# Dimensional Analysis

- $[L]$ - length
- $[M]$ - mass
- $[T]$ - time
- $[J]$ - candela (luminosity)
- $[\theta]$ - temperature
- $[I]$ - current
- $[N]$ - mole

## Height of a Projectile

$$X_m = f(g, m, v_0) \longrightarrow [X_m] = [m^a v_0^b g^c]$$

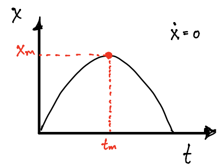
$$T^0 M^0 L = M^a \left(\frac{L}{T}\right)^b \left(\frac{L}{T^2}\right)^c = M^a L^{b+c} T^{-b-2c}$$

$$\left.\begin{aligned} L&: b+c=1 \\ T&: -b-2c=0 \\ M&: a=0 \end{aligned}\right\} \begin{aligned} c&=-1 \\ b&=2 \\ a&=0 \end{aligned} \longrightarrow \boxed{X_m \sim \frac{V_0^2}{g}} \longrightarrow X_m = \alpha \frac{V_0^2}{g}$$

$$\underbrace{\ddot{x}(t)}_{\frac{d^2x}{dt^2}} = -\frac{gR^2}{(R+x)^2} \qquad x(0)=0 \qquad \dot{x}(0)=V_0$$

$R + X \approx R$ i.e. $\frac{x}{R} \ll 1$

$$\ddot{x}(t) = -g \longrightarrow \boxed{x(t) = -\tfrac{1}{2}gt^2 + v_0 t}$$




$$\dot{x} = 0 \longrightarrow t = V_0/g \quad \therefore \quad \boxed{x_m = \frac{V_0^2}{2g}} \quad \therefore \alpha = \frac{1}{2}$$

## Drag on a Sphere

- depends on: Radius $(R)$, velocity $(v)$, density $(\rho)$, viscosity $(\mu)$

$$D_f = f(R, v, \rho, \mu) \longrightarrow [D_f] = [R^a v^b \rho^c \mu^d]$$

$$MLT^{-2} = L^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d = L^{a+b-3c-d} T^{-b-d} M^{c+d}$$

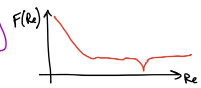
$$\left.\begin{aligned} L&: a+b-3c-d=1 \\ T&: -b-d=-2 \\ M&: c+d=1 \end{aligned}\right\} \begin{aligned} b&=2-d \\ c&=1-d \\ a&=2-d \end{aligned} \longrightarrow D_f \sim R^{2-d} v^{2-d} \rho^{1-d} \mu^d \sim R^2 v^2 \rho \Big(\underbrace{\overbrace{\frac{\mu}{Rv\rho}}^{Re \equiv \frac{Rv\rho}{\mu}}}_{\text{dimensionless product/group}}\Big)^d$$

$\Pi^d$

physical Similarity

model

$$\frac{\mu_m}{R_m V_m \rho_m} = \frac{\mu}{Rv\rho} \longrightarrow \boxed{V_m = \frac{\mu_m R \rho}{\mu R_m \rho_m} V}$$




$$\boxed{D_f = \rho R^2 v^2 F(\Pi)}$$

$$\begin{cases} D_f = \alpha R^2 v^2 \rho \Pi^d \\ D_f = \alpha_1 R^2 v^2 \rho \Pi^{d_1} + \alpha_2 R^2 v^2 \rho \Pi^{d_1} \\ D_f = \rho R^2 v^2 \underbrace{(\alpha_1 \Pi^{d_1} + \alpha_2 \Pi^{d_2} + \cdots)}_{F(\Pi)} \end{cases}$$