Tsunami modeling

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PASI 2013: Tsunamis and storm surges
Valparaiso, Chile
January 2-13, 2013

Analytical solutions of shallow water equations for wave runup

Analytical solutions

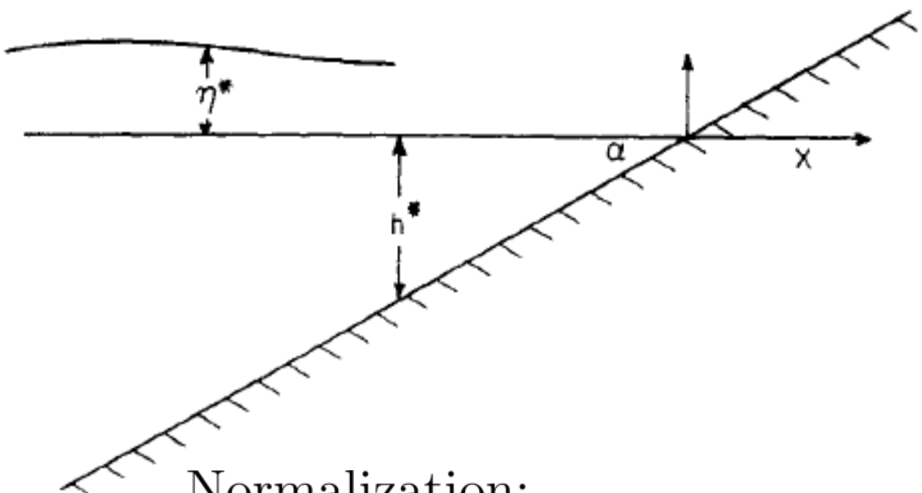
- Two-dimensional shallow water equations
  - o Linear model
  - o Nonlinear model
- Simply geometry
  - o A uniform beach
  - o A sloping beach connected to a constant depth
- Non-breaking waves
  - o Theoretical criterion: $\frac{\partial \eta}{\partial x} \to \infty$
- Frictionless sea bottom

Analytical solution by the transform technique

Water waves of finite amplitude on a sloping beach

By G. F. CARRIER and H. P. GREENSPAN
Pierce Hall, Harvard University

(*Received* 2 *December* 1957)



$$\eta_t + [(\eta - x)u]_x = 0$$
$$u_t + uu_x + \eta_x = 0$$

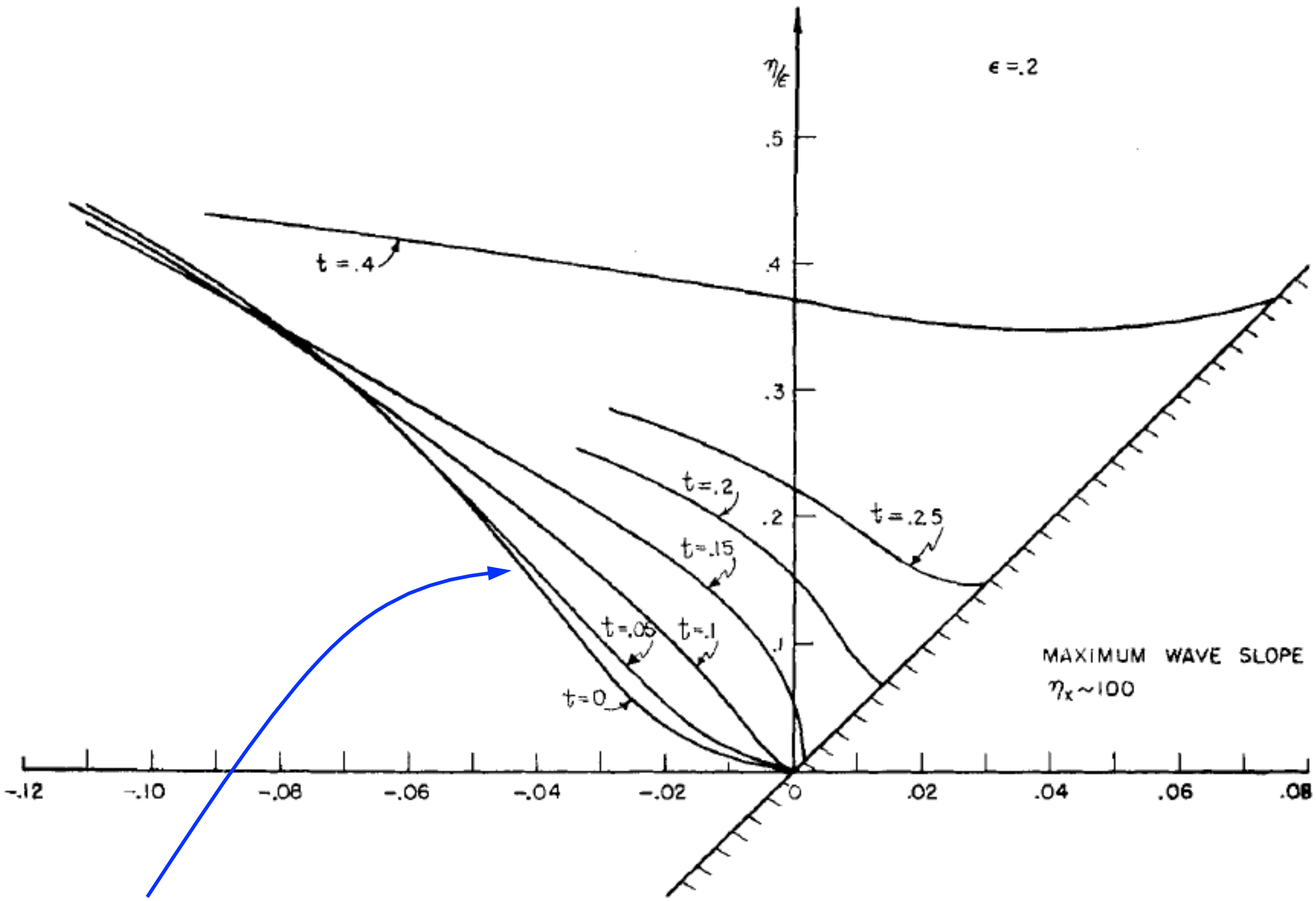
$$\begin{cases} \lambda = 2(u+t) \\ \sigma = 4\sqrt{\eta - x} \end{cases} + \begin{cases} t = \dfrac{\lambda}{2} - u \\ x = \dfrac{\phi_\lambda}{4} - \dfrac{\sigma^2}{16} - \dfrac{u^2}{2} \\ \eta = x + \dfrac{\sigma^2}{16} \end{cases}$$

Normalization:

$$x = x^*/L_0, \quad \eta = \eta^*/\alpha L_0$$
$$T_0 = L_0/\alpha g, \quad U_0 = \sqrt{\alpha g L_0}$$

$$(\sigma\phi_\sigma)_\sigma - \sigma\phi_{\lambda\lambda} = 0$$

Carrier & Greenspan (1958)



$$\eta = \epsilon \left[1 - \frac{5}{2} \frac{a^3}{(a^2+\sigma^2)^{1.5}} + \frac{3}{2} \frac{a^5}{(a^2+\sigma^2)^{2.5}} \right], \quad a = 1.5\sqrt{1+0.9\epsilon}$$

Linear waves running up a simple beach

WATER WAVE RUN-UP ON A BEACH

Joseph B. Keller and Herbert B. Keller

COPY 2

SERVICE BUREAU CORPORATION
New York, N. Y.

June, 1964

OFFICE OF NAVAL RESEARCH
DEPARTMENT OF THE NAVY
Washington, D. C.

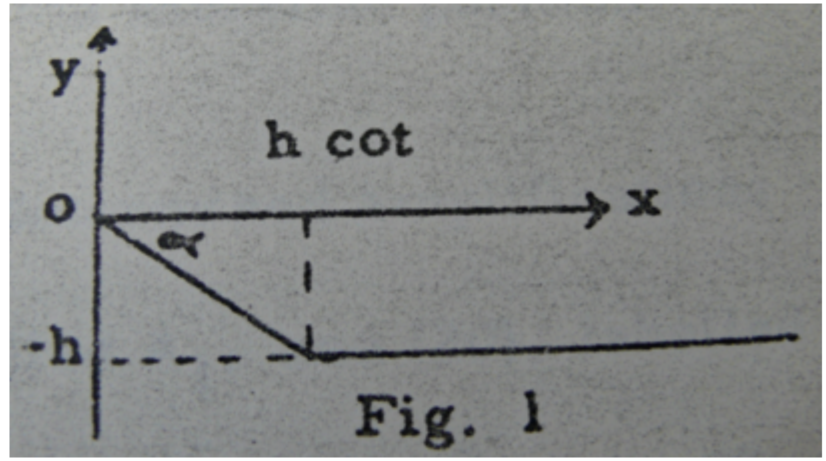


Fig. 1

- Linear shallow water equations:

$$\eta_{xx} - (\eta_x h)_x = 0$$

- Solutions:

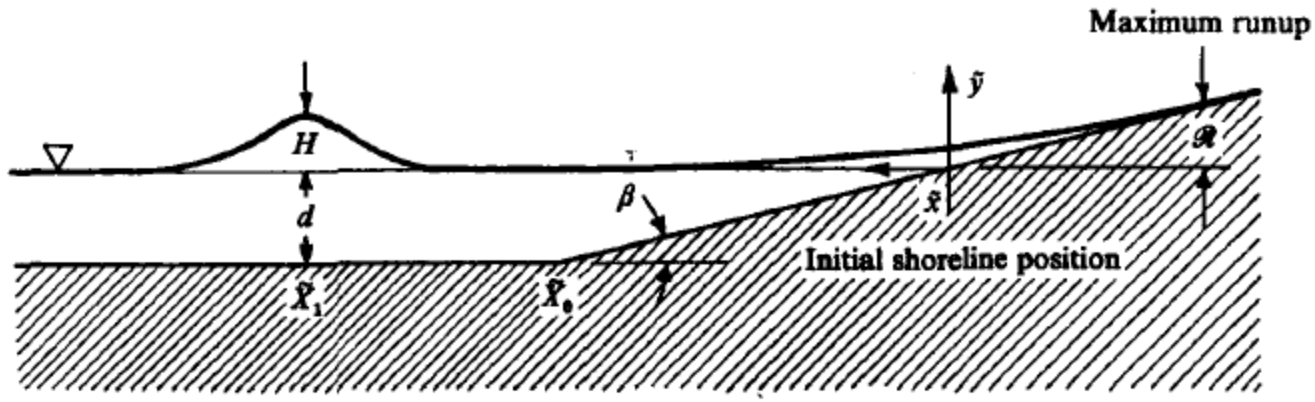
$$\eta(x,t) = \begin{cases} A_i e^{-ik(x+t)} + A_r e^{ik(x-t)}, & h = 1 \\ B J_0\left(2k\sqrt{x\cot\alpha}\right) e^{-ikt}, & h = x\tan\alpha \end{cases}$$

  - Constants A_r and B are functions of α, k, and A_i
  - $x \geq 0 \;\Rightarrow$ fixed shoreline

J. Fluid Mech. (1987), *vol.* 185, *pp.* 523–545
Printed in Great Britain

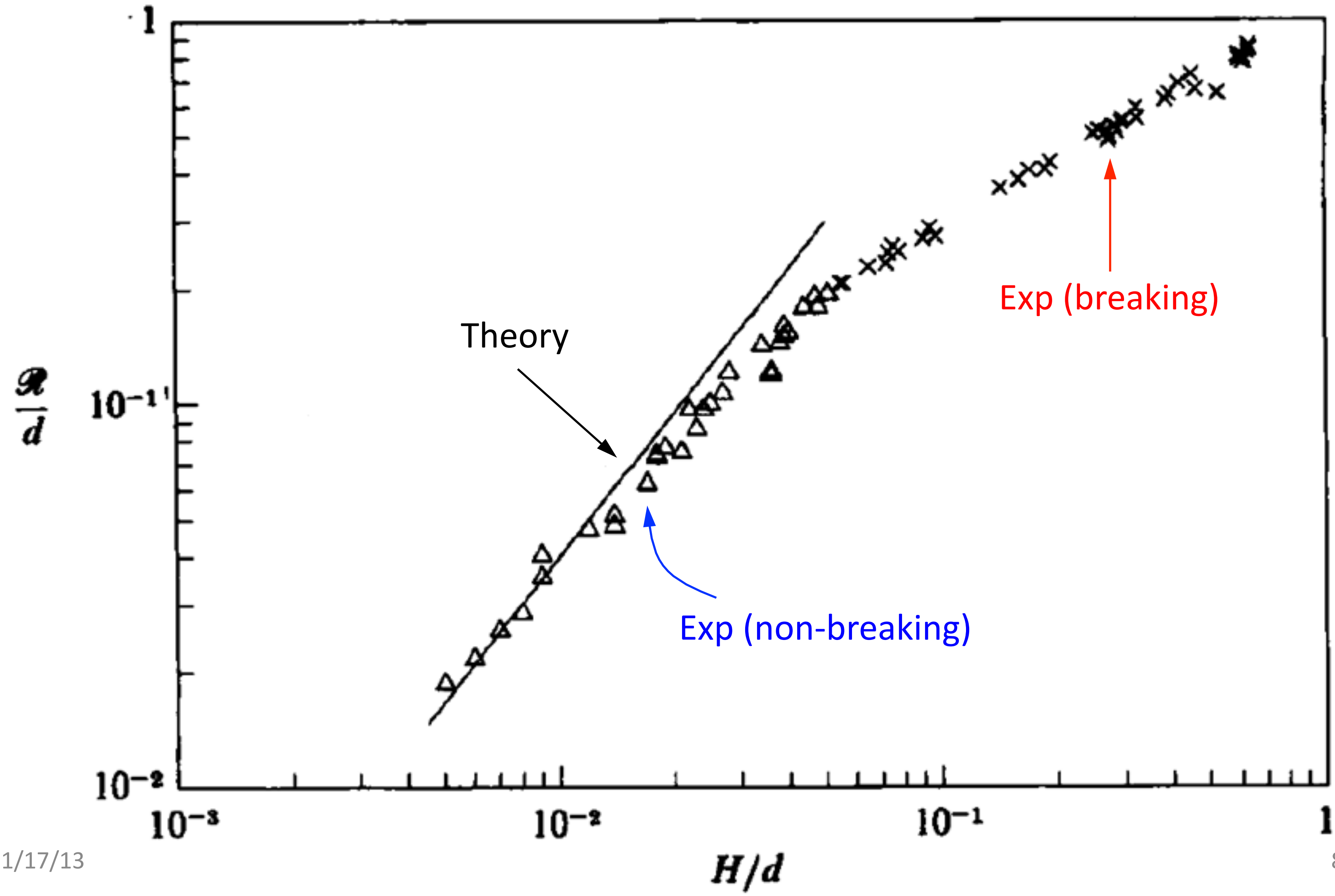
The runup of solitary waves

By COSTAS EMMANUEL SYNOLAKIS
School of Engineering, University of Southern California, Los Angeles, California 90089-0242, USA



- Shallow water equations
- Non-breaking solitary waves
- Basic idea: Keller & Keller + Carrier & Greenspan
- Analytical solutions on the sloping beach
- Run-up laws: $\frac{\mathcal{R}}{d} = 2.831\sqrt{\cot\beta}\,(H/d)^{1.25}$

Synolakis (1987): maximum runup height of solitary waves on a 1:19.85 beach



Analytical solutions

- Bathymetry
  - The 1+1 beach (e.g. Synolakis) is more useful than the uniform sloping beach (Carrier & Greenspan)
- For the 1+1 beach
  - Linear wave model: no shoreline solutions
  - Nonlinear wave model: restricted to solitary waves
- Questions…
  - Is solitary wave a good model wave for tsunamis?
  - Extension of the analytical approach to a more general (non-breaking) waveform?

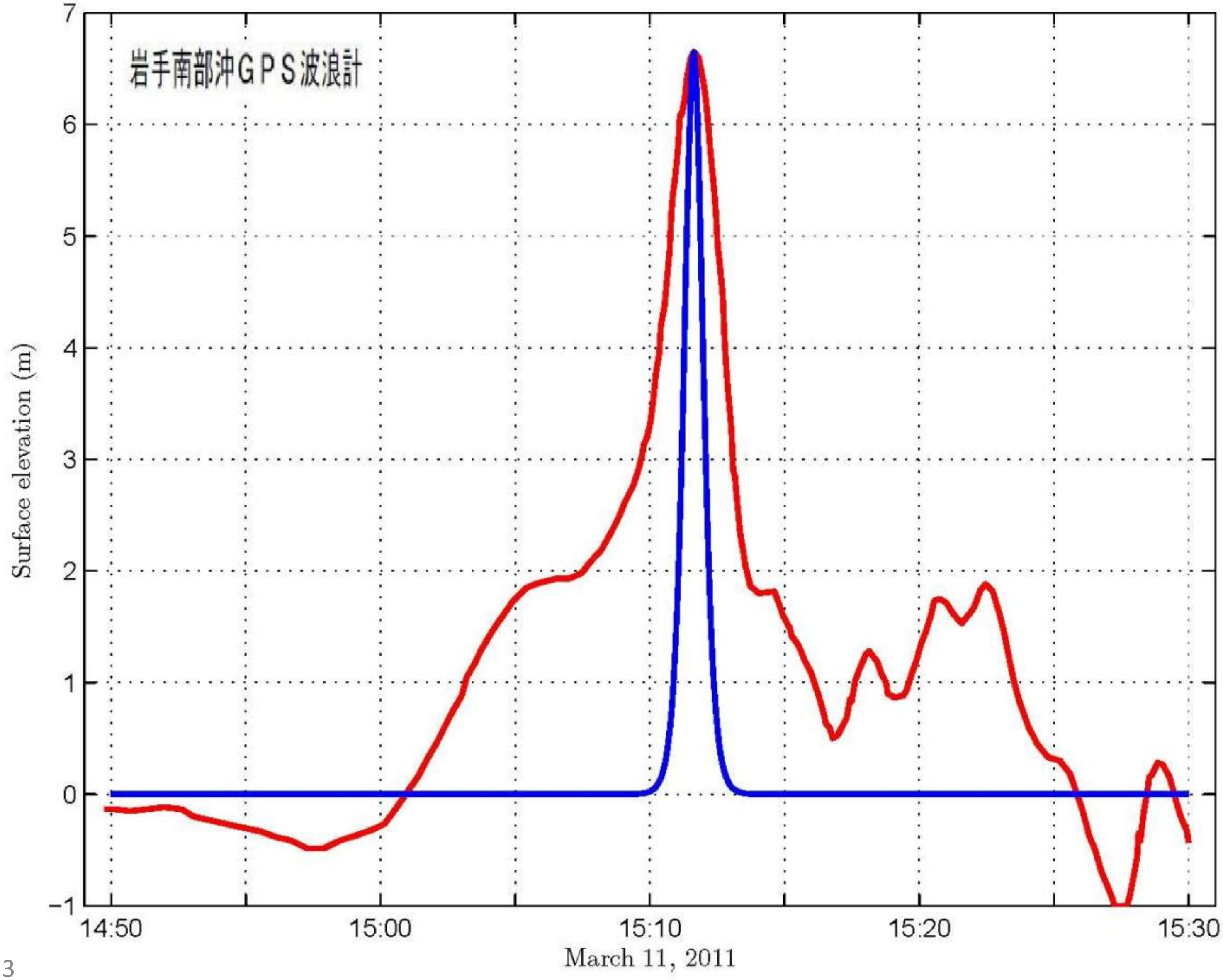
Solitary waves vs. leading tsunami waves

- Solitary wave
  - Solution of the KdV equation
  - Permanent form

$$\eta(x,t) = H\text{sech}^2\left[\frac{1}{h}\sqrt{\frac{3H}{4h}}\left(x - t\sqrt{g(h+H)}\right)\right]$$

  - Very attractive to analytical and laboratory studies
- Solitary waves and leading tsunami waves are generally not comparable
  - Field observations
  - Analytical arguments

2011 Japan Tsunami: GPS wave gauge data



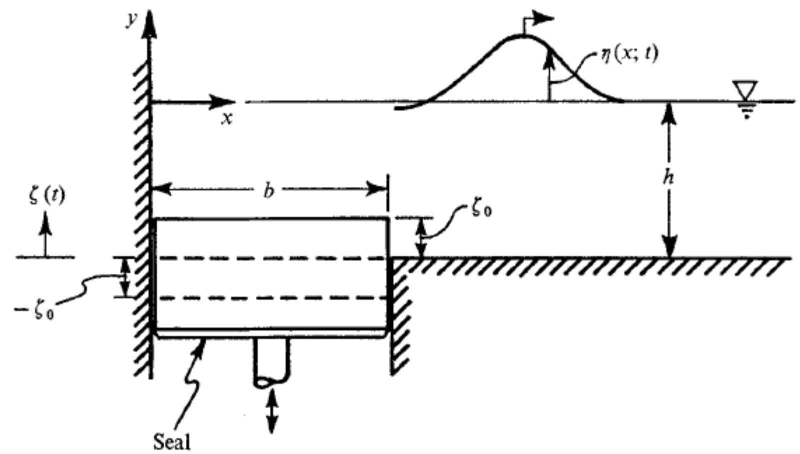
Evolutions of solitary waves?

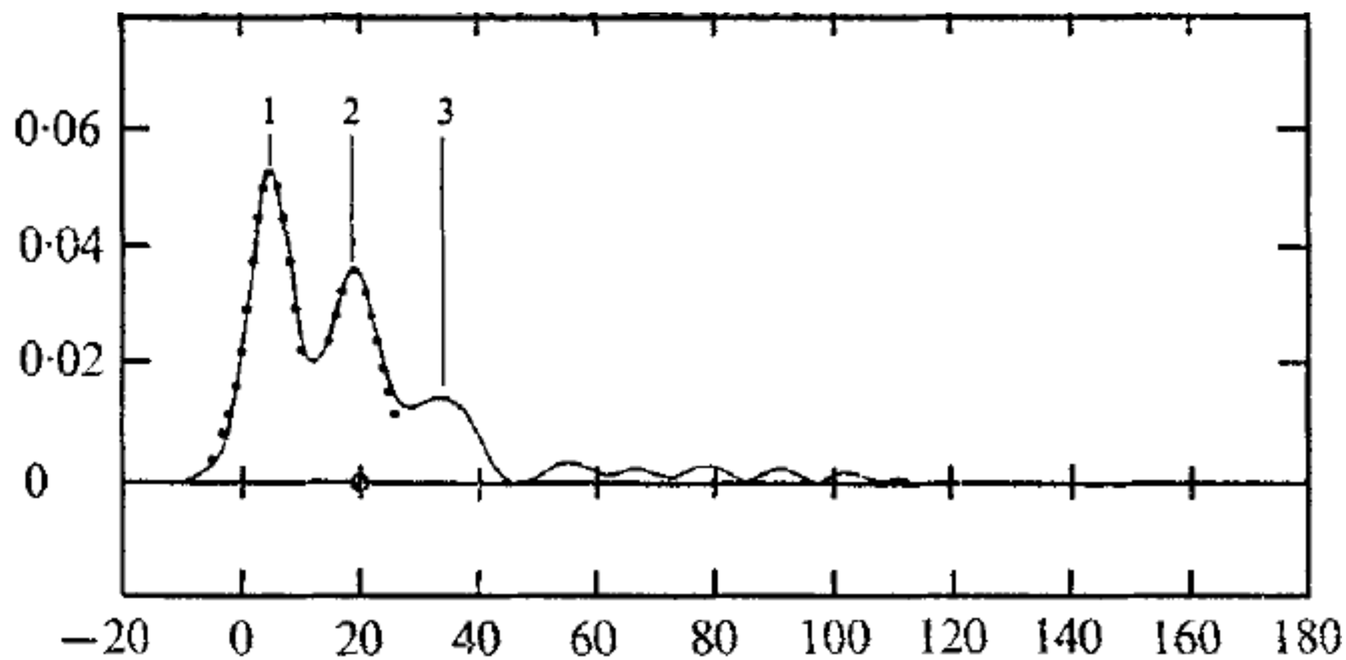
J. Fluid Mech. (1974), *vol.* 65, *part* 2, *pp.* 289–314 289
Printed in Great Britain

The Korteweg–de Vries equation and water waves. Part 2. Comparison with experiments

By JOSEPH L. HAMMACK
W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena

AND HARVEY SEGUR
Department of Mathematics, Clarkson College of Technology, Potsdam, New York



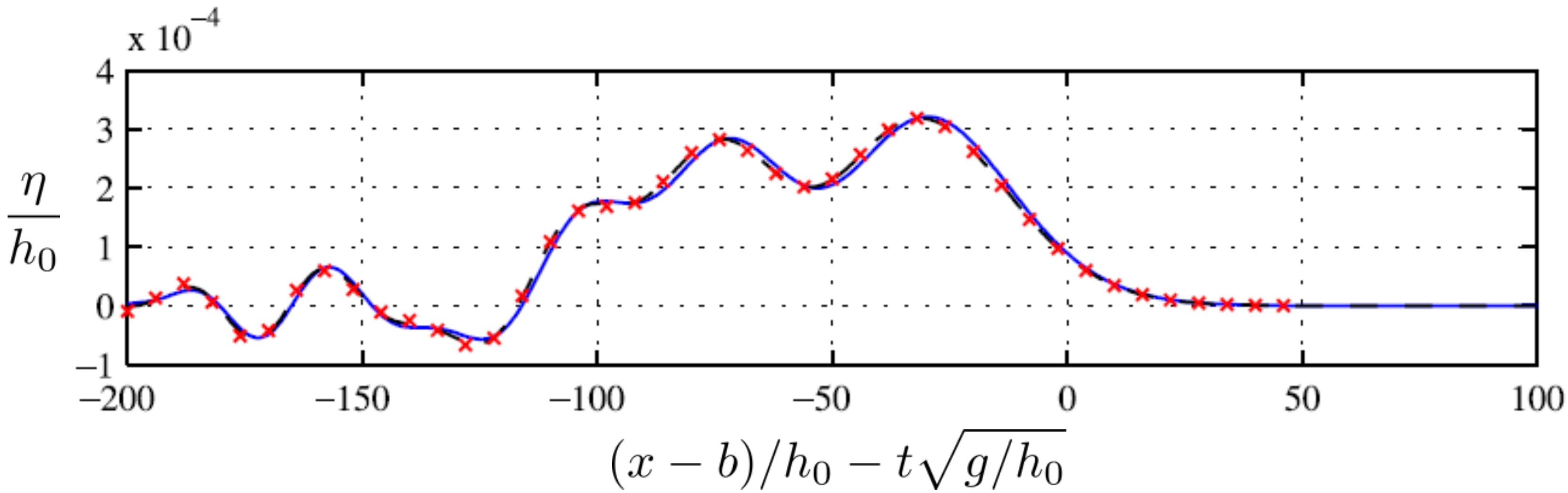


Leading tsunami waves evolve into solitary waves?......NO!

JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 113, C12012, doi:10.1029/2008JC004932, 2008

On the solitary wave paradigm for tsunamis

Per A. Madsen,[1] David R. Fuhrman,[1] and Hemming A. Schäffer[1]



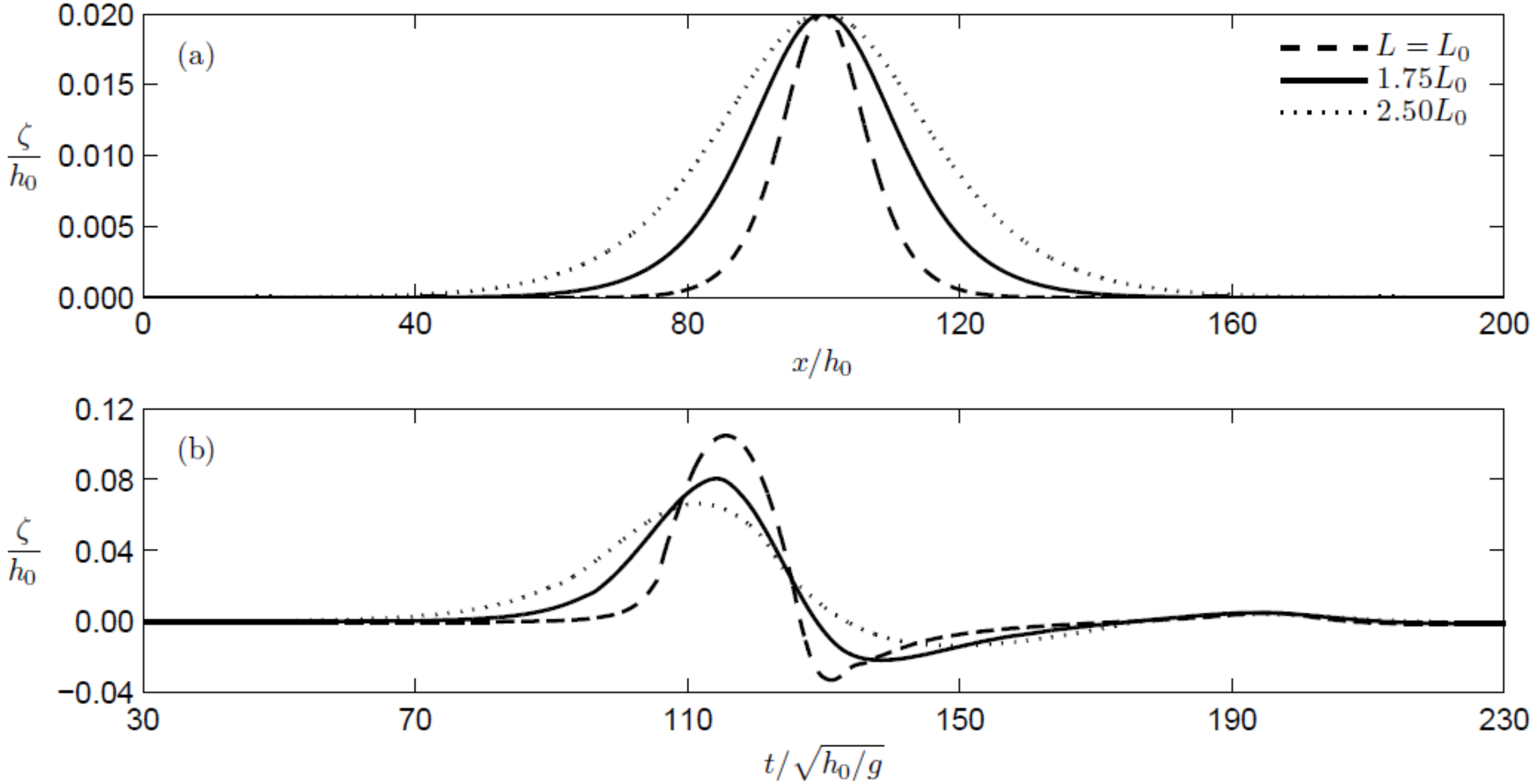
- Initial rectangular hump: $a \times 2b$
- Parameters: $\frac{a}{h_0} = 0.0005$, $\frac{b}{h_0} = 50$, $\frac{t}{\sqrt{h_0/g}} =$ **4900**

A better model wave for tsunamis?

- Solitary wave paradigm
  - Extensive studies have been based on the solitary wave theory
    - They are still important
    - They are still useful: use with care
  - How good (or bad) of these past findings?
  - Improvement on the analytical approach?
  - Suggestion on the laboratory experiments?

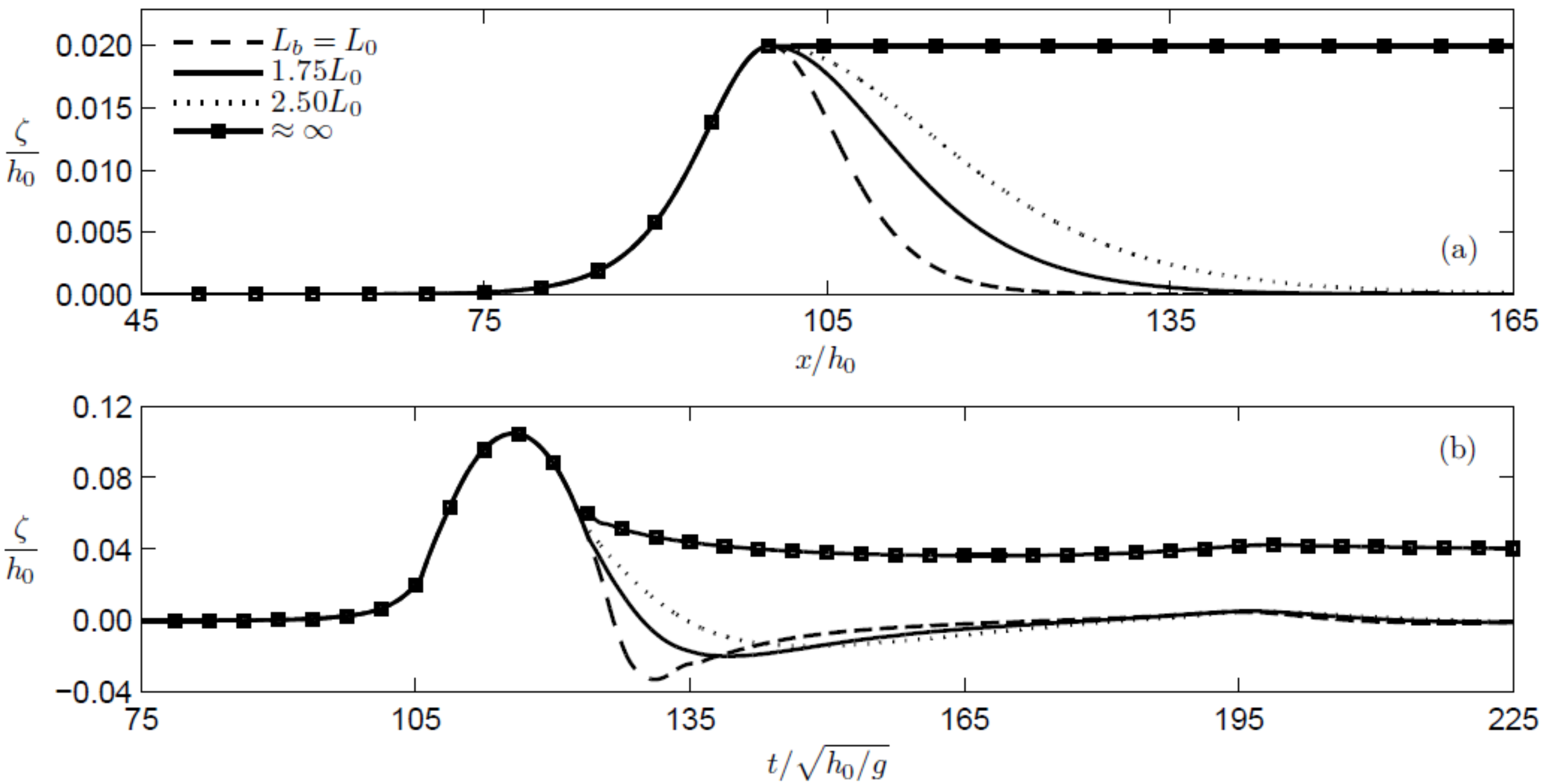
Effect of incident waveform on the runup height

Effect of the incident waveform on the runup height



Runup of three different sech$^2(\cdot)$-shape waves with the same wave height but different wavelengths. (a) Incident wave forms; (b) Evolutions of the shoreline tips.

Same accelerating phase; different decelerating phase



(a) Incident wave forms; (b) Evolutions of the shoreline.

Analytical estimation of the runup height

Extended analytical (asymptotic) solutions

J. Fluid Mech. (2010), *vol.* 645, *pp.* 27–57. © Cambridge University Press 2010 27
doi:10.1017/S0022112009992485

Analytical solutions for tsunami runup on a plane beach: single waves, *N*-waves and transient waves

PER A. MADSEN† AND HEMMING A. SCHÄFFER‡
Department of Mechanical Engineering, Technical University of Denmark, 2800 Kgs Lyngby, Denmark

- 1+1 beach model (beach slope *s*)
- Extend the approach by Synolakis (JFM 1987)
- Shoreline solutions in the transformed domain

$$U(\tau) = -\frac{2\sqrt{t_0}}{s}\int_{2t_0}^{\tau}\frac{F_u(\tau - t)}{\sqrt{\tau - 2t_0}}dt,$$

$$R(\tau) = 2\sqrt{t_0}\int_{2t_0}^{\tau}\frac{F_\eta(\tau - t)}{\sqrt{\tau - 2t_0}}dt - \frac{U^2}{2g},$$

where

$$\tau = t - \frac{U(\tau)}{sg}$$

$$t_0 = \frac{h_0/s}{\sqrt{gh_0}}$$

$$F_u(t) = \frac{\partial}{\partial t}F_\eta(t) = \frac{\partial^2}{\partial t^2}\,\eta(t)\big|_{\text{toe}}$$

New runup solutions for cnoidal waves
(Chan & Liu 2012)

- Alternative expressions for the cnoidal waves

$$\eta(x,t) = \sum_{n=1}^{\infty} \widetilde{A}_n \cos\left(\widetilde{\Omega}_n \left(t - \widetilde{t}\right)\right) \quad \text{or} \quad \eta(x,t) = \widehat{\zeta}_2 + \sum_{n=-\infty}^{\infty} \widehat{H}\mathrm{sech}^2\left[\widehat{\Omega}(t - \widehat{t}_n)\right]$$

- Runup solutions

$$U(\tau) = \sum_{n=1}^{\infty} \frac{2\widetilde{A}_n}{s} \sqrt{\pi \widetilde{\Omega}_n^3 t_0} \sin\left(\widetilde{\theta}_n + \frac{\pi}{4}\right)$$

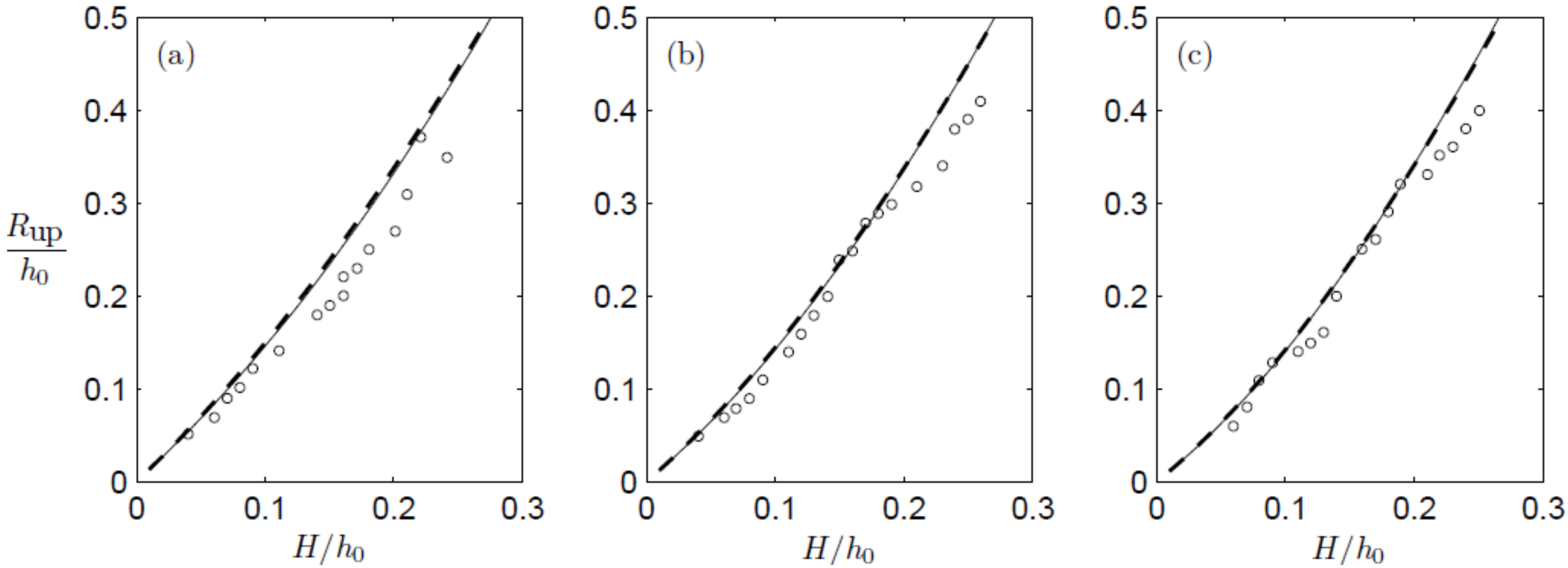
$$R(\tau) = \sum_{n=1}^{\infty} 2\widetilde{A}_n \sqrt{\pi \widetilde{\Omega}_n t_0} \cos\left(\widetilde{\theta}_n + \frac{\pi}{4}\right) - \frac{U^2}{2g}$$

$$U(\tau) = \sum_{n=-\infty}^{\infty} \frac{16}{s} \widehat{H} \sqrt{2\pi \widehat{\Omega}^3 t_0} \mathrm{Li}\left(-\frac{5}{2}, z_n\right)$$

$$R(\tau) = \sum_{n=\infty}^{\infty} -8\widehat{H} \sqrt{2\pi \widehat{\Omega} t_0} \mathrm{Li}\left(-\frac{3}{2}, z_n\right) - \frac{U^2}{2g}$$

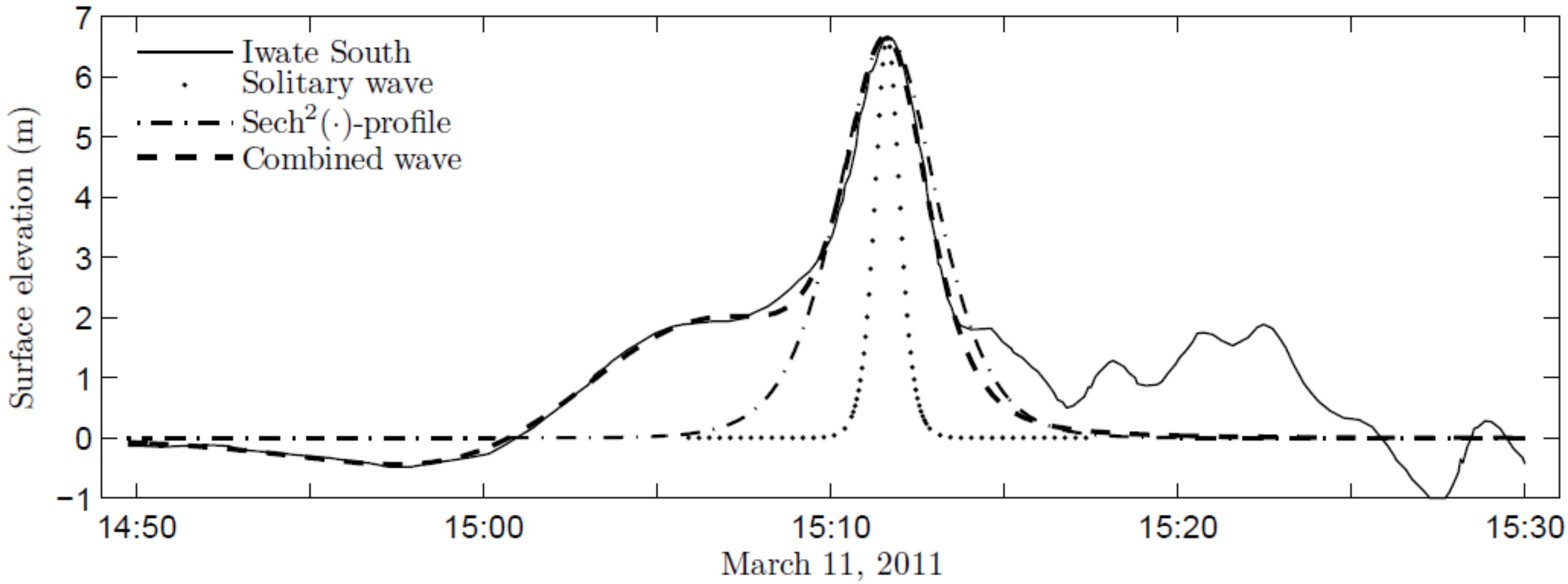
Runup of cnoidal waves

Maximum runup height on a 1:1 slope.



Symbols are the experimental data of Ohyama (1987). Solid lines plot the new analytical solutions. Water depth is fixed at 0.2 m. Wave frequencies are different in cases (a) to (c).

Extension of the analytical approach



- Solid line: 2011 Japan Tsunami record
- Dot: solitary wave profile
- Dashed-dotted line: $\text{sech}^2(\cdot)$-profile with a relaxed wavelength
- Dashed line: $\eta(t) = \sum_{n=1}^{3} h_n \text{sech}^2 \Omega_n (t - t_n)$
- Extension of the existing analytical solutions for solitary waves?

Runup of leading tsunami waves

- Analytical solution
  - A quick estimation on the runup height
- Use the 2011 Japan Tsunami as an example
  - Record at a GPS-based station as the input
- Assuming a 1+1 beach with a 1-on-10 slope
  - Analytical approach works only for a simple beach
  - Estimation on the runup height: the final slope (the closest to the shoreline) has the dominant effect on the runup (Kanoglu & Synolakis 1998)

Estimation of the runup height