits for IBM's five-qubit Quantum Computers*, 2018 21ST EUROMICRO CONF. ON DIGITAL SYS. DESIGN 680 (2018).

C. Quantum Software

i. Optimization

Optimization refers to a computer program selecting the best element from some set of available alternatives.[83] Optimization problems arise in quantitative disciplines including computer science, engineering, and economics.[84] In the simplest case, an optimization problem consists of maximizing or minimizing a function by systematically choosing input values from within an allowed set and computing the function's value.[85] The two most common quantum optimization algorithms are Quadratic Unconstrained Binary Optimization (QUBO) and Quantum Approximate Optimization Algorithm (QAOA).

A. Quadratic Unconstrained Binary Optimization

Quadratic unconstrained binary optimization (QUBO) algorithms are a common quantum optimization algorithm.[86] QUBOs are used for solving a variety of optimization problems and lie at the heart of experimentation carried out with AQCs.[87] For example, QUBO models are being explored in initiatives by organizations such as IBM, Google, Amazon, Microsoft, D-Wave and Lockheed Martin, Los Alamos National Laboratory, Oak Ridge National Laboratory, Lawrence Livermore National Laboratory, and NASA's Ames Research Center.[88] In short, the QUBO algorithm minimizes binary Boolean variables[89] to solve optimization problems.[90]

To illustrate, one-way scientists solve problems using AQCs, is to first express the problem as a QUBO.[91] Then, the scientists embed the logical problem

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[83] *See* Zahedinejad & Zaribafiyan, *supra* note 31, at 1.

[84] Generating a Control Sequence for Quantum Control, U.S. Patent No. 10,587,277 col. 1 l. 17-25 (filed Sep. 23, 2015); *see also* '990 Patent, *supra* note 47, col. 1 l. 6-8.

[85] '333 Patent, *supra* note 49, col. 4 l. 12-14; *see also* Multi-State Quantum Optimization Engine, U.S. Patent No. 10,095,981, at [57] (filed Mar. 22, 2017).

[86] *See* Quantum Processor Based Systems and Methods that Minimize an Objective Function, U.S. Patent No. 10,467,543 col. 4 l. 31-45 (filed Oct. 22, 2015); *see also* Brian Seamus Haney, *Leap*, GITHUB (Jul. 19, 2019), https://github.com/Bhaney44/Leap/blob/master/q.py [https://perma.cc/RJB3-FFRT] (for computer code executing QUBO algorithms on D-Wave's quantum computer).

[87] Fred Glover, Gary Kochenberger & Yu Du, *Quantum Bridge Analytics I: A Tutorial on Formulating and Using QUBO Models*, SPRINGER NATURE 335, 363-364 (2019).

[88] *Id.* at 336.

[89] Boolean variables correspond to TRUE or FALSE, states corresponding to 1 and 0 values. Chris Drake, *Boolean Algebra*, PYTHON EDA 1, 4-5 (2012), https://pyeda.readthedocs.io/en/latest/boolalg.html [https://perma.cc/LY9C-ATPU].

[90] Underpinning the success of QUBO models is the concept of magnetism in materials, which stems from the near zero Kelvin state of the quantum chip.

[91] Cruz-Santos et al., *supra* note 59, at 1 (a recent report presenting a QUBO transformation to solve the Minimum Multicut Problem, an important problem in both theoretical computer science and computer vision).

in the AQC's physical architecture by mapping logical variables and qubits.[92] The mapping for QUBO problems utilizes the Boolean state variables, TRUE and FALSE,[93] which are represented in a triangular matrix.[94] In the triangular matrix, the diagonal terms are "the linear coefficients, and the non-zero off-diagonal terms are the quadratic coefficients."[95] Finally, the problem is sent to the AQC, which performs the annealing process[96] and returns the results.[97]

B. Quantum Approximate Optimization

A second popular quantum optimization algorithm is the Quantum Approximate Optimization Algorithm (QAOA).[98] QAOA models are being explored in initiatives at IBM, Google, D-Wave and Lockheed Martin, and NASA's Ames

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[92] *Id.*

[93] D-Wave System Inc., *Problem Formulation: Key Concepts*, D-WAVE SYS. DOCUMENTATION, https://docs.dwavesys.com/docs/latest/c_gs_3.html#qubo [https://perma.cc/UCD3-WE6L] (last visited September 21, 2020).

[94] *Id.* ("A QUBO problem is defined using an upper-diagonal matrix Q, which is an N x N upper-triangular matrix of real weights x, and a vector of binary variables, minimizing the function:

$$f(x) = \sum_i Q_{i,i} X_i + \sum_{i<j} Q_{i,j} x_i x_j.$$

[95] *Id.* ("This can be expressed more concisely:

$$\min_w w_j x_i x_j + \sum_i b_i x_i.$$

[96] *See* Systems, Methods, and Apparatus for Automatic Image Recognition, U.S. Patent No. 8,073,808 col. 5 l. 35 (filed Apr. 18, 2008) ("[Q]uantum annealing is a computation method that may be used to find a low-energy state of a system. Similar to classical annealing, the method relies on the principle that natural systems tend towards lower energy states because lower energy states are more stable. However, while classical annealing uses classical thermal fluctuations to guide a system to its global energy minimum, quantum annealing may use natural quantum fluctuations, such as quantum tunneling, to reach a global energy minimum more accurately or more quickly"); *see also*'543 Patent, *supra* note 86, col. 1 l. 55.

[97] Systems, Methods, and Apparatus for Recursive Quantum Computing Algorithms, U.S. Patent No. 8,244,650 col. 6 l. 26 (filed Aug. 14, 2012); *see also* Constructing and Programming Quantum Hardware for Quantum Annealing Processes, U.S. Patent No. 10,510,015 col. 1 l. 20 (filed Dec. 17, 2019).

[98] Edward Farhi, Jeffrey Goldstone & Sam Gutmann, *A Quantum Approximate Optimization Algorithm*, Cornell U. ARXIV.ORG (NOV. 14, 2014), https://arxiv.org/abs/1411.4028v1 [https://perma.cc/PP9C-Z9FW] (presenting a general quantum algorithm for approximate optimization); *see also* V. Akshay, H. Philathong, M.E.S. Morales & J.D. Biamonte, *Reachability Deficits in Quantum Approximate Optimization*, 124(9) PHYSICAL REV. LETTERS 090504-1 (2020) (reporting that QAOA exhibits a strong dependence on the ratio of a problems constraint to variables, which limits the algorithm's capacity to minimize a corresponding objective function).

Research Center.[99] In short, the QAOA produces approximate solutions for optimization problems.[100] Further, the extent to which QAOA provides a possibility for speedup relative to classical optimization algorithms has yet to be explored with depth.[101]

First, scientists define an optimization problem's parameters and a cost function.[102] Then, the QAOA maps a cost function to a Hamiltonian function.[103] In turn, this transforms the classical optimization problem to a quantum optimization problem, because Hamiltonian energy measurement is a quantum problem.[104] Next, each state begins in a quantum superposition.[105] Finally, QAOA iteratively applies unitary operations to the state.[106] The function's value gradually moves toward optimality and eventually converges with a global minimum.[107] The global minimum corresponds to the problem's optimal solution.[108]

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[99] *See* Zhihui Wang, Stuart Hadfield, Zhang Jiang & Eleanor G. Rieffel, *The Quantum Approximation Optimization Algorithm for MaxCut: A Fermionic View*, 97 PHYSICAL REV. A 022304-1 (2017); *see also* '990 Patent, *supra* note 47, at [57], [73]; *see also* '466 Patent, *supra* note 37, at [73] col. 1 l. 6.

[100] Farhi et al., *supra* note 98, at 2 ("Combinational optimization problems are specified by *n* bits and *m* clauses. Each clause is a constraint on a subset of bits which is satisfied for certain assignments of those bits and unsatisfied for the other bit assignments. The objective function, defined on *n* bit strings, is the number of satisfied clauses,

$$C(z) = \sum_{\alpha}^{m} C_\alpha(z)$$

where $z = z_1 z_2 \cdots z_n$ is the bit string and $C_\alpha(z) = 1$ if z satisfies clause α and 0 otherwise.")..

[101] *See, e.g.,* Leo Zhou, Sheng-Tao Wang, Soonwon Choi, Hannes Pichler & Mikhail D. Lukin, *Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices*, 10 PHYSICAL REV. X 021067-1-2 (2020) ("It thus remains a critical problem to assess QAOA at intermediate depths where one may hope for a quantum computational advantage.").

[102] *See* Wang et. al., *supra* note 99, at 1 (in QAOA, the problem Hamiltonian encodes the optimization algorithm's cost function and a Hamiltonian are applied alternately).

[103] Zhou et al., *supra* note 101, at 2, converting the problem to the Hamiltonian:

$$H_C = C(\sigma_1^z, \sigma_2^z, \cdots, \sigma_N^z).$$

[104] *See* '990 Patent, *supra* note 47, col. 3 l. 38.

[105] *See, e.g.,* RIEFFEL & POLAK, *supra* note 23, at 178.

[106] For QAOA, the unitary operator is:

$$U(C,\gamma) = e^{-i\gamma C} = \prod_{\alpha=1}^{m} e^{-i\gamma C_\alpha}.$$

See, e.g., Farhi et al., *supra* note 98, at 2.

[107] Zhou et al., *supra* note 101, at 4 (the gradual movement is typically accomplished by gradient based optimization methods).

[108] *Id.* at 3 ("for certain cases one can prove a guaranteed minimum approximation . . . .").

ii. Machine Learning

The driving force of computing technology is the realization that every piece of information can be represented as numbers.[109] But, one problem with classical machine learning systems is that data processing is computationally expensive.[110] In other words, processing the world's information with machine learning algorithms takes a tremendous amount of computational power.[111] Quantum computing offers a solution to this problem, making "use of quantum algorithms as part of a larger implementation."[112] Two quantum machine learning frameworks are most prominent, Boltzmann Machines and Markov Models.[113]

A. Boltzmann Deep Learning

Perhaps the most important character in quantum machine learning is Austrian Physicist Ludwig Boltzmann.[114] Boltzmann provided a statistical description of isolated systems with a definite energy.[115] Boltzmann's models form the foundation for the modern Quantum Boltzmann Machine (QBM) – which is the

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[109] *See* ETHEM ALPAYDIN, INTRODUCTION TO MACHINE LEARNING, 2 (3rd ed. 2014).

[110] *See, e.g.,* Brian S. Haney, *The Perils and Promises of Artificial General Intelligence*, 45 J. LEGIS. 151, 162 (2018); *see also* Tabrez Y. Ebrahim, *Computational Experimentation*, 21 VAND. J. OF ENT. & TECH. L. 591, 602 (2019) ("Machine learning is simply a form of data analysis that uses algorithms to continuously learn from data by recognizing hidden patterns without being programmed to do so.").

[111] *See, e.g.,* MAXIM LAPAN, DEEP REINFORCEMENT LEARNING HANDS-ON, 125 (2018).

[112] Biamonte et. al., *supra* note 19, at 195; Amin et al., *supra* note 48, at 021050-1 (proposing "a quantum probabilistic model for machine learning based on a Boltzmann distribution of a quantum Hamiltonian," a Quantum Boltzmann Machine).

[113] *See* Maria Schuld, Ilya Sinayskiy & Francesco Petruccione, An Introduction to Quantum Machine Learning 4 (2014), https://arxiv.org/pdf/1409.3097.pdf [https://perma.cc/74XZ-E8U5] ("Reinforcement learning is a central mechanism in the development and study of intelligent agents."); *see also* Christa Zoufal, Aurélien Lucchi & Stefan Woerner, Variational Quantum Boltzmann Machines 1 (2020), https://arxiv.org/pdf/2006.06004.pdf [https://perma.cc/3H6C-T5Z6] ("Boltzmann Machines offer a powerful framework for modelling probability distributions. These types of neural networks use an undirected graph structure to encode relevant information.")

[114] Ali Eftekhar, Ludwig Boltzmann 20-21 (unpublished manuscript) (on file at the University of Pittsburgh Philosophy of Science Archive) (Boltzmann achieved fame in the late nineteenth century for the developing statistical mechanics models, explaining and predicting how the properties of atoms determine the properties of matter); *See also* GREENE, *supra* note 75, at 151 (etched in Boltzmann's tombstone in Zentralfriedhof in Vienna, is the equation:

$$S = k \log W.$$

The equation expresses the mathematical formalism for entropy – the tendency of the universe to move order to disorder. Entropy is Boltzmann's most famous contribution to physics).

[115] RAMAMURTI. SHANKAR, QUANTUM FIELD THEORY AND CONDENSED MATTER 10 (2017). *See also* Geoffrey Hinton, *Restricted Boltzmann Machines*, Lecture on Advanced Machine Learning, University of Toronto, Lecture 4, (2013), https://www.cs.toronto.edu/~hinton/csc2535/notes/lec4new.pdf [https://perma.cc/AKG9-DXZT].

center of attention for innovative research relating to the intersection of quantum computing and machine learning.[116] A QBM is a "network of symmetrically coupled stochastic binary units."[117] In other words, a QBM is a model representing a probability distribution over a set of binary variables.[118] QBMs use two types of binary variables – visible variables, *v*, and hidden variables, *h*.[119] *Figure 3* presents a QBM model.

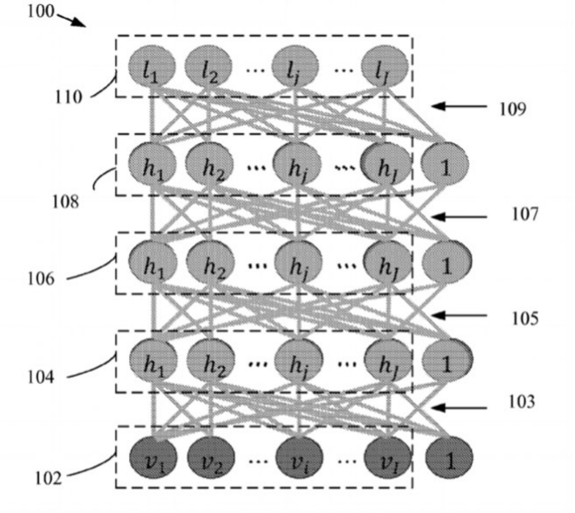

Figure 3[120]

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[116] *See, e.g.,* '466 Patent, *supra* note 37, col. 1 l. 16-39; *see also* Quantum-Assisted Training of Neural Networks, U.S. Patent No. 10,417,553 (filed May 1, 2015).

[117] Ruslan Salakhutdinov & Geoffrey Hinton, *Deep Boltzmann Machines*, 5 PROC. OF MACHINE LEARNING RES. 448 (2009) (the QBM "contains a set of visible units $v \in \{0,1\}^D$ and a set of hidden units $h \in \{0,1\}^P$"); *see* Operating a Quantum Processor in a Heterogeneous Computing Architecture, U.S. Patent No. 10,402,743 col. 17 l. 38-41 (filed Oct. 25, 2018) (issued Sept. 3, 2019) ("A Boltzmann Machine (BM) can be described as a graph where each node (or unit) is equipped with a parameter and each edge is equipped with a (coupling) parameter.").

[118] *See* Fabian Ruehle, *Data Science Applications to String Theory*, 839 PHYSICS REP. 1, 42 (2020).

[119] *See* Volodymyr Mnih, Hugo Larochelle & Geoffrey E. Hinton, Conditional Restricted Boltzmann Machines for Structured Output Prediction, Presented at the Twenty-Seventh Uncertainty in Artificial Intelligence Conference, (July 14-17, 2011), *in* UNCERTAINTY IN ARTIFICIAL INTELLIGENCE 514, 515 (2011); Geoffrey Hinton, *Restricted Boltzmann Machines*, Lecture on Advanced Machine Learning, University of Toronto, Lecture 4, (2013), https://www.cs.toronto.edu/~hinton/csc2535/notes/lec4new.pdf [https://perma.cc/AKG9-DXZT].

[120] Quantum Deep Learning, WO 2016/089711 Al fig. 1 (filed Nov. 2, 2015) (illustrating a deep Boltzmann machine, with an input layer, three hidden layers, and a visible layer).

The visible variables correspond to the important variables of a system – for example, the inputs and outputs.[121] The hidden variables enable the encoding more complex relationships among the visible variables.[122]

There are several QBM variations.[123] However, for machine learning purposes, the most important is the Deep Boltzmann Machine (DBM). The DBM is a multilayer neural network in which each layer captures a different abstraction of information.[124] In the DBM, additional hidden nodes are added to create a multi-layered network, deriving deeper abstractions for statistical inference and meaning.[125] In essence, deep learning algorithms can be run on quantum hardware by re-framing neural network architectures through a Boltzmann Formalism.[126] Thus, QBMs provide opportunity for quantum speedup at the intersection of deep learning and quantum hardware.[127]

B. Markovian Reinforcement Learning

The roots of reinforcement learning date back to the early twentieth century and the work of Russian mathematician, Andrei Markov.[128] And, the *Markov Decision Process* (MDP) remains the foundation of reinforcement learning.[129]

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[121] *Id.*

[122] *See id.*; ‘553 Patent, *supra* note 116, col. 3 l. 46-60.

[123] One QBM variation is the Restricted Boltzmann Machine (RBM). An RBM is a two-layer neural network, where the hidden units are conditionally independent given the visible states. The RBM is modeled as a bigraph: a set of graph vertices with two distinct sets. In the RBM, there are no connections between hidden nodes, or between visible nodes. Mnih et al., *supra* note 119, at 515; *see also* ‘466 Patent, *supra* note 37. A second variation is the Semi-Restricted Boltzmann Machine. A Semi-RBM is a two-layer neural network, where the hidden variables are in conditional equilibrium with the visible variables. However, the visible units may not be in conditional equilibrium with the hidden variables. In other words, there are connections between the hidden variables, but not the visible variables. Hinton, *supra* note 115.

[124] *See* Ruehle, *supra* note 118, at 44 (the DBM is a feed-forward neural network trained with input-out pairs).

[125] *See* Ruslan Salakhutdinov & Geoffrey Hinton, *An Efficient Learning Procedure for Deep Boltzmann Machines*, 24 NEURAL COMPUTATION 1967, 1970 (2012).

[126] For example, QBM’s may be executed on D-Wave’s AQC. ‘553 Patent, *supra* note 116, col. 4 l. 4-9.

[127] *See* ‘553 Patent, *supra* note 116, col. 5 1. 11-16.

[128] Markov’s work in probability theory resulted in one of the twentieth century’s most important ideas, the Markov Decision Process (MDP). *See* Gely P. Basharin, Amy N. Langville & Valeriy A. Naumov, *The Life and Work of A.A. Markov*, 386 LINEAR ALGEBRA AND ITS APPLICATIONS 3, 15-16 (2004); *see also* Application Analytics Reporting, U.S. Patent No. 9,858,171 col. 10-11 (filed Mar. 30, 2016); GEORGE GILDER, LIFE AFTER GOOGLE 82-85 (2018) (Markov’s models are used in search algorithms, machine translation, and financial trading).

[129] In short, the MDP is a statistical tool for predicting future behavior. MDPs trace the probabilistic transitions from one state to another through time. Brian S. Haney, *Applied*

Reinforcement learning is a type of machine learning concerned with learning how an agent should behave in an environment to maximize a reward.[130] The purpose of reinforcement learning algorithms is to learn how an agent should make decisions.[131] Reinforcement learning algorithms contain three elements: (1) model: the description of the agent-environment relationship;[132] (2) reward: the agent's goal;[133] and (3) policy: the way in which the agent makes decisions.[134]

In reinforcement learning, the environment[135] represents the problem.[136] Formally, reinforcement learning is described through an agent-environment

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Artificial Intelligence in Modern Warfare & National Security Policy, 11 HASTINGS SCI. & TECH. L.J. 61 (2020); *see, e.g.,* Controlling Dynamical Systems with Bounded Probability Failure, U.S. Patent No. 10,423,129 (filed July 26, 2017) (issued Sept. 24, 2019); Contextual Services in a Network Using a Deep Learning Agent, U.S. Patent No. 10,498,855 (filed June 17, 2016) (issued Dec. 3, 2019).

[130] EUGENE CHARNIAK, INTRODUCTION TO DEEP LEARNING 113 (2018); *see* MYKEL J. KOCHENDERFER, DECISION MAKING UNDER UNCERTAINTY 77 (2015); Asynchronous Deep Reinforcement Learning, U.S. Patent No. 10,346,741 col.1 l. 17-34 (filed May 11, 2018) (issued July 9, 2019).

[131] CHARNIAK, *supra* note 130.

[132] *Id.* ("*Reinforcement learning* (abbreviated *RL*) is the branch of machine learning concerned with how an *agent* should behave in an *environment* in order to maximize a *reward*."); *see* Katerina Fragkiadaki, *Deep Q Learning* (2018), https://www.cs.cmu.edu/~katef/DeepRL-Fall2018/lecture_DQL_katef2018.pdf [https://perma.cc/57V9-YTX8] (Lecture on Deep Reinforcement Learning, Carnegie Mellon School of Computer Science).

[133] MAXIM LAPAN, DEEP REINFORCEMENT LEARNING HANDS-ON 5-6 (Frank Pohlmann et al. eds., 2018).

[134] *See* Method and Apparatus for Improved Reward-Based Learning Using Adaptive Distance Metrics, U.S. Patent No. 9,298,172 (filed Oct. 11, 2007) (issued Mar. 29, 2016); *see also* Fragkiadaki, *supra* note 132.

[135] Environments are made up of two types of spaces, state spaces and action spaces. There are two types of state spaces. The first is fully observable. In a fully observable environment, the agent directly senses the total state at each time step in the environment. In contrast, the second type of environment is partially observable. In a partially observable environment, the agent senses a fraction of the environment. The total of all the states in an environment is called the episode, which concludes with the last state, the terminal state. *See* Jennifer Barry, Daniel T. Barry & Scott Aaronson, *Quantum Partially Observable Markov Decision Processes*, 90 PHYSICAL REV. A, 032311-1, 2 (Sept. 9, 2014); *see also* Method and Apparatus for Improved Reward-Based Learning Using Nonlinear Dimensionality Reduction, U.S. Patent No. 8,060,454 (filed Oct. 11, 2007) (issued Nov. 15, 2011); Alex Kendall, Jeffery Hawke, David Janz, Przemyslaw Mazur, Daniele Reda, John-Mark Allen, Vinh-Dieu Lam, Alex Bewley & Amar Shah, *Learning to Drive in A Day* (2018), https://arxiv.org/abs/1807.00412 [https://perma.cc/F4Q8-MDMY]; KOCHENDERFER, *supra* note 130, at 134-135; LAPAN, *supra* note 111, at 19-20; OpenAI, *Key Concepts in RL*, OPENAI SPINNING UP (2018), https://spinningup.openai.com/en/latest/spinningup/rl_intro.html [https://perma.cc/6BS2-USZF].

[136] LAPAN, *supra* note 111, at 8; *see also* '172 Patent, *supra* note 134, col. 1 l. 15-30.

interaction, with the MDP.[137] For example, in robotics control systems, the environment is made up of states for moments in time in which the environment exists.[138] In the quantum context, the environment is a Quantum Observable Markov Decision Process (QOMDP).[139] In other words, the state-space is described with a Hamiltonian, rather than a classical state measurement.[140] The agent is an algorithm solving the environment or problem.[141] In an QOMDP, the interaction begins when an agent chooses an action in the environment's initial quantum state.[142] The model continues to the next quantum state, where the agent receives a reward and a set of actions from which to choose, the agent selects an action, the environment returns a reward and the next quantum state.[143] Ultimately, in reinforcement learning an agent learns to take actions optimizing a reward.[144]

The goal for an agent in a QOMDP is to maximize its expected rewarded during the episode.[145] In other words, the agent's goal is to maximize its total reward, rather than the reward for its immediate state.[146] The agent's policy determines the value the agent returns over the course of an episode.[147] A policy is

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[137] *See* Ruehle, *supra* note 118, at 64-65.

[138] Kendall et al., *supra* note 135, at 2.

[139] *See* Barry et al., *supra* note 135, at 1.

[140] Systems and Methods for Problem Solving, Useful for Example in Quantum Computing, U.S. Patent No. 9,881,256 col. 2 l. 13-18 (filed Aug. 21, 2015) ("The quantum processor systems 10 may be implemented to physically realize quantum annealing (QA) and/or adiabatic quantum computing (AQC) by initializing the system in an initial state preferred by an initial Hamiltonian and evolving the system to a final state preferred by a problem Hamiltonian.").

[141] The agent may iterate over the action space, selecting actions according to a defined policy. *See* CHARNIAK, *supra* note 130, at 113; *see also* '741 Patent, *supra* note 130, col. 2 l. 4-48; '855 Patent, *supra* note 129.

[142] Barry et al., *supra* note 135, at 4 ("in a QOMDP, the agent can keep track of the quantum state using Eq. (12) each time it takes an action and receives an observation.").

[143] CHARNIAK, *supra* note 130, at 113.

[144] Barry et al., *supra* note 135, at 2*; see also* Controlling Dynamical Systems with Bounded Probability Failure U.S. Patent No. 10,423,129 (filed July 26, 2017) (issued Sept. 24, 2019).

[145] Episode refers to the total experience of an agent progressing through an environment a terminal state. *See* KOCHENDERFER, *supra* note 130, at 77; *see also* '855 Patent, *supra* note 129.

[146] *See* CHARNIAK, *supra* note 130, at 113.

[147] Formally, the policy is represented as the goal for a given environment is to find the optimal policy, which maximizes the agent's reward in an episode. Interestingly, an agent may use both policies to learn an optimal strategy for an environment. *See* Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A. Rusu, Joel Veness, Marc G. Bellemare, Alex Graves, Andreas K. Fidjeland, Georg Ostrovski, Stig Petersen, Charles Beattie, Amir Sadir, Joannis Antonogolou, Helen King, Dharshan Kumaran, Daan Wierstra, Shane Legg & Demis Hassahis, *Human-Level Control Through Deep Reinforcement Learning*, 518 NATURE 529, 529 (2015); *see also* Dynamically Updating a Power Management Policy of a Processor, U.S. Patent No. 10,146,286 (filed Jan. 14, 2016); System, Method and Device for Predicting

a mapping from states to probabilities for selecting actions.[148] In other words, a policy is the way in which an agent makes decisions.[149] Therefore, the goal for quantum reinforcement learning is to identify and select the policy which maximizes expected reward for an agent acting in an environment.[150]

iii. Search

Search is the algorithmic process for finding something.[151] Search is a popular problem in finance,[152] advertising,[153] and telecommunications.[154] In fact, several software engineering tasks can be formalized as search problems.[155] Perhaps the most notorious search algorithm is the Google page rank algorithm,[156] which utilizes Markovian techniques.[157] The Google page rank algorithm made Larry

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Navigational Decision-Making Behavior, U.S. Patent No. 8,478,642 (filed Oct. 20, 2009); KOCHENDERFER, *supra* note 130, at 79-80; Fragkiadaki, *supra* note 132.

[148] KOCHENDERFER, *supra* note 130, at 79-80; *see also* '454 Patent, *supra* note 135, col. 2 l. 33-45.

[149] KOCHENDERFER, *supra* note 130, at 80.

[150] Barry et al., *supra* note 135, at 032311-2.

[151] BRADLEY N. MILLER & DAVID L. RANUM, PROBLEM SOLVING WITH ALGORITHMS AND DATA STRUCTURES USING PYTHON 187 (Franklin Beedle & Associates, 2nd ed. 2011).

[152] *See* Louis Tessler & Tim Byrnes, *Bitcoin and Quantum Computing* (Jan. 9, 2018) (unpublished manuscript) https://arxiv.org/abs/1711.04235 [https://perma.cc/S85J-8P4L].

[153] Entity-Based Searching with Content Selection, U.S. Patent No. 10,580,039, at [57] (filed Nov. 16, 2016) ("Systems and methods for entity-based searching with content selection include receiving a search query and determining that the search query corresponds to a search entity."); *see also* Identification and Ranking of News Stories of Interest, U.S. Patent No. 8,667,037 (filed Feb. 8, 2013); *see also* Methods for Refining Search Results in an Application, U.S. Patent No. 10,565,262 (filed Sept. 23, 2016).

[154] *See* Devices and Methods for Locating Accessories of an Electronic Device, U.S. Patent No. 10,410,485 (filed Oct. 2, 2017).

[155] *See* Methods, Apparatus, and Computer Program Products for Quantum Searching for Multiple Search Targets, U.S. Patent No. 9,697,252 col. 1 l. 20 (filed Oct. 5, 2015) ("For example, a test generator searches for sets of inputs that result in branches or paths being covered, a finite state machine (FSM) verifier searches for inputs that lead to states where a given property is violated, and a synthesis tool searches for compositions of library components that have a specified behavior.").

[156] Ranking Search Results, U.S. Patent No. 8,682,892, at [57] (filed Sept. 28, 2012) (assigned to Google Inc.) ("Methods, systems, and apparatus, including computer programs encoded on computer storage media, for ranking search results"); *see also* Query Ranking Based on Query Clustering and Categorization, U.S. Patent No. 8,145,623 (filed May 1, 2009) (assigned to Google, Inc.).

[157] GILDER, *supra* note 128, at 90-91; *see also* Pattern Recognizing Engine, U.S. Patent No. 9,336,774 col. 1 l. 33-36 (filed Apr. 22, 2013) (assigned to Google) ("This specification describes systems that generate, configure, and use a pattern recognizing system that includes a dynamic hierarchy of connected pattern recognizer processors.").

Page and Sergey Brin a fortune,[158] promising high hopes and economic incentives for superior quantum search algorithms. As such, recently, there have been several attempts to develop quantum search algorithms[159] offering a quantum speedup.[160]

A. Grover's Algorithm

Grover's Algorithm is a quantum search algorithm, superior to any possible classical search algorithm.[161] Some scholars focus research in quantum search on structured data forms.[162] Thus, it is important to note, the Grover's Algorithm only offers a speedup on unstructured search problems.[163] Indeed, Grover's Algorithm uses amplitude amplification to search an unstructured set of elements.[164] According to Grover, "[t]he result in this paper is a subtle consequence of the fact that quantum mechanical amplitudes can be negative, whereas the associated classical quantities are probabilities which are required to be positive."[165]

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[158] Larry Page and Sergey Brin are Google's Cofounders. GILDER, *supra* note 128; *see also* Alphabet Inc., Annual Report (Form 10-K) (Dec. 31, 2018).

[159] *See, e.g.,* Pulak Ranjan Giri &Vladimir E. Korepin, A Review on Quantum Search Algorithms, 16 Quantum Info. Process 1, 2 (2017); *see also* Edward Farhi & Jeffrey Goldstone, Quantum Computation by Adiabatic Evolution 1 (Jan 28, 2000) (unpublished manuscript), https://arxiv.org/abs/quant-ph/0001106v1 [https://perma.cc/YG5C-9C65].

[160] Biamonte et al., *supra* note 21, at 195 ("A quantum speedup results if the number of queries needed to solve the problem is lower for the quantum algorithm compared to the classical algorithm.").

[161] Lov K. Grover, *Quantum Computers Can Search Arbitrarily Large Databases by a Single Query*, 79 Physical Review Letters 4709, 4711 (1997).

[162] Giri & Korepin, *supra* note 159, at 2 (providing a detailed description of database quantum search algorithms).

[163] RIEFFEL & POLAK, *supra* note 23, at 117 ("Grover's Algorithm succeeds in finding a solution with $O(\sqrt{N})$ calls to an oracle, whereas the best possible classical approaches require $O(N)$ calls.").

[164] Haney, *supra* note 29, at 140 ("The algorithm begins with an equal superposition:

$$|\psi\rangle = \frac{1}{\sqrt{N}} \sum_x |x\rangle,$$

of all N values of the search space and repeatedly performs the same sequence of transformations: (1) Apply U_p to $|\psi\rangle$; (2) Flip the sign of all basis vectors that represent a solution; (3) and Perform an inversion about the average, a transformation mapping every amplitude:

$$A - \delta \text{ to } A + \delta,$$

where A is the average of the amplitudes. The sequence of transformations iterates, until the final measurement returns a value x with a high probability of interest.") (footnote omitted).

[165] Grover, *supra* note 161, at 4711.

While Grover Search is limited to unstructured data, the underlying algorithm is used in many quantum search variants and descendants.[166] In sum, Grover argues his algorithm is a demonstration of yet another way in which quantum computers can outperform their classical counterparts.[167]

B. Search Sample Equivalence

At this point, the scale at which quantum search algorithms provide speedups is unclear. Some early attempts to implement Grover's Algorithm on D-Wave's AQC have been unsuccessful.[168] However, D-Wave's system is capable of effectively solving sampling problems.[169] And, Scott Aaronson has mathematically proven search sample equivalence.[170] According to Aaronson, "[i]f classical computers can efficiently solve any search problem that quantum computers can solve, then they can also approximately sample the output distribution of any quantum circuit."[171] If Aaronson's proof is valid, then D-Wave's AQC can run any search algorithm by reframing the search problem as a sampling problem. Yet, some scholars contend a quantum computer would need at least one-million qubits to be commercially useful.[172]

Quantum machine learning literature suggests quantum search speedups have already been achieved.[173] Multiple D-Wave patents include claims for quantum

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[166] *See, e.g.,* '252 Patent, *supra* note 155; *see also* Router with De-centralized Processing Using Intelligent Ports, U.S. Patent No. 6,078,963 (filed Jan. 16, 1998); *see also* Methods, Apparatus, and Computer Program Products for Quantum Searching for Multiple Search Targets, U.S. Patent No. 9,152,922 col. 4 l. 13 (filed Dec. 16, 2009) (explaining computer program code for carrying out Grover Search may be "written in a high-level programming language, such as Java, C, and/or C++, for development convenience").

[167] Grover, *supra* note 161, at 4711.

[168] Aamir Mandviwalla, Keita Ohsiro & Bo Ji, Implementing Grover's Algorithm on the IBM Quantum Computers, IEEE International Conference on Big Data at 1 (2018) (implementing Grover's Algorithm on IBM's GMQC, however, finding that the results were relatively disappointing for the possibility of practical implementations in the near future).

[169] Re-Equilibrated Quantum Sampling, U.S. Patent No. 10,346,508 (filed Jan. 18, 2018) (sampling from low-energy states and characterizing the energy landscape is useful for machine learning problems building a probabilistic model of reality); *see also* Yaroslav Koshka & M.A. Novotny, Towards Sampling from Nondirected Probabilistic Graphical models using a D-Wave Quantum Annealer 10 (2019), https://arxiv.org/abs/1905.00159 [perma.cc/X8J7-YYK7].

[170] Scott Aaronson, *The Equivalence of Sampling and Searching*, 55 Theory Comput. Syst. 281, 287-291 (Sept. 15, 2013) (formally defining sampling and search problems and offering a proof of equivalence); *see also* '252 Patent, *supra* at note 153.

[171] Aaronson, supra note 170, at 282.

[172] JOHN D. KELLEHER, DEEP LEARNING 244 (2019).

[173] Amin et al., *supra* note 48, at 021050-1 ("Machine learning is a rapidly growing field in computer science with applications in computer vision, voice recognition, medical diagnosis spam filtering, search engines, etc."); *see also* Biamonte et al., *supra* note 21, at 195.

search applications utilizing optimization algorithms.[174] As such, the future is promising for quantum search and quantum software more generally. Indeed, quantum optimization and machine learning software systems already solve difficult real-world problems, like robotics control.[175] Moreover, the IP rights protecting these new technologies may mean the difference between rags and riches for quantum computing firms.

III. PATENTS

A patent[176] is the most traditional legal form of IP protection for new technologies.[177] A patent provides the holder the legal right to prohibit others from using, making, or selling an invention without permission.[178] Notre Dame Law Professor Stephen Yelderman argues the U.S. patent system's central goal is to provide adequate incentive to innovators to publish their invention in exchange for rights.[179] As such, in conferring the exclusive right to discoveries to its inventors, a patent confers an essential temporary monopoly to the holder.[180] This

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[174] '543 Patent, *supra* note 86 (assigned to D-Wave Systems Inc.); *see also* Systems and Methods for Finding Quantum Binary Optimization Problems, U.S. Patent No. 10,275,422 (filed Mar. 27, 2015) (assigned to D-Wave Systems Inc.); Systems and Methods that Formulate Problems for Solving by a Quantum Processor Using hardware Graph Decomposition, U.S. Patent No. 9,875,215 (filed Dec. 17, 2013) (assigned to D-Wave Systems Inc.).

[175] '499 Patent, *supra* note 54.

[176] The United States Patent and Trademark Office ("USPTO") reviews applications to determine whether a claimed invention is: (1) is statutory subject matter; (2) is useful; (3) is novel; (4) would not be considered obvious by a hypothetical person of ordinary skill in the field; and (5) is described well enough that those in the field can make and use the invention. *See* 35 U.S.C. §§ 101-103 (requiring statutory subject matter, including any new process, machine, manufacture, or composition of matter, or any new and useful improvement thereof); *see also* 35 U.S.C. § 112.

[177] *See* JOHN PALFREY, INTELLECTUAL PROPERTY STRATEGY 56 (MIT Press 2012).

[178] Stephen Yelderman, *The Value of Accuracy in The Patent System*, 84 U. CHI. L. REV. 1217, 1263 (2017); *see also* U.S. CONST. art. I, § 8, cl. 8 (providing the constitutional basis for patents: "[t]he Congress shall have Power...To promote the Progress of Science and useful Arts, by securing for limited Times to Authors and Inventors the exclusive Right to their respective Writings and Discoveries . . . .").

[179] In other words, the patent statute promotes technological progress through the monopoly it offers for the creation and disclosure of something new. *See* Yelderman, *supra* note 178, at 1262-63, 1270; *see also* Max Stul Oppenheimer, *Patents 101: Patentable Subject Matter and Separation of Powers*, 15 VAND. J. ENT. & TECH. L. 1, 8 (2012) ("The system Congress created provides a delicate balance. In exchange for monopoly rights, the innovator must provide a description of how to make and use the invention.").

[180] In short, a patent awards the exclusive rights to use and profit from an invention to the holder, backed by the Government. *See* Bryce C. Pilz, *Student Intellectual Property Issues on the Entrepreneurial Campus*, 2 MICH. J. PRIV. EQUITY & VENTURE CAP. L. 1, 16 (2012); *see also* Andrew Beckerman-Rodau, *The Problem with Intellectual Property Rights: Subject Matter Expansion*, 13 YALE J. L. & TECH. 35, 54-55 (2010-2011) ("[The USPTO's g]ranting of a patent provides typical property rights [including] the right of the patent owner 'to exclude

Article empirically analyzes Quantum Patents in three parts: first, we introduce and model the novel, Quantum Patent Dataset; second, we discuss Quantum Patent claim drafting, providing critique to specific patents; and third, we develop data models and metrics for Quantum Patent valuation.

A. Data

The Quantum Patent Dataset includes 192 Quantum Patents. Data was gathered on all patents granted by the USPTO containing the following eight phrases as of December 31, 2019:

Quantum Computer;
Adiabatic Quantum Computing;
Quantum Computing AND Optimization;
Quantum AND QUBO;
Quantum AND Machine Learning;
Quantum AND Boltzmann Machine;
Quantum AND Markov; and
Quantum Computing AND Search.

This article's assumption is patents with the phrases in the claims are a meaningful Quantum Patent sample, including both software and hardware inventions. The assumption is made because if a patent contains one of these eight phrases in its claims, it is likely the patent relates in some way to quantum computing hardware or software.

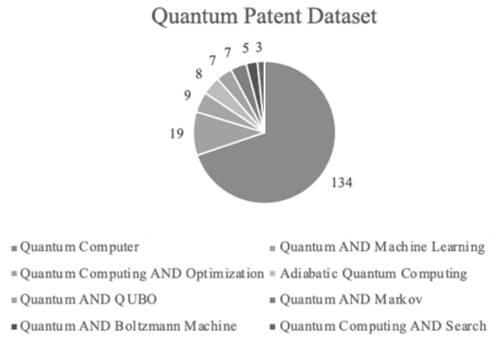

Figure 4[181]

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others from making, using, offering for sale, or selling the invention throughout the United States or importing the invention into the United States.'") (quoting 35 U.S.C. § 154 (a)(1)).

[181] The Quantum Patent Dataset discussed in this section was compiled by the author using data gathered from the United States Patent and Trademark Office website and is on file with the author. *See Search for Patents*, USPTO (Oct. 18, 2018, 9:50 AM), https://www.uspto.gov/patents/search.

While the dataset is not intended to be completely comprehensive, generally, the dataset is intended to mirror the discussion in Part II.

i. Year

The Quantum Patent Dataset includes each patent's date, measured by the year the patent was granted. *Figure 5* graphs the rate at which Quantum Patents are granted each year.

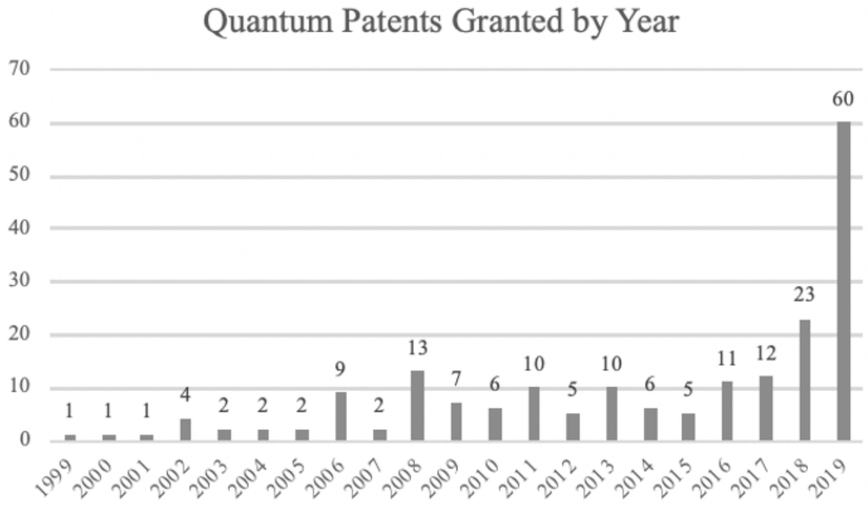

Figure 5[182]

In the year 2019, a total of sixty Quantum Patents were granted, an apparently anomalous occurrence.[183] However, when viewed in conjunction with the twenty-three Quantum Patents granted in the year 2018 – relative to the Quantum Patents granted from 1999 to 2017 – it is more likely the USPTO will continue granting higher numbers of Quantum Patents, following a similar trajectory as patents for high technologies and mirroring the Law of Accelerating Returns (LOAR).[184]

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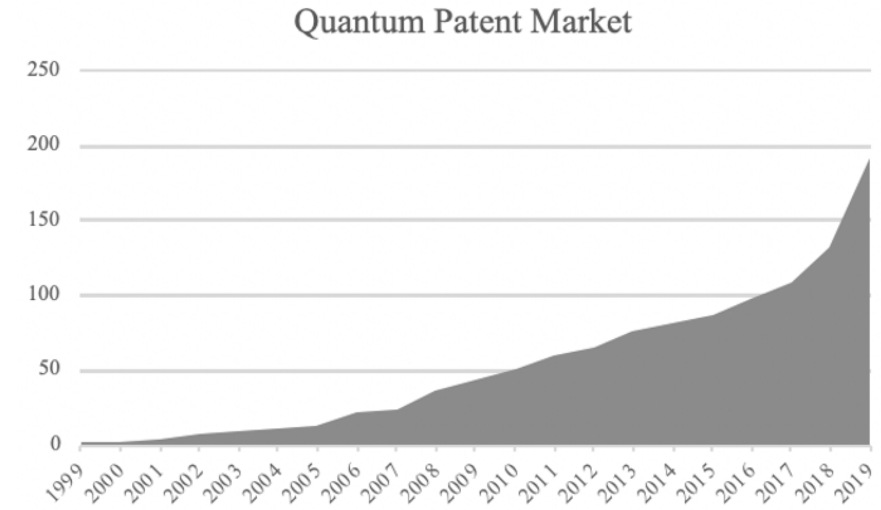
[182] *Id.*

[183] *Id.*

[184] Brian S. Haney, *AI Patents: A Data Driven Approach*, 19 CHI.-KENT J. INTELL. PROP. 407, 424 (2020) (citing RAY KURZWEIL, HOW TO CREATE A MIND 250 (2012)) (mapping trajectories of AI Patents granted by year and explaining that "[T]he Law of Accelerating Returns… states the price and performance of information technology follows a predictable exponential trajectory.").



ii. Market

The Quantum Patent Dataset measures the Quantum Patent Market by volume (the total number of Quantum Patents). *Figure 6* graphs the Quantum Patent Market from 1999 – 2019.

Figure 6[185]

The Quantum Patent market was born in 1999, when U.S. Patent No. 5,970,445 was awarded to the Japanese optics firm, Cannon.[186] Now, the Quantum Patent Market is growing at an accelerating rate.[187]

iii. Owners

The Quantum Patent Dataset includes information relating to Quantum Patent owners, which includes a patent's inventors and assignees. *Figure 7* graphs depicts a sample of Quantum Patent owners, measured by total patents.

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[185] Haney, *supra* note 181.

[186] Speech Recognition Using Equal Division Quantization, U.S. Patent No. 5,970,445 (issued Oct. 19, 1999) (assigned to Canon Kabushiki Kaisha).

[187] Haney, *supra* note 181 (in the year 2004, the market included 11 patents; in the year 2009, the market included 44 patents; in the year 2014, the market included 81 patents; and in the year 2019, the market included 192 total patents).

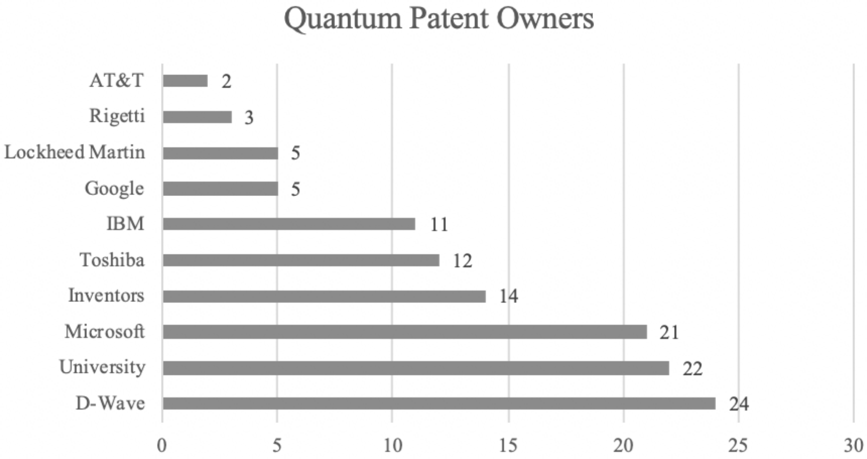

Figure 7[188]

D-Wave owns the most Quantum Patents in the dataset (24), with Microsoft (21) and IBM (11) owning the second and fourth most for any firm.[189] Further, in the aggregate Universities (22) and Inventors (14) own second and fourth most patents in the Quantum Patent Dataset overall.[190] However, total patent volume is only one market metric. Importantly, the legal rights associated with a patent are crystalized in the claims.

B. Claims

Patent Claims mark the invention's boundaries, defining the particular thing invented and making the public aware of the invention.[191] Patent claims the protected bounds for an invention, defining devices, structures, or methods.[192] Moreover, the USPTO only issues patents for claims it decides satisfy the statutory requirements.[193] Further, "courts construe patent claims by starting with the plain meaning of their terms as they would be understood by a person having ordinary skill in the art."[194] As such, claims are the most important part of a

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[188] *Id.*

[189] *Id.*

[190] *Id.*

[191] KEVIN F. O'MALLEY, JAY E. GRENIG, & HON. WILLIAM C. LEE, 3A FED. JURY PRAC. & INSTR. CIVIL 628 (6th ed. 2012).

[192] Mark A. Lemley, *The Changing Meaning of Patent Claim Terms,* 104 MICH. L. REV. 101, 107 (2005).

[193] Oppenheimer, *supra* note 179, at 4. ("…and a challenge to an issued patent will succeed if the challenger can show that any of these requirements have not been met.")

[194] Lemley, *supra* note 192, at 101-02.

patent[195] because claims are the only part of the patent that can be infringed.[196] Three essential factors for Quantum Patent claim drafting are definiteness, non-obviousness, and scope.[197]

i. Patentability

A. Definiteness

The definiteness requirement in 35 U.S.C. § 112(b), requires that the "specification shall conclude with one or more claims particularly pointing out and distinctly claiming the subject matter" of the invention.[198] The Supreme Court recently ruled on the definiteness requirement stating, "a patent is invalid for indefiniteness if its claims, read in light of the specification delineating the patent, and the prosecution history, fail to inform, with reasonable certainty, those skilled in the art about the scope of the invention."[199] The "reasonable certainty" standard balances two interests.[200]

"Patent claims should provide the public with clear notice of the exclusionary rights provided by the patent."[201] Distinct claims guard against "unreasonable advantages to the patentee and disadvantages to others arising from uncertainty as to their respective rights."[202]

At the same time, the "definiteness requirement must take into account the inherent limitations of language."[203] In other words, some uncertainty is necessary to ensure the appropriate incentives for innovation.[204]

Claim definiteness is a question of law that courts review without deference,[205] flowing from a court's obligation to construe claims *de novo*.[206] Further, "the definiteness inquiry trains on the understanding of a skilled artisan at the

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[195] *See id.*

[196] O'MALLEY ET AL., *supra* note 191, at 628.

[197] 35 U.S.C. §§ 101-103; 35 U.S.C. § 112.

[198] 35 U.S.C. § 112(b) (2012).

[199] Nautilus, Inc. v. Biosig Instruments, Inc., 572 U.S. 898, 901 (2014); *see also* Mark A. Lemley, *Software Patents and The Return of Functional Claiming*, 2013 WIS. L. REV. 905, 930 (2013) ("A related problem is the uncertainty associated with the meaning and scope of a software patent.").

[200] Dean Alderucci, *The Automation of Legal Reasoning: Customized AI Techniques for the Patent Field*, 58 DUQ. L.R. 50, 77 (2020).

[201] *Id.* at 77 ("Clear notice is necessary to avoid '[a] zone of uncertainty which enterprise and experimentation may enter only at the risk of infringement claims.'") (quoting *Nautilus*, 572 U.S. at 909-10).

[202] Gen. Elec. Co. v. Wabash Appliance Corp., 304 U.S. 364, 369 (1938).

[203] *Nautilus*, 572 U.S. at 899.

[204] *See id.* at 909 (quoting Festo Corp. v. Shoketsu Kinzoku Kogyo Kabushiki Co., 535 U. S. 722, 732 (1996)).

[205] Star Sci., Inc. v. R.J. Reynolds Tobacco Co., 655 F.3d 1371, 1373 (Fed. Cir. 2011).

[206] Kinetic Concepts, Inc. v. Blue Sky Med. Grp., Inc., 554 F.3d 1010, 1022 (Fed. Cir. 2009).

time of the patent application, not that of a court viewing matters post hoc."[207] Thus, the standard associated with a person having ordinary skill in the art is relevant to definiteness.[208]

There are objective measures for indefiniteness.[209] For example, a claim's definiteness depends on whether the terms used in the claim have ascertainable meanings, so an inspection of claim terms is useful to indefiniteness analysis.[210] Thus, the presence of a definition for a claim term is useful in analysis.[211] If a claim term is not defined in the specification, then this suggests that the claim is less likely to be definite because the patent's specification might not provide the person of ordinary skill with enough information to understand the meaning of the term.[212]

A second example of objective metrics correlating with definiteness is the presence of a coined term, when the patent drafter is permitted to use claim terms of her own devising.[213] If the term has never appeared in any previous publication then it is possible that the person of ordinary skill would not ascribe a definite meaning to the term.[214] Thus, if a claim term is both coined and undefined, the claim is less likely to be definite.[215]

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[207] *Nautilus*, 572 U.S. at 911.

[208] AllVoice Comput. PLC, v. Nuance Commc'n, Inc., 504 F.3d 1236, 1240 (Fed. Cir. 2007).

[209] *See* Dean Alderucci, *The Automation of Legal Reasoning: Customized AI Techniques for the Patent Field*, 58 DUQ. L.R. 50, 78-79 (2020).

[210] *See* Cox Commc'ns, Inc. v. Sprint Commc'n Co. LP, 838 F.3d 1224, 1232 (Fed. Cir. 2016).

[211] Alderucci, *supra* note 209, at 78.

[212] *See* Bancorp Servs., L.L.C. v. Hartford Life Ins. Co., 359 F.3d 1367, 1373 (Fed. Cir. 2004).

[213] *See* Vitronics Corp., v. Conceptronic, Inc., 90 F.3d 1576, 1582 (Fed. Cir. 1996) (holding that the patent drafter may invent a new term rather than use the term known in the relevant technical literature).

[214] *See* Advanced Ground Info. Sys., Inc. v. Life360, Inc., 830 F.3d 1341, 1348 (Fed. Cir. 2016) (holding that claim term "symbol generator" was not a term of art and was indefinite).

[215] *See* Capital Sec. Sys. v. NCR Corp., 725 Fed. Appx. 952, 959 (Fed. Cir. 2018) (affirming district court's holding of indefiniteness because the claim term 'transactional operator' "has no commonly-accepted definition and its scope is unclear in view of the intrinsic evidence").

Finally, descriptive claim terms relating to unspecified limits, terms of degree,[216] and adjectives[217] all correlate with a higher probability of indefiniteness.[218] According to Dean Alderucci, the Research Director at Carnegie Mellon's Center for AI and Patent analysis, "The inclusion of such words increases the likelihood that the claim does not have the requisite amount of certainty to satisfy the definiteness requirement."[219] For example, "the claim may include a term of degree," such as a temporal distance between actions that must be "substantially equal."[220] Consider claim 1 in U.S. Patent 9,882,112:

> 1. A multi-qubit device comprising: a first layer structure disposed on a substrate in a vertical direction of the multi-quit device and comprising an array of a plurality of qubits; and a second layer structure disposed between the substrate and the first layer structure and comprising a plurality of flux generating elements that apply flux to the plurality of qubits in the vertical direction, wherein each of the plurality of qubits and each of the plurality of flux generating elements corresponding to the plurality of qubits have centers that are aligned on substantially a same axis in the vertical direction.[221]

The claim states "the plurality of qubits have centers that are aligned on substantially a same axis in the vertical direction."[222] However, the claim is

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[216] Definiteness does not require that the claim provide mathematical precision, and terms of degree without numerical limits can be considered definite, particularly if the relevant field of technology admits no more precise way of specifying the invention. The key issue is whether the specification provides some standard for measuring that degree. *See* Rosemount, Inc. v. Beckman Instruments, Inc., 727 F.2d 1540, 1547 (Fed. Cir. 1984) (ruling term used was precise enough although recognizing that some subject matters have a limit to their terms' preciseness which complicates analyses for judges); *see also* Datamize, LLC v. Plumtree Software, Inc., 417 F.3d 1342, 1351 (Fed. Cir. 2005); Biosig Instruments, Inc. v. Nautilus, Inc., 783 F.3d 1374, 1378 (Fed. Cir. 2015) (quoting Interval Licensing LLC v. AOL Inc., 766 F.3d 1364, 1370 (Fed. Cir. 2014)).

[217] Adjectives are problematic because they induce vagueness. For example, adjectives such as "agile" can be ambiguous as to a requisite degree of software agility, thus rendering the term indefinite. *See* Halliburton Energy Servs. v. M-I LLC, 514 F.3d 1244, 1256 (Fed. Cir. 2008).

[218] Dean Alderucci, *The Automation of Legal Reasoning: Customized AI Techniques for the Patent Field*, Duq. L.R. (2020) (forthcoming) (on file with author).

[219] *Id.* at 79.

[220] *Id.* at 79-80.

[221] Multi-Qubit Device and Quantum Computer Including the Same, U.S. Patent No. 9,882,112 col. 1 l. 63-65 (filed Dec. 22, 2016) (issued Jan. 30, 2018).

[222] *Id.; see, e.g.,* Nuclear Magnetic Resonance Quantum Computing Method with Improved Solvents, U.S. Patent No. 6,218,832 col. 8 l. 43-48 (filed Feb. 16, 1999) (issued Apr. 17, 2001) (providing another example of indefinite terms of degree, claim 11 states, "[T]he quantum computer molecule includes two nuclei, each with a spin having a resonant frequency, and wherein the difference in the resonant frequencies of the two nuclear spins is substantially larger than strength of the magnetic coupling between the spins of the two nuclei.").

indefinite as to the precise location for the qubit center alignment and as to whether that location is actually the same axis. Thus, to avoid indefiniteness there should be some standard for measuring degree.[223]

In short, if the claim provides enough certainty to one skilled in the art when read in the invention's context, then the claim is definite.[224] Objective factors correlating with claim definiteness include: "(1) whether the terms in the claims are defined or used in the patent; (2) whether the claim term appears to be coined rather than in common usage; and (3) whether any claim terms are inherently vague words."[225] One could aggregate this information by developing a scoring algorithm, "counting the percentage of claim terms that lack a definition, or counting the number of vague terms in the claims."[226] However, the scoring algorithm's reliability and consistency would improve with more factors and objective claim data examples.

B. Obviousness

The Patent Act's Section §103 states that a patent claim is invalid "if the differences between the claimed invention and the prior art are such that the claimed invention as a whole would have been obvious . . . to a person having ordinary skill in the art."[227] The goal of the non-obviousness requirement is to limit patents for only those inventions representing a "sufficiently large advance" over previously known technology.[228] In fact, the statute states that obviousness should be judged from the perspective of the 'person having ordinary skill in the art'.[229]

According to Alderucci, "a full legal analysis of the obviousness of a patent claim requires understanding [(1)] the patent's technology, [(2)] the state-of-the-art in the technology's field, and [(3)] the differences between the two."[230] For example, consider the relationship between IBM's '854 patent and Google's '015 patent. The '854 patent claims:

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[223] *See* Seattle Box Co., Inc. v. Indus. Crating & Packing, Inc., 731 F.2d 818, 826 (Fed. Cir. 1984).

[224] Alderucci, *supra* note 200, at 71 (citing Nautilus, Inc. v. Biosig Instruments, Inc., 572 U.S. 898, 901 (2014)).

[225] *Id.* at 80-81.

[226] *Id.* at 81.

[227] 35 U.S.C. § 103. *See also* Jeanne C. Fromer, *The Layers of Obviousness in Patent Law*, 22 HARV. J. OF L. & TECH. 75, 79 (2008) ("The nonobviousness doctrine seeks to ensure that patents are granted only for technologically significant advances to foster the patent system's goal of stimulating useful innovation.").

[228] Lee Petherbridge, *On the Development of Patent Law*, 43 LOY. L.A. L. REV. 893, 907-08 (2010); *see* Sensonics, Inc. v. Aerosonic Corp., 81 F.3d 1566, 1570 (Fed. Cir. 1996).

[229] Alderucci, *supra* note 200, at 70 (citing Endress + Hauser Inc. v. Hawk Meas. Sys. Pty., 122 F.3d 1040, 1042 (Fed. Cir. 1997)) (analogizing the person having ordinary skill in the art to the reasonable man in criminal law).

[230] *Id.*; *see also* Colleen Chien, *Reforming Software Patents*, 50 HOUSTON L. R. 2, 24-26 (2012) ("Throughout history, the Supreme Court has redefined obviousness standards.").

> A d-wave qubit structure comprising: a qubit comprising a multicrystal junction of superconducting crystalline structures; and a superconducting screening structure surrounding said qubit, wherein said qubit comprises one of a superconducting ring and a superconducting multi-crystal junction disk, wherein said multi-crystal junction disk includes a junction of differently aligned high temperature superconductor crystalline structures, and wherein relative orientations of hexagonal grains of said crystalline structures are chosen such that said qubit generates a half-integer quantum of flux at each grain boundary interjection point.[231]

Where, the ‘015 patent claims:

> An apparatus comprising: a first pair of logical superconducting units for use in computation; a first pair of control superconducting units for use in assisting the computation; a first coupler between a first logical superconducting unit and a second logical superconducting unit; a second coupler between a first control superconducting unit and a second control superconducting unit, a third coupler between the first logical superconducting unit and the second control superconducting unit; and a fourth coupler between the second logical superconducting unit and the first control superconducting unit.[232]

Both patents relate to novel hardware architectures for quantum computation. The ‘854 patent, which was granted in 2002, describes a disk with various junctions for superconducting crystalline structures.[233] By contrast, the ‘015 patent, which was granted in 2019, describes the logical relationship between various superconducting units connected with couplers.[234] Here, the ‘015 patent presents a non-obvious innovation because a person skilled-in-the-art of quantum computation would not obviously think to evolve quantum computing technology from superconducting crystalline structures toward a logical formalism-based control system.

However, “the assessment is complicated by the fact that it involves considerations with very ill-defined boundaries.”[235] For example, the ‘854 patent’s junctions, and the ‘015 patent’s couplers share similar functionality in connecting superconducting units.[236] To that end, one could argue the ‘015 patent fails the non-obvious test because one skilled-in-the-art would recognize the obvious relationship between the junctions and couplers and conclude that the ‘015 patent’s couplers are an obvious variant of the ‘854 patent’s junctions.[237] Thus, the

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[231] Quantum Computing with D-Wave Superconductors, U.S. Patent No. 6,495,854, at [57] (filed Dec. 30, 1999) (assigned to the International Business Machines Corp.).

[232] Constructing and Programming Quantum Hardware for Quantum Annealing Processes, U.S. Patent No. 10,510,015 col. 12 l. 65 (filed Mar. 7, 2018) (assigned to Google LLC).

[233] *See* ‘854 Patent, *supra* note 231, at [45], [57].

[234] *See* ‘015 Patent, *supra* note 232, at [45], [57].

[235] Alderucci, *supra* note 200, at 70.

[236] ‘854 Patent, *supra* note 231; ‘Patent 015, *supra* note 232.

[237] *See* Alderucci, *supra* note 200, at 70.

obviousness requirement produces considerable uncertainty in Quantum Patent ownership rights.

Ultimately, obviousness is legal question,[238] relying on factual analysis "including: [(1)] the scope and content of the prior art, [(2)] the differences between the prior art and the claims of the patent, [(3)] and the level of ordinary skill in the art."[239] Further, these factual considerations may be objectively measured to identify probabilistic correlation.[240] Indeed, the relationship between the prior art and the patent could be objectively measured according to the relative syntactic similarity between the prior art claims and patent claims.[241] For example, there is a high degree of syntactic difference between the '854 patent and the '015 patent.[242] In other words, the two patent's claims use a completely different vocabulary. Regardless, a claim's scope is both a critical and complex assessment for Quantum Patents.

ii. Scope

A patent's scope defines the protectable property rights.[243] The scope question is not limited to validity or infringement.[244] Rather, scope refers to the range of things patent rights protect against competition.[245] However, "patent law too has gaps resulting from its [conceptual] separation of validity and infringement," as well as defenses.[246] For example, because of the separation between validity, infringement, and defenses, a party may often successfully argue that an IP right means one thing in one context and something very different in another.[247]

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[238] *Id. (*citing Graham v. John Deere Co., 383 U.S. 1, 17 (1966)).

[239] *See id.* ("Additional facts such as commercial success of the invention, long felt but unsolved needs solved by the invention, and the failure of others to create the invention can also be relevant to determining whether a patent claim is obvious.").

[240] *See* Alderucci, *supra* note 200, at 80-81.

[241] *Id.*

[242] *See* '854 Patent, *supra* note 231; '015 Patent, *supra* note 232.

[243] Mark A. Lemley & Mark P. McKenna, *Scope*, 57 WM. & MARY L. REV 2197, 2209 (2015) ("[T]he IP regimes require, at least in theory, not just similarity between the defendant's and plaintiff's works, but similarity with respect to the protectable elements.").

[244] *Id.* at 2202.

[245] *See* Dan L. Burk & Mark A. Lemley, *Policy Levers in Patent Law*, 89 VA. L. REV. 1575, 1675 (2003); *see also* Colleen Chien, *Software Patents as a Currency, Not Tax, on Innovation*, 31 BERKELEY TECH. L.J. 1669, 1681 (2017) ("The boundaries of patent rights are also more readily ascertainable than trade secrets, defining the duration of the right and the scope of the claims so that the parties do not have to do so.").

[246] Lemley & McKenna, *supra* note 243, at 2240 (arguing patent owners can and do exploit these gaps with some regularity. For example, patentees in computer software, have sought broader patent claim interpretation, to the point where many claims are not limited either to a particular computer algorithm or approach or to a particular hardware implementation).

[247] *See id.* at 2220-21.

Further, sometimes different actors decide different doctrines, at different times,[248] creating gaps through often contradicting, near arbitrary decisions.[249]

As such, claim drafting involves a balancing of interest. On one hand, patent "rights that claim too broad a scope are more likely to be invalid because they may tread on the rights" of prior art.[250] On the other hand, patent rights with a narrower scope are more likely to be valid, but a narrower scope may limit the firm's freedom of action in engineering and design as a result.[251] The balance of interest in claim drafting is not a dichotomy, but rather a continuous scale, which may be measured with objective metrics.[252] Appreciation for the balancing of validity and ownership rights is critical for both quantum hardware and quantum software claim drafting.

A. Quantum Hardware Claims

In this new, but competitive market, the firm's ability to protect and defend technology rights is often the difference between market dominance and irrelevance, as, a patent's scope defines the boundaries of a firm's legal protections and defenses.[253] For example, consider the IBM '605 patent's first claim:

> A quantum computer system comprising: a plurality of quantum circuits arranged in a two-dimensional layout; and wherein the plurality of quantum circuits includes at least one interior quantum circuit that is not along a perimeter of the two-dimensional layout, wherein the at least one interior quantum circuit comprises a plurality of layers, a top layer of the plurality of layers including a through hole to a bottom layer of the plurality of layers; and a signal wire positioned at least partially within the through hole and connecting the bottom layer to the top layer.[254]

The '605 patent's claim 1 discusses a plurality of quantum circuits, with at least one interior circuit with a plurality of layers.[255] Further, the interior circuit's

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[248] *See id.* at 2222 ("Even if the decisionmaker is the same, validity, infringement, and defenses often come with different burdens of proof.").

[249] Some IP issues are decided by judges, others by juries. *See* Eileen M. Herlihy, *The Ripple Effect of Seventh Amendment Decisions on the Development of Substantive Patent Law*, 27 SANTA CLARA COMPUTER & HIGH TECH L.J. 333, 343 (2011) (quoting Markman v. Westview Instruments, Inc., 527 U.S. 370, 372 (1996)) (holding "the construction of a patent, including terms of art within its claim, is exclusively within the province of the court").

[250] Lemley & McKenna, *supra* note 243, at 2202.

[251] *See* PALFREY, *supra* note 177, at 77.

[252] John R. Allison, Mark A. Lemley, Kimberley A. Moore & R. Derek Trunkey, *Valuable Patents*, 92 GEO. L.J. 435, 438 (2004).

[253] *See* PALFREY, *supra* note 177, at 77.

[254] Sys. & Method for Routing Signals in Complex Quantum Sys., U.S. Patent No. 10,347,605 col. 6 l. 31-43 (filed Nov. 28, 2017) (assigned to International Business Machines Corporation).

[255] *Id.*

layers are connected with a wire. Now consider, the Rigetti '365 patent's first claim:

> A quantum computing system comprising: a multi-dimensional array of qubit devices, each qubit device having a respective qubit operating frequency that is independent of an offset electromagnetic field experienced by the qubit device; and coupler devices residing at intervals between neighboring pairs of the qubit devices in the multi-dimensional array, each coupler device being configured to receive coupler control signals that produce an electromagnetic interaction between the respective neighboring pair of qubit devices, each coupler device configured to vary a coupling strength of the electromagnetic interaction according to an offset electromagnetic field experienced by the coupler device, wherein each of the coupler devices has a respective coupler operating frequency that varies with the offset electromagnetic field experienced by the coupler device, and the coupling strength varies according to the coupler operating frequency, wherein each of the neighboring pairs of qubit devices comprises a first qubit device having a first qubit operating frequency and a second qubit device having a second, distinct qubit operating frequency, wherein the coupler control signals each comprise a DC component and an AC component, and the AC component drives the coupler devices at a drive frequency that corresponds to a sum or difference of the first qubit operating frequency and the second qubit operating frequency.[256]

The '365 patent's claim 1 discusses a multi-dimensional array of qubit devices.[257] Each qubit has an independent and respective frequency.[258] The qubits are connected via couplers, which send and receive control signals producing an electromagnetic reaction among the qubits.[259] Further, the strength of the control signals varies according to coupling strength.[260]

Now, compare the '605 patent's claim scope with the '365 patent's claim scope. The '605 has a much broader legal claim than the '365 because the '605 describes a much more general design. Indeed, the '605 claims a simple circuit model for a quantum computer, where the '365 claims the explicit design details relating to the quantum computer's gate arrays and electromagnetic control flow. Thus, the '605 has the stronger legal claim to a broader art compared to the '365 because of the way in which the claims were drafted. Shakespeare said it best: "Brevity is the soul of wit."[261]

However, the '365 patent has a stronger claim to validity because of the explicit engineering details Rigetti disclosed. By contrast, it is more likely the '605 patent would be invalidated for obviousness. The person having ordinarily skill

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[256] Operating a Multi-Dimensional Array of Qubit Devices, U.S. Patent No. 9,892,365 col. 104 l. 2-31 (filed Feb. 27, 2015) (assigned to Rigetti & Co., Inc.).

[257] *Id.*

[258] *Id.*

[259] *Id.*

[260] *Id.*

[261] WILLIAM SHAKESPEARE, HAMLET act 2, sc. 2.

in the art could conclude the arrangement disclosed in the '605 patent's first claim is obvious because D-Wave discloses a 2-dimensional quantum circuit layout with a niobium wire in its '701 and '283 patents.[262] Therefore, simply rearranging the physical circuit layout and describing qubits as circuits instead of couplings is not only indefinite but also obvious when read in light of the prior art.

Quantum hardware complexity presents an interesting challenge for claim construction because the nuances between different types of quantum computers only manifests in the machines' meticulously crafted details. Perhaps the most daunting issue is not determining the category of quantum computer to which a machine belongs, but rather understanding what makes the particular machine completely unique. For example, in addition to AQCs and GMQCs new research continues to result in new hardware architectures.[263] However, the technical innovation and difference between these models is arguably *de minimis*.

In fact, in 2002, IBM was granted a patent specifically titled, *Quantum Computing with D-Wave Superconductors*, which claimed a narrow derivation of "a qubit structure comprising: a qubit comprising a multi-crystal junction of superconducting crystalline structures…"[264] Further comparing AQCs and GMQCs – patents for both quantum machines consistently describe quantum gates as a critical component[265]– suggesting now, the ultimate issue is one of scale. In any event, both the validity and ownership rights for any legal claims to quantum technology will be determined on a fact-by-fact basis, resulting in market uncertainty, risk, and potential.

B. Quantum Software Claims

Software inventions are patentable subject matter.[266] "Once a software patent has been issued, the literal scope of its claims will be construed by the court as

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[262] *See* '701 Patent, *supra* note 16; s*ee also* '283 Patent, *supra* note 41.

[263] *See*, *e.g.*, Frank Arute et al., *Quantum Supremacy Using a Programmable Superconducting Processor*, 574 NATURE, 505, 505-06 (2019) (describing the Quantum Sycamore Processor and argument for quantum supremacy).

[264] '854 Patent, *supra* note 231, col. 12 l. 24-38 ("A d-wave qubit structure comprising: a qubit comprising a multicrystal junction of superconducting crystalline structures; and a superconducting screening structure surrounding said qubit, wherein said qubit comprises one of a superconducting ring and a superconducting multi-crystal junction disk, wherein said multi-crystal junction disk includes a junction of differently aligned high temperature superconductor crystalline structures, and wherein relative orientations of hexagonal grains of said crystalline structures are chosen such that said qubit generates a half-integer quantum of flux at each grain boundary interjection point.").

[265] *See* '701 Patent, *supra* note 16; *see also* Quantum Logic Using Three Energy Levels, U.S. Patent No. 6,943,368 (filed Mar. 28, 2005); '854 Patent, *supra* note 231; '024 Patent, *supra* note 20; *but see* '497 Patent, *supra* note 36.

[266] Julie E. Cohen & Mark A. Lemley, *Patent Scope and Innovation in the Software Industry*, 89 CALIF. L. REV. 1, 8 (2001).

a matter of law in any infringement suit."[267] Further, the rapid introduction of quantum software patents into the patent system means that within a relatively short time, the background conditions for quantum software innovation have been configured.[268] This is particularly noticeable in areas like network security,[269] machine learning,[270] and robotics control.[271]

For example, consider the similarities and differences between Google's '466 patent and Rigetti's '743 patent, both of which claim variant Quantum Boltzmann Machines (QBMs) for machine learning.[272] Google's '466 patent claims:

> A method performed by a system of one or more computers for probabilistic inference in a model for use in machine learning, the method comprising: receiving data for training the model, the data comprising observed data for training and validating the model, and wherein the model is a modified restricted Boltzmann machine that includes interactions among hidden units of the restricted Boltzmann machine, wherein the interactions are based on hardware connections of a quantum oracle implemented using a quantum machine comprising an adiabatic quantum computing system, the hardware connections comprising couplers that connect qubits included in the quantum oracle; deriving input to the quantum oracle using the received data and a state of the model, the input mapping at least some interactions of different interconnected units of the model to connections between qubits in the quantum oracle; providing the input to the quantum oracle for learning the inference in the model; and receiving from the quantum oracle data representing the learned inference."[273]

The '466 patent claims a method for probabilistic inference, data processing, and machine learning utilizing a Restricted Boltzmann Machine (RBM)[274] and a

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[267] *Id.* at 37.

[268] *See id.* at 14 ("The rapid introduction of large numbers of software patents into the patent system means that within a relatively short time, the background conditions for software innovation have been substantially reconfigured.").

[269] *See* Peter W. Shor, *Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer* 20 (1996), https://arxiv.org/abs/quant-ph/9508027 [https://perma.cc/G6LE-P8VG] (describing a solution to the discrete logarithm problem on a GMQC); *see also* Scott Aaronson, *Shor, I'll do it*, SHTETL-OPTIMIZED, THE BLOG OF SCOTT AARONSON (Feb. 24, 2007), https://www.scottaaronson.com/blog/?p=208 [https://perma.cc/RYM4-WZHD] (describing Shor's algorithm and its practical implications for cybersecurity).

[270] *See* Processing of Comm. Signals Using Machine Learning, U.S. Patent No. 10,396,919 (filed May 14, 2018).

[271] *See* '499 Patent, *supra* note 54.

[272] '466 Patent, *supra* note 37; '743 Patent, *supra* note 117.

[273] *See* '466 Patent, *supra* note 37, col. 12 l. 27-48.

[274] An RBM is a two-layer neural network, where the hidden units are conditionally independent given the visible states. The RBM has no lateral edges with its visible or hidden variables and is modeled as a bigraph graph. *See id.*; *see also* Hinton, supra note 119; Mnih et al., *supra* note 119.

quantum oracle, implemented on an AQC.[275] The claim narrowly describes the relationship between a quantum oracle, qubit connectivity, and data flow.

Consider the scope of Google's '466 patent's claim 1 compared to the scope of Rigetti's '743 patent's claim 7. Rigetti's '743 patent claims:

> 6. The heterogeneous computing method of claim 1, wherein the computer program code is configured to execute a training algorithm, and the second computing task comprises gradient estimation by quantum sampling.
>
> 7. The heterogeneous computing method of claim 6, comprising: by operation of the host processor unit, obtaining a Boltzmann machine state and a training vector; by operation of the host processor unit, generating the set of instructions for the quantum processor unit based on the Boltzmann machine state and training vector, the set of instructions configured to cause the quantum processor unit to perform a gradient estimation by quantum sampling algorithm based on the Boltzmann machine state and training vector; by operation of the quantum processor unit executing the set of instructions, generating a set of gradient values by executing the set of instructions.[276]

The '743 patent's claim 7 describes a heterogeneous computing method utilizing gradient estimation, quantum sampling, and a Boltzmann machine state and training vector.[277] Further, the method's operation as a set of instructions using a quantum processor is claimed.[278]

Julie Cohen and Mark Lemley argue that the "characteristics of the software industry requires a narrow approach to questions of patent scope."[279] According to Cohen and Lemley, "once a software patent is issued, the literal scope of its claims are construed by the court as a matter of law in any infringement suit."[280] In other words, the "process of claim construction determines a patent's scope."[281] Such an approach is useful in considering the '743 and '466 patents because literal readings of both patents are subject to narrow interpretations relating to legal claims. While both patents discuss quantum processing methods utilizing a QBM, the claims' structures differ enough to identify their legal and technical independence.

In sum, three important considerations for Quantum Patent claim drafting are definiteness, non-obviousness, and scope. Avoiding terms of degree improves the probability that a claim will be interpreted as definite. Obviousness is heavily dependent on a highly complex and fact intensive analysis for Quantum Patents, and considerations as to scope balance the legal claim's breadth and the higher

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[275] '283 Patent, *supra* note 41.

[276] '743 Patent, *supra* note 117, col. 21 l. 38-55.

[277] *Id.*

[278] *Id.*

[279] Cohen & Lemley, *supra* note 266, at 37.

[280] *Id.*

[281] *Id.*

probability that narrow patents are ruled valid. Perhaps most importantly, each of these three considerations contributes to the patent's economic value.

C. Valuation

The way in which patents are valued is a crucial consideration for a firm's strategic planning, growth strategy, and bottom line. "The patent system is designed to encourage innovation by offering a temporary monopoly over inventions or works of authorship."[282] However, one problem with the patent system is that it lacks standard models for valuation. As a result, some argue that patent valuation involves a degree of speculation.[283] Thus, insights, which help improve patent value objectivity, add value to an organization.[284] Further, knowledge about patent value can inform patent strategy decisions. For example, in 2009, Nokia and Samsung paid a small semiconductor firm[285] in King of Prussia, Pennsylvania, called InterDigital, a combined $653 million for a portfolio of patents for smart phone technology.[286]

As a general rule, a speculation argument as to the validity of patent value is fallaciously prescriptive because value is inherently subjective rather than intrinsic.[287] Indeed, value is a concept ascribed by people to things in their environment, not something intrinsic emanating from objects. Even in the corporate context, value is a balancing of present and future assets, most commonly understood through modeling techniques.[288] In other words, an informed, transparent, and data-driven decision[289] within a defined model is not any more

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[282] Benjamin N. Roin, *Intellectual Property Versus Prizes: Reframing the Debate*, 81 U. CHI. L. REV. 999, 1001 (2014).

[283] Robert Pitkethly, *The Value of Patents*, 3 (Judge Inst., Working Paper No. WP 21/97, 1997) (noting that patent valuation requires "making judgements about the future in much the same way that stock market prices have embedded in them judgements of investors about the future performance of a company").

[284] *Id.*

[285] A semiconductor is a solid substance that has a conductivity between that of an insulator and most other metals. Silicon semiconductors are essential components of most electronic circuits. BRITANNICA, https://www.britannica.com/science/semiconductor [https://perma.cc/KXF4-YPN8] (last visited Sept. 15, 2020).

[286] PALFREY, *supra* note 177, at 18; *see also* In Matter of Arbitration Between InterDigital Commc'ns Corp., 528 F. Supp. 2d 340 (S.D.N.Y. 2007); InterDigital Commc'ns Corp. v. Nokia Corp., 407 F. Supp. 2d 522 (S.D.N.Y. 2005).

[287] Haney, *supra* note 29, at 150.

[288] *See* PETER THIEL, ZERO TO ONE 44-45 (2014) (arguing that "the value of a business today is the sum of all the money it will make in the future"); *see also* Brett M. Frischmann & Mark P. McKenna, *Systems of Human and Intellectual Capital*, 93 TEX. L. REV. 231, 236 (2015) ("The point is simply to highlight the range of complex characteristics that frustrate simple models and continue to plague descriptive accounts of intellectual capital law.").

[289] Andrew Campbell, Jo Whitehead & Sydney Finkelstein, *Why Good Leaders Make Bad Decisions*, HARV. BUS. REV. 1, 2-5 (2009) (discussing the fact that most daily decisions are made unconsciously).

speculative than other asset valuations.[290] A review of patent valuation literature reveals three overarching models for patent valuation: income models, cost models, and market models.[291]

i. Models

Income models value assets based on the economic benefit the asset may receive over its life.[292] The theory is that the extent to which patents affect a technologies ability to generate income influences the patent's valuation.[293] Factors incorporated in income models include future profits, reasonable royalties, and cash flow analyses.[294] Income models are particularly popular for determining damages in patent litigation, making income models a strong persuasive authority for determining patent value.[295] Indeed, the Reasonable Royalty Model,[296] a specific type of income model, is a historic bedrock technique in patent license valuation.[297] However, income models struggle to account for investment costs, which mature over time and are subject to market uncertainties.

Cost models are based on the idea that technology is worth the amount it costs the technology's owner to develop and protect it.[298] The assumption underlying cost models is that the cost of developing a new asset is commensurate with the economic value the asset can provide during its life.[299] Cost models are favorable to quantum technology, which has most of its value in the future. Cost models incentivize firms to keep good accounts of research and development (R&D) spending, making the model appealing for its precision.[300] Recent reports suggest global non-classified investment in quantum computing R&D total more

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[290] JAMES W. CORTADA, INFORMATION AND THE MODERN CORPORATION 4-5 (2011) (discussing knowledge as a vital asset class for corporations).

[291] Brian S. Haney, *Rocket Patent Strategies*, 24 UNIV. S.F. INTELL. PROP. & TECH. L.J. (forthcoming 2020) (analyzing the three patent valuation models in the context of reusable rocket technology).

[292] Ted Hagelin, *A New Method to Value Intellectual Property,* 30 AIPLA Q.J. 353, 363 (2002) (stating that the net future income stream is discounted to the asset's present value).

[293] *Id.* at 364.

[294] Gavin C. Reid, Nicola Searle & Saurabh Vishnubhakat, *What's it Worth to Keep a Secret?*, 13 DUKE L. & TECH. REV. 116, 137 (2015).

[295] Amy L. Landers, *Patent Valuation Theory and the Economics of Improvement*, 88 TEX. L. REV. SEE ALSO 163, 166 (2009) (noting that "patent damages are a make-whole remedy, intended to restore the patentee to the same position as before the infringement").

[296] *See* Mark A. Lemley, *Distinguishing Lost Profits from Reasonable Royalties*, 51 WM. & MARY L. REV. 655, 669 (2009) (noting that under a reasonable-royalty model, patent law aims to provide patentees with payment for the "rate that would have both compensated patentees and allowed users of the technology to make a reasonable profit").

[297] *See id.* (noting that patent law aims to provide patentees with payment for lost profits and other competitive harm suffered through infringement).

[298] Reid et al., *supra* note 294, at 139.

[299] Hagelin, *supra* note 292, at 359.

[300] *See* Haney, *supra* note 291.

than €1.5 billion (approximately $US1.8 billion) annually.[301] However, it is likely the majority of quantum computing research is classified due to the technology's potential for cybersecurity[302] and defense applications.[303] Regardless for quantum computers, costs models could include a variety of factors including, R&D costs, patent prosecution fees,[304] and engineering fees[305] for the technology.

Market models define fair market value for a technology.[306] Generally, market models value assets based upon comparable transactions between unrelated parties.[307] In essence, the fair market value is determined by assessing the price a buyer would pay a seller for the technology.[308] Intimately intertwined with a

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[301] THE NAT'L ACAD. OF SCI., ENG'G, MED., QUANTUM COMPUTING: PROGRESS AND PROSPECTS 179-180 (Emily Grumbling & Mark Horowitz eds., 2019).

[302] In the aggregate, "U.S. payment, clearing, and settlement systems process approximately 600 million transactions per day, valued at over $12.6 trillion." *See* Haney, *supra* note 29, at 126. Many of these systems rely on Public-Private Key Cryptography, and, according to a Royal Society Open Science Report, quantum computers are "capable of deducing the private key from a formerly revealed public key with little effort." Stewart et al., *supra* note 58, at 5.

[303] Brian S. Haney, *Automated Source Selection & FAR Compliance*, 48 PUB. CONT. L.J. 751, 752 (2019) (the United States annual defense budget exceeds $700 billion).

[304] Stuart J.H. Graham & Ted Sichelman, *Why Do Start-Ups Patent?* 23 BERKELEY TECH. L.J. 1063, 1085 (2008) ("Simple economics suggest that the high cost of patenting will deter some inventors from filing.").

[305] One factor which may be considered in a cost model is a patent's inventorship. *See* Malcom T. Meeks & Charles A. Eldering, PhD, *Patent Valuation: Aren't We Forgetting Something? Making the Case for Claims Analysis in Patent Valuation by Proposing a Patent Valuation Method and a Patent-Specific Discount Rating Using the CAPM*, 9 NW. J. TECH. TECH. & INTELL. PROP. 194, 196-203 (2010); *see* Heather Hamel, *Valuing the Intangible: Mission Impossible? An Analysis of The Intellectual Property Valuation Process*, 5 CYBARIS AN INTELL. PROP. L. REV. 183, 187-188 (2014) (acknowledging the argument that the greater the number and prestige of the inventors on a patent, the higher the patent quality because more intelligence and time was dedicated to the patent, and counterarguing such estimations may overlook inventions by a single previously unknown inventor which took substantial time and effort); *see also* R. Polk Wagner, *Understanding Patent-Quality Mechanisms*, 157 U. PA. L. REV. 2135, 2138 (2009) (prestige and time may also correlate with the capacity of a granted patent to meet the statutory standards of patentability – most importantly, to be novel, nonobvious, and clearly and sufficiently described).

[306] Reid et al., *supra* note 294 at 140.

[307] Ted Hagelin, *A New Method to Value Intellectual Property*, 30 AIPLA Q.J. 353, 362 (2002).

[308] Hamel, *supra* note 305, at 204-205 (Market models generate the widest range of valuations. One reason for market model's higher variance is the subjectivity in measuring market value compared to other models. A second reason for the higher variance is dependent upon whether market analysis is conducted prospectively or retroactively. Indeed, prospective market valuations tend to be more grounded with the support of financial data as opposed to retroactive valuations.); *see also* W. Michael Shuster, *Artificial Intelligence and Patent*

technology's market value is the technology's commercialization.[309] In addition to the revenue from licensing, a patent's ability to trigger sales is also relevant in technology valuation.[310] Indeed a patent's ability to influence consumers to buy a product or a newer version of an existing product correlates with increase in value.[311] For example, ownership rights in the latest technology for quantum computers increase firm value.[312] Another example is a patent's ability to trigger sales in the entirely new market for quantum computers.[313] The global quantum computing market's value remains difficult to define, but recent reports suggest in the aggregate the market is receiving at least $8 billion in both public and private investment annually.[314]

Further, the IP rights resulting from this investment are even less clear because patent value is largely revealed through "rare but highly conspicuous transactions and litigation"[315] yet, no bright-line rule exists for technology valuation.[316] A patent's direct financial value is the potential extra profits obtainable

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Ownership, 75 WASH. & LEE L. REV. 1945, 1987 (2018) ("[M]arket participants may have the technical knowledge necessary for licensing, which creates efficiencies and increases a patent's licensing value for market participants.").

[309] Hamel*, supra* note 305, at 191; *see also* Shuster, *supra* note 308, at 1985-6 (stating that patents are most valuable to market participants).

[310] Meeks & Eldering, *supra* note 305, at 202; *see also* Shuster, *supra* note 308, at 1987 ("[A]ny patentee can attempt to monetize its patents by selling the rights to practice the technology; the business model only requires patent ownership and startup funds.").

[311] Hamel, *supra* note 305, at 191-192; *see also* Shuster, *supra* note 308, at 1987 ("[A]ll relevant benefits arising from patent ownership are most valuable when the patentee participates in the relevant marketplace.").

[312] *See* Hamel, *supra* note 305, at 191-192.

[313] *See* Shubha Ghosh, *Decoding and Recoding Natural Monopoly, Deregulation, and Intellectual Property*, 2008 U. ILL. L. REV. 1125, 1170-1171 (2008). One example of patent correlation with sales in an entirely new market is Edison's electricity empire. *See* Apparatus for The Electrical Transmission of Power, U.S. Patent No. 265,786 (filed Aug. 7, 1882); *see also* Electric-Lamp, U.S. Patent No. 223,898 (issued Jan 27, 1880).

[314] THE NAT'L ACAD. OF SCI., ENG'G, MED., *supra* note 301, at 181 (defining investments by China, the UK, Australia, Sweden, and the EU). The broader technology market accounts for more than $12 trillion in annual economic activity. *See* HUAWEI & OXFORD ECON, DIGITAL SPILLOVER, MEASURING THE TRUE IMPACT OF THE DIGITAL ECONOMY 2-6 (2017) (measuring market in 2016 as $11.5 trillion, growing at 2.5x the rate of global GDP); *see also* PALFREY, *supra* note 177, at 126.

[315] Robert Pitkethly, *The Valuation of Patents* 1 (Judge Inst. Working Paper, Paper No. 21/97, 1997); *see also* John R. Allison, Mark A. Lemley & Joshua Walker, *Extreme Value or Trolls on Top? The Characteristics of the Most-Litigated Patents*, 158 U. PA. L. REV. 1, 3 (2009) (empirical evidence suggests that the most-litigated patents are software patents, filed by non-practicing entities, and unsuccessful in court).

[316] *See* Amy L. Landers, *Patent Valuation Theory and the Economics of Improvement*, 88 TEX. L. REV. 163, 165 (2009).

from fully exploiting the invention defined by the patent's claims.[317] However, "the aim in valuing patents… is to enable those managing them to know their value sufficiently accurately and objectively to make well-founded decisions concerning their management."[318] Therefore, there exists a need for formalized and objective patent value metrics to improve efficiency, objectivity, and transparency in technology transactions.

ii. Metrics

Some view patents as economic assets, *per se*.[319] Yet, many patents turn out to be worthless.[320] Others argue that any valuation method is merely a starting point towards better decision making.[321] By defining objective patent value metrics with reference to the three valuation models, this Article aims to provide a concrete framework for Quantum Patent valuation. Interestingly, Professor Allison argues that, at least in the aggregate, valuable patents can be identified.[322]Allison explains valuable patents cite more prior art, make more claims, and have more inventors.[323] Allison's work provides strong support for general correlations drawn from contrasting valuable and non-valuable patents.[324]

According to Allison, valuable patents make more claims.[325] *Figure 8* is a graph depicting the average number of claims per patent, for Quantum Patents owned by firms, inventors, and the Quantum Patent dataset total.

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[317] Robert Pitkethly, *The Valuation of Patents* 2 (Judge Inst. Working Paper, Paper No. 21/97, 1997).

[318] *Id.* ("[F]or example, to decide how much to pay for or invest in a business as part of the firms overall financial planning.").

[319] *See, e.g.*, Meeks & Eldering, supra note 305, at 194.

[320] Allison et al., *supra* note 252, at 437 ("May patents are not worth enforcing — either because the inventions they cover turn out to be worthless, or because even if the invention has economic value the patent does not.") (citations omitted).

[321] Pitkethly, *supra* note 315.

[322] Allison et al., *supra* note 252, at 438 (arguing that the author's "data conclusively demonstrate[s] that valuable patents differ in substantial ways from ordinary patents both at the time the applications are filed and during their prosecution").

[323] *Id.* (Allison argues six key characteristics of litigated patents are: "(1) They tend to be young—that is, litigated soon after they are obtained. (2) They tend to be owned by domestic rather than foreign firms. (3) They tend to be issued to inventors or small companies, not to large companies. (4) They cite more prior art than non-litigated patents, and in turn are more likely to be cited by others. (5) They spend longer in prosecution than ordinary patents. (6) They contain more claims than ordinary patents"); *see also* Hamel, supra note 305, at 187; *see also* Wagner, *supra* note 305, at 2138 (prestige and time may also correlate with the "capacity of a granted patent to meet the statutory standards of patentability – most importantly, to be novel, nonobvious, and clearly and sufficiently described").

[324] *See* Allison et al., *supra* note 252, at 438.

[325] *Id.*

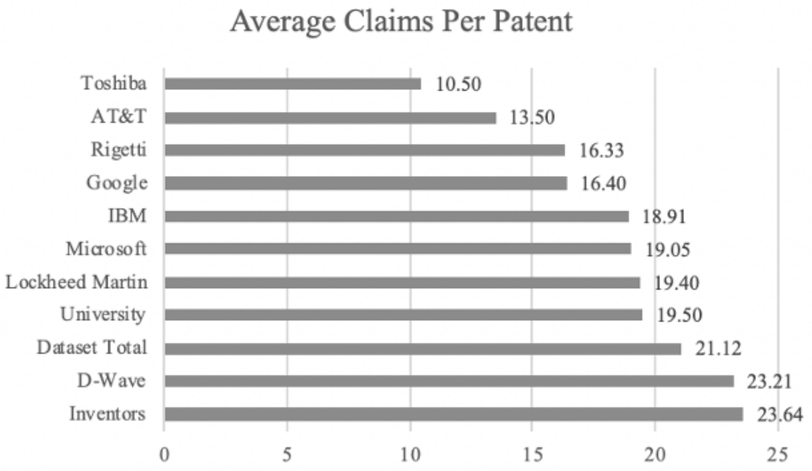

Figure 8[326]

Interestingly, Quantum Patents owned by the original inventors (23.64 avg. claims per patent) averaged more claims per patent than any firm. D-Wave (23.21 avg. claims per patent) came in close second, with Quantum Patents owned by universities (19.50 avg. claims per patent) finishing in third. While there is not an inherently causal relationship between the number of claims and the patent's value, there is a correlation.[327] As a result, Quantum Patents owned by the original inventors are likely more valuable than Quantum Patents owned by firms.[328] Therefore, D-Wave likely has a more valuable Quantum Patent portfolio when compared to the other firms listed.

Further, Allison explains valuable patents cite more prior art.[329] *Figure 9* is a graph depicting the average number of prior art citations per patent, for Quantum Patents owned by firms, inventors, and the Quantum Patent dataset total.

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[326] Haney, *supra* note 184.

[327] Allison et al., *supra* note 252, at 451 (valuable patents include significantly more claims).

[328] *Id.* at 468 (suggesting a possible reason patents owned by small inventors are more valuable is that small rather than large entities are the real wellsprings of innovation in the United States).

[329] *Id.* at 438.

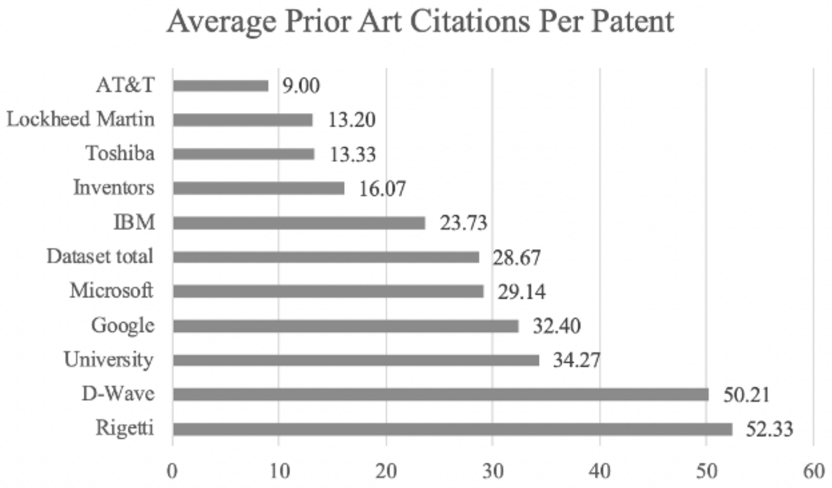

Figure 9[330]

Rigetti's U.S. Patent No. 10,402,743 and 10,127,499 both cite 73 examples of prior art, which may slightly skew the sample for the firm.[331] Regardless, it is likely the '743 and '499 are more valuable patents as a result.[332]

Third, some may contend valuable patents have more inventors.[333] *Figure 10* is a graph depicting the average number of inventors per patent, for Quantum Patents owned by firms, inventors, and the Quantum Patent dataset total.

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[330] Haney, *supra* note 184.

[331] '743 Patent, *supra* note 117; '499 Patent, *supra* note 54.

[332] *See* Allison et al., *supra* note 252, at 451 (valuable patents cite more prior art than generally issued patents).

[333] *See* Hamel, *supra* note 305, at 187 ("While this analysis [of examining the number of inventors] may seem logical, such considerations seem to undermine and overlook patents that could be of extremely high worth or quality but name only one inventor."); *See also* Wagner, *supra* note 305, at 2138.

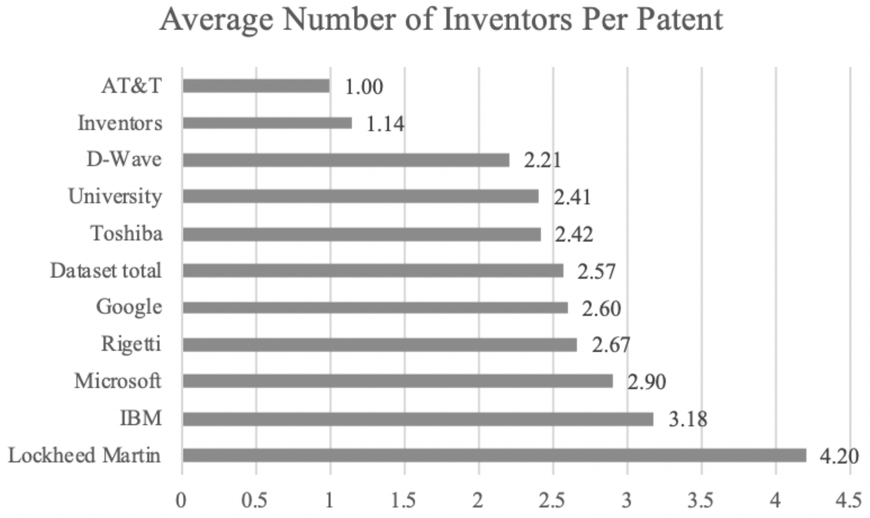

Figure 10[334]

Lockheed Martin's Quantum Patent portfolio has the highest number of inventors per patent (4.20 avg. inventors per patent). Thus, the Lockheed Martin Quantum Patent portfolio may be more valuable because, as Health Hamel mentions, "the greater the number inventors on a patent, the higher the patent quality because more intelligence and time were dedicated to the patent."[335]

However, one problem that exists is how to use this information to make patent strategy decisions more effectively. However, using these and other factors correlating with patent value, an expert system may be developed to formalize the decision-making process altogether. In other words, the expert system can assign a dollar value to any patent or group of patents. Ron Dolin argues one method of formalizing human intuition in decision making is a weighted geometric mean.[336] While patent valuation is inherently subjective, Dolin's

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[334] Haney, *supra* note 184.

[335] However, this not to say that patents with fewer inventors are necessarily less valuable. But, there is at a positive a correlation between the number of inventors and patent value. *See* Hamel, supra note 305, at 187.

[336] Ron Dolin, Measuring Legal Quality: Purposes, Principles, Properties, Procedures, and Problems (June 2017) (unpublished manuscript) (on file with Harvard Law School, Center on the Legal Profession) (accessible at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2988647) Formally the weighted geometric mean is described:

$$s = \sqrt[\sum_{j=1}^{n} w_j]{\prod_{i=1}^{n} F_i^{\,w_i}}.$$

algorithm provides a method to more objectively measure patent value by flexibly combining a variety of objective metrics.[337] In short, Dolin's algorithm is simply an adjusted weighted average, but it is incredibly effective due to its heterogeneous property.[338] As such, the utility gained from the algorithm is a standardized method for proactive Quantum Patent value optimization. In other words, firms and inventors can optimize the algorithm's metrics while writing a patent and optimize their patent value.

IV. CONCLUSION

Conventional wisdom teaches that technological progress is driven by the LOAR.[339] The LOAR's application to information technology, Moore's Law, projects exponential trends in technological progress toward an ultimate technological singularity.[340] However, a contrarian perspective on technological progression permeates among the world's greatest innovators. For example, Paul Allen, the late co-founder of Microsoft, deeply believed that scientific progress is irregular.[341] Similarly, according to Peter Thiel, "[o]ur ancestors lived in static, zero-sum societies where success meant seizing things from others."[342] "Then, after 10,000 years of fitful advance from primitive agriculture to medieval windmills and 16th-century astrolabes, the modern world suddenly experienced relentless technological progress."[343] Society moved "from primitive agriculture to medieval windmills" to "steam engine[s] in the 1760s," accelerating technological progress through the industrial revolution until the 1970s.[344] But, no matter how predictable nor swift its trajectory, technology today runs the world, and quantum computers are the world's most powerful technology.

In the above equation *s* is the document score; *n* represents the number of factors Fi; and Wi, is the per factor weight. The square root is a summation equation designed to calculate the total weight for all factors.

[337] *Id.*

[338] *See* Brian S. Haney, Calculating Corporate Compliance & The Foreign Corrupt Practices Act, 19 U. PITT. J. TECH. L. & POL'Y 1, 25 (2018) (discussing algorithmic applications for automated compliance functions).

[339] Haney, *supra* note 110, at 155.

[340] *See* RAY KURZWEIL, HOW TO CREATE A MIND 249 (2012).

[341] Paul G. Allen & Mark Greaves, *The Singularity Isn't Near*, MIT TECH. REV. (Oct. 12, 2011), https://www.technologyreview.com/s/425733/paul-allen-the-singularity-isnt-near/ [https://perma.cc/HU3D-U2HE].

[342] Thiel, *supra* note 288, at 9.

[343] *Id.*

[344] *Id.*

APPENDICES

Appendix A. Summary of Notation

Notation	**Meaning**
$H_s(s)$	The energy of system.
$-\frac{1}{2}\sum_i \Delta(s)\sigma_i^x$	The Initial Hamiltonian.
$\varepsilon(s)\left(-\sum_i h_i\sigma_i^z + \sum_{i<j} J_{ij}\sigma_i^z\sigma_j^z\right)$	The Final Hamiltonian.
σ^z	Pauli matrices.
I	Identity transformation.
X	Negation.
$\otimes$	Tensor product.
$\vert x\rangle\langle y\vert$	The outer product of $\vert x\rangle$ and $\langle y\vert$.
$O(N)$	Linear time.
$\vert\psi\rangle$	Superposition.
U_p	Quantum Oracle.
A	The average of the amplitudes
X	A value of w at which X takes its minimum value.
x_j	Binary variable
x_i	Binary Variable

b_i	Bias
w_i	Interaction strength coefficient
$U(C,\gamma)$	Unitary operator.
γ	Angle.

Appendix B. Quantum Gate Matrices

Gate Name	**Number of Qubits**	**Matrix**[345]
Hadamard	1	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Pauli-Z	1	$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
NOT	1	$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
CNOT	2	$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}$
Toffoli	3	$\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$

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[345] THE NAT'L ACAD. OF SCI., ENG'G, MED., *supra* note 301, at 2-16.

Appendix C. Top Ten Most Valuable Quantum Patents

Rank	U.S. Patent No.	Title	Owner	Year
1	7,135,701	Adiabatic quantum computation with superconducting qubits	D-Wave	2006
2	9,432,024	Multiple-qubit wave-activated controlled gate	IBM	2016
3	9,400,499	Systems and methods for real-time quantum computer-based control of mobile systems	D-Wave	2016
4	10,339,466	Probabilistic inference in machine learning using a quantum oracle	Google	2019
5	10,402,743	Operating a quantum processor in a heterogeneous computing architecture	Rigetti	2019
6	10,417,553	Quantum-assisted training of neural networks	Lockheed Martin	2019
7	6,495,854	Quantum computing with d-wave superconductors	IBM	2002
8	8,504,497	Methods of adiabatic quantum computation	D-Wave	2013
9	9,697,252	Methods, apparatus, and computer program products for quantum searching for multiple search targets	AT&T	2017
10	9,836,698	Method and system for decomposing single-qubit quantum circuits into a discrete basis	Microsoft	2017

Appendix D. Honorable Mention Valuable Quantum Patents

Rank	U.S. Patent No.	Title	Owner	Year
HM	9,633,313	Method and system that implement a V-gate quantum circuit	Microsoft	2017
HM	10,474,960	Approximate gate and supercontrolled unitary gate decompositions for two-qubit operations	IBM	2019
HM	10,469,087	Bayesian tuning for quantum logic gates	Microsoft	2019
HM	9,858,531	Fault tolerant scalable modular quantum computer architecture with an enhanced control of multimode couplings between trapped ion qubits	University of Maryland; Duke University; and University of British Columbia	2018
HM	8,832,165	Computer systems and methods for quantum verification and validation	Lockheed Martin	2014
HM	7,411,187	Ion trap in a semiconductor chip	The Regents of the University of Michigan	2008

HM	10,346,760	Constructing and programming quantum hardware for robust quantum annealing processes	Google	2019
HM	10,318,881	Systems and methods for quantum processing of data	D-Wave	2019
HM	10,346,508	Re-equilibrated quantum sampling	D-Wave	2019
HM	10,176,433	Training a quantum optimizer	Microsoft	2019
HM	8,190,553	Methods and systems for quantum search, computation and memory	Thomas J. Routt	2012
HM	7,660,533	Quantum Fourier transform based information transmission system and method	The United States of America as represented by the Secretary of the Army	2010
HM	7,620,672	Method for performing classical Bayesian net calculations using a quantum computer	Tucci; Robert R.	2009
HM	7,126,106	Quantum computer and	Toshiba	2006

		quantum computation method		
HM	7,015,499	Permanent readout superconducting qubit	D-Wave	2006
HM	9,660,859	Methods and systems for quantum ready computations on the cloud	1QB Information Technologies Inc.	2017
HM	9,537,953	Methods and systems for quantum ready computations on the cloud	1QB Information Technologies Inc.	2017
HM	10346348	Quantum computing methods and devices for Majorana Tetron qubits	Microsoft	2019
HM	10,127,499	Operating a quantum processor in a heterogeneous computing architecture	Rigetti	2019
HM	10,229,355	Quantum processor and its use for implementing a neural network	1QB Information Technologies Inc.	2019
HM	10,484,479	Integration of quantum processing devices with distributed computers	QC Ware Corp.	2019