GeoClaw and Tsunami Modeling: unique numerical aspects and future directions

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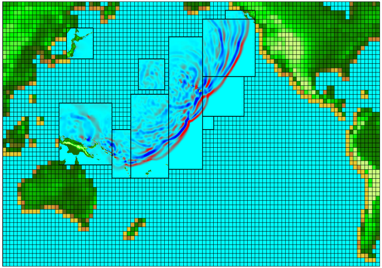
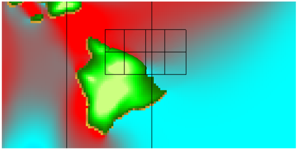
PASI, Valparaiso, Chile, Jan. 2013





Acknowledgments and Collaborators

Numerical methods and software development:

- Randall LeVeque, University of Washington.
- Marsha Berger, Courant Institute, NYU.
- Kyle Mandli, University of Texas, Austin.
- Donna Calhoun, Boise State University.
- David Ketcheson, KAUST.
- Dave Yuen, University of Minnesota.

Geophysical modeling:

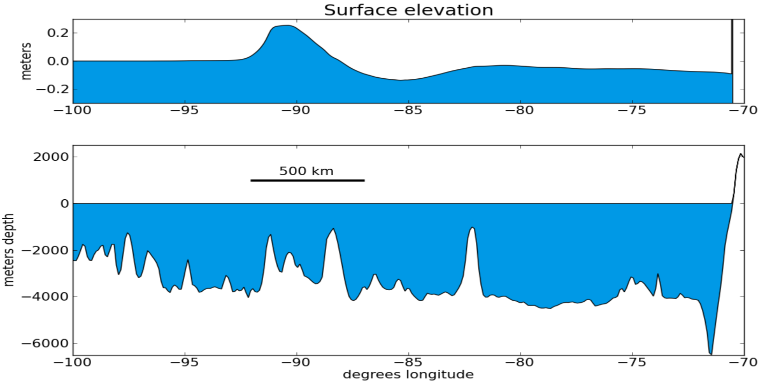
- Richard Iverson, U.S. Geological Survey.
- Roger Denlinger, U.S. Geological Survey.

Overview

- Survey of some unique numerical issues for tsunami modeling
  - well-balancing for ocean propagation
  - depth-positivity (wet/dry problem)
  - shoreline instabilities
  - ill-posedness and instabilities (*e.g.*, resonance, roll waves, etc.)
- Introduction to granular-fluid flows
  - modeling multiphase "non-rheological" flows
  - coupling with tsunamis?

Steady-states and well balancing

- well-balancing has to do with numerically maintaining particular steady-states
- required to resolve small perturbations to those steady-states
- cannot be determined by order of accuracy or convergence
  $\rightarrow$ should be satisfied on coarse grids



Steady-states and well balancing

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0,$$

$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}\left(hu^2 + \frac{1}{2}gh^2\right) + \frac{\partial (huv)}{\partial y} = -gh\frac{\partial b}{\partial x},$$

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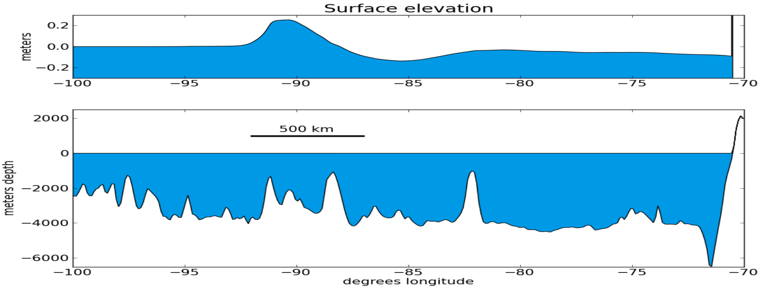

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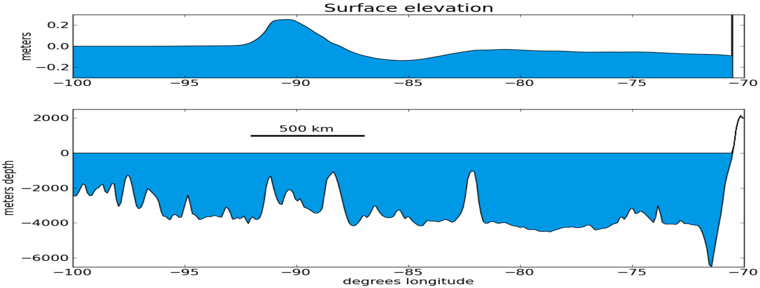
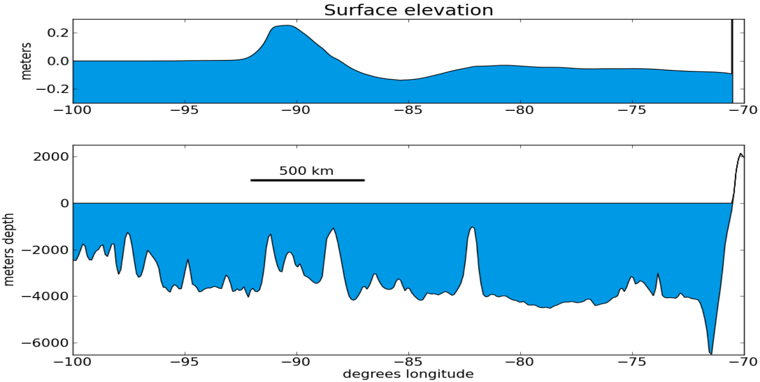


Steady-states and well balancing

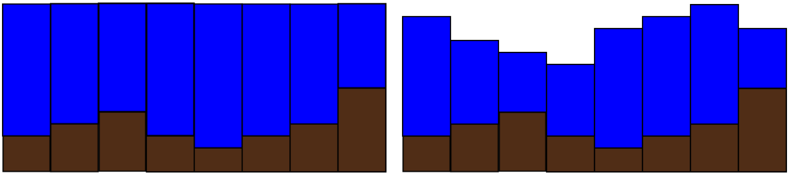
Flow dynamics are small perturbations to a balanced steady state:

$$q_t + \mathcal{A}(q)q_x = \psi(q,x,y),$$
$$\mathcal{A}(q)q_x \approx \psi(q,x,y)$$



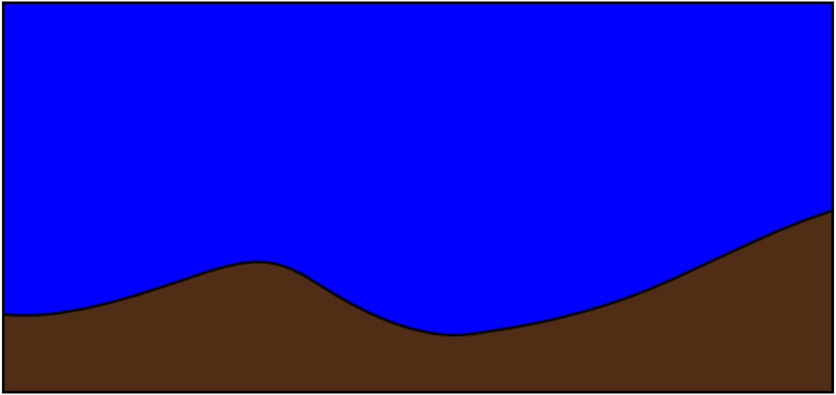
Steady-states and well balancing

- applications with delicate balanced steady states require specialized methods.
- a numerical scheme is well-balanced if it exactly preserves discontinuous steady-state weak solutions.
- traditional schemes do not satisfy this property.



Balance law $q_t + f(q)_x = \psi(q, x)$

- Steady states arise when $f(q)_x = \psi(q, x)$



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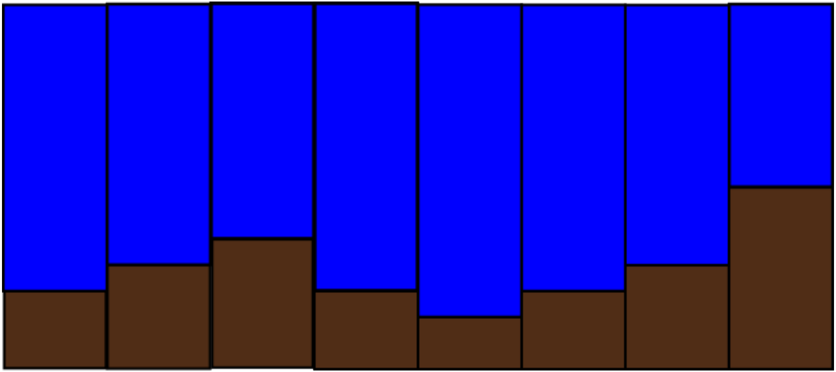

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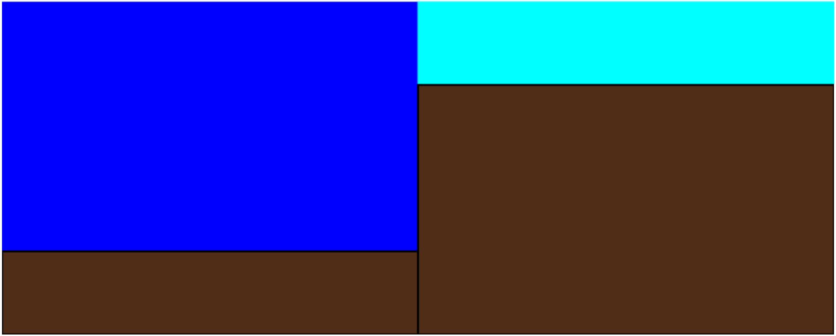
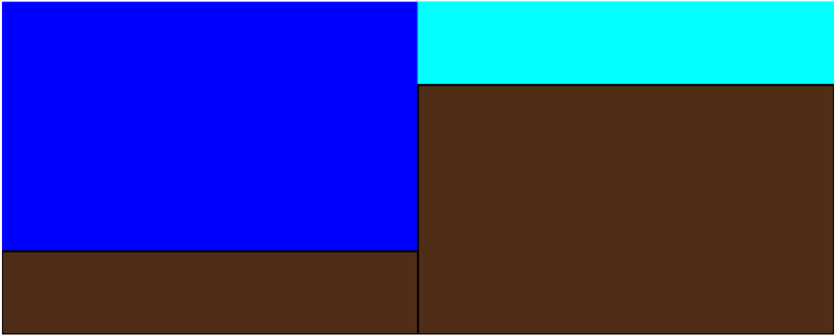


Balance law $q_t + f(q)_x = \psi(q, x)$: fractional step method

- step 1: $Q^n \to Q^*$: solve homogeneous problem: $q_t + f(q)_x = 0$.



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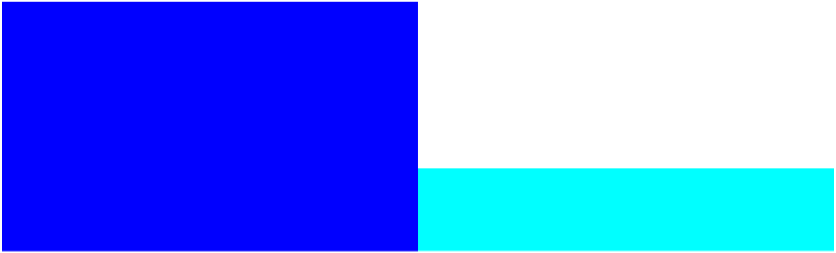

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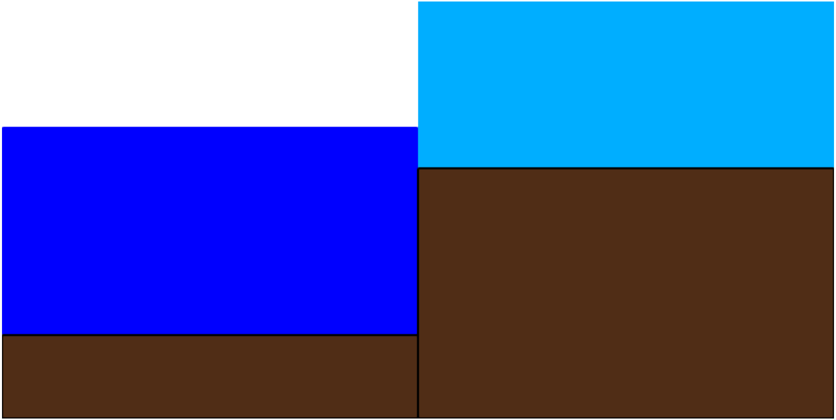

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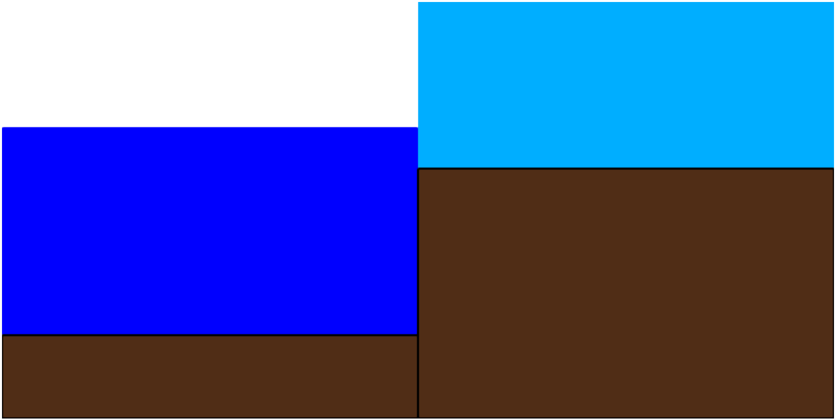
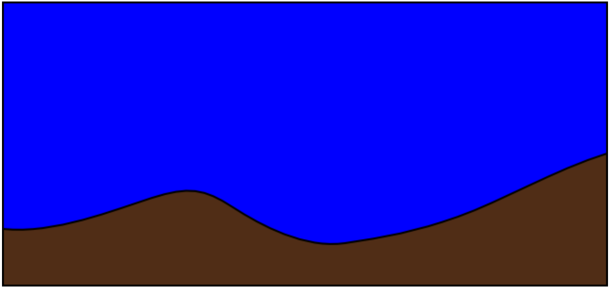
- step 2: $Q^* \to Q^{n+1}$: solve ODEs for the source term: $q_t = \psi(q, x)$



Balancing steady states: $q_t + f(q)_x = \psi(q,x)$

- Preserving steady states $q_t + f(q)_x = \psi(q,x)$.
- $\mathcal{W} = [q, \varphi(q), b]^T$: $\mathcal{W}_t + \mathcal{A}(\mathcal{W})\mathcal{W}_x = 0$.
- The steady state field

$$\mathcal{A}(\mathcal{W})\mathcal{W}_x = 0, \quad \Rightarrow \mathcal{W}_x = r^0(\mathcal{W}), \quad \lambda^0(\mathcal{W}) = 0$$



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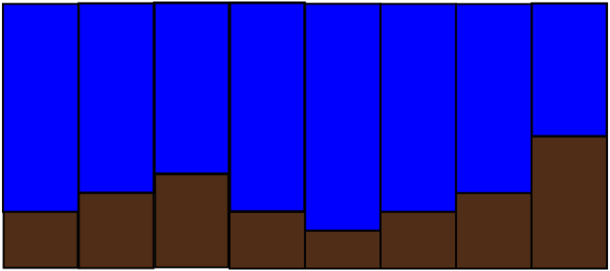

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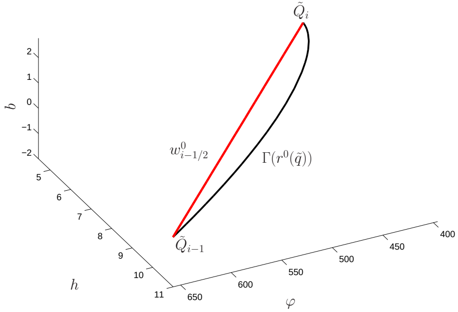

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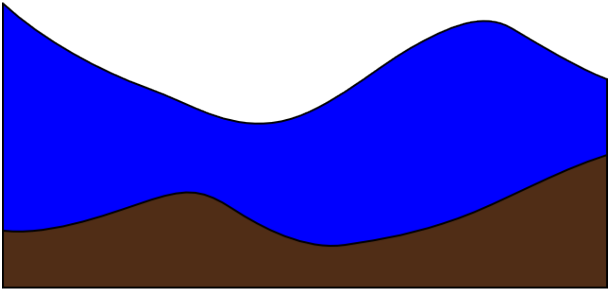

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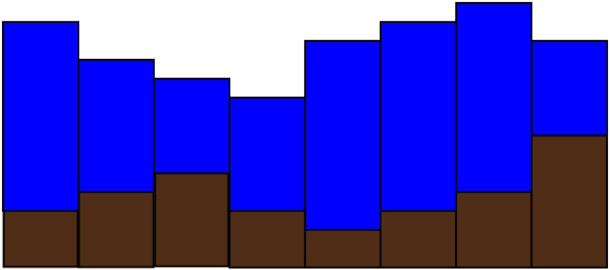
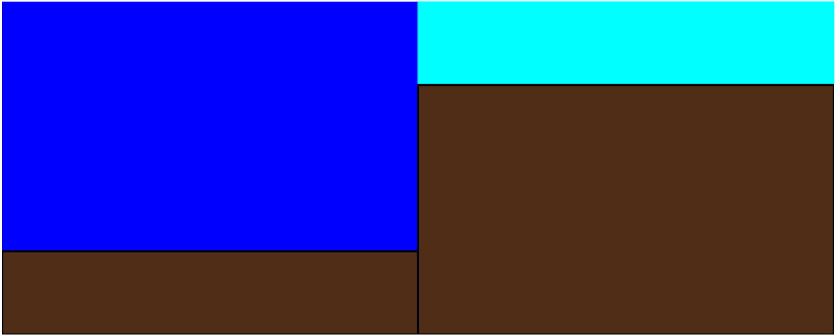


Balance law $q_t + f(q)_x = \psi(q, x)$: steady state wave

- solve augmented homogeneous system: $\mathcal{W}_t + \mathcal{A}(\mathcal{W})\mathcal{W}_x = 0$.
- motionless steady states: stationary steady state wave only



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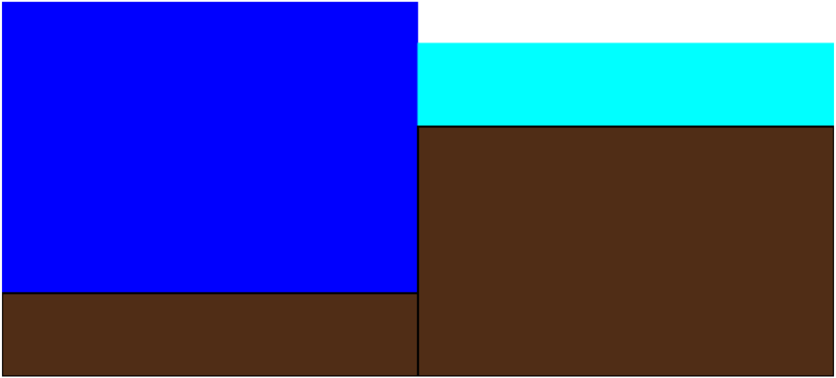

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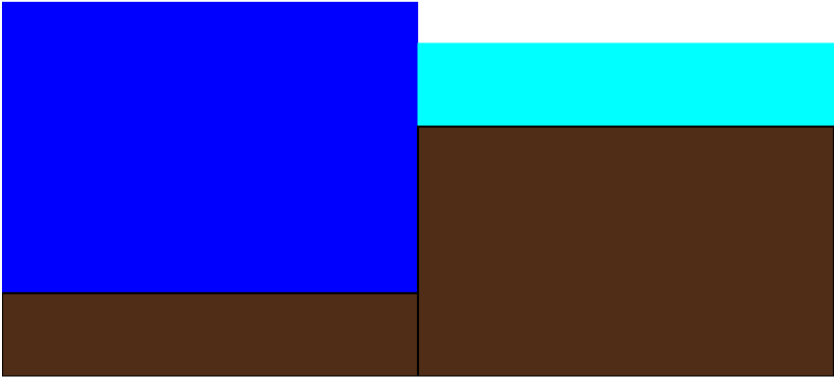

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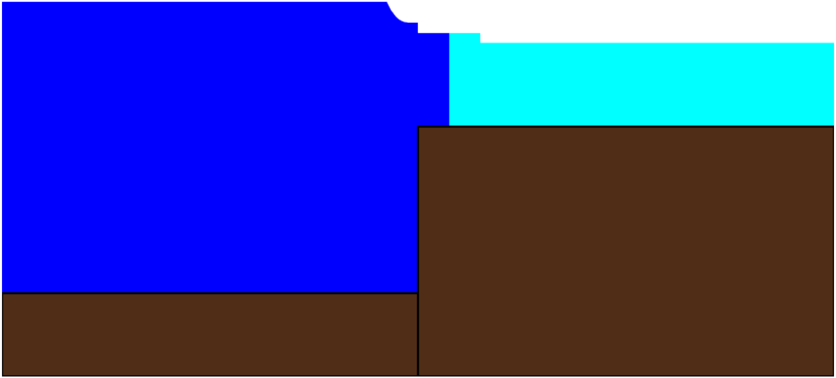

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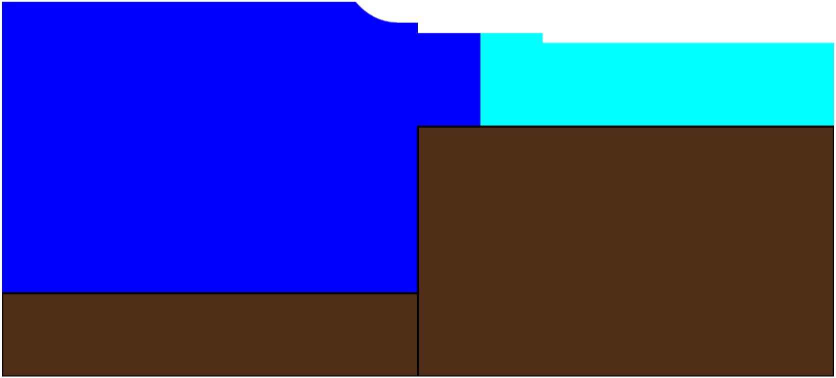

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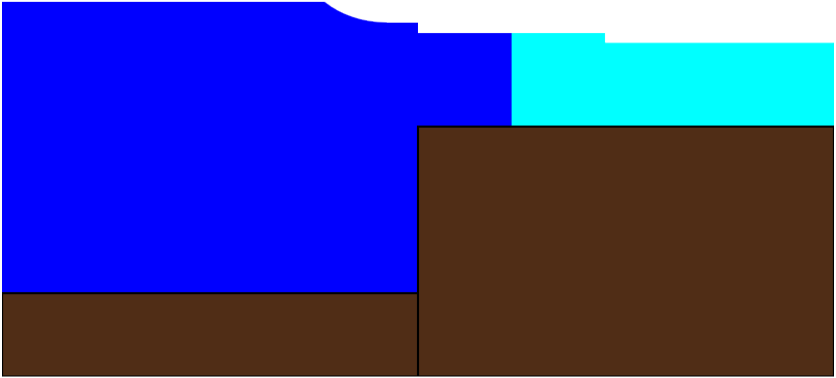

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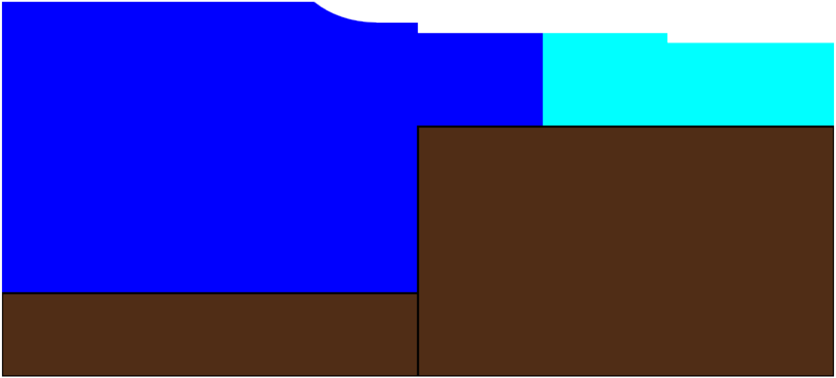

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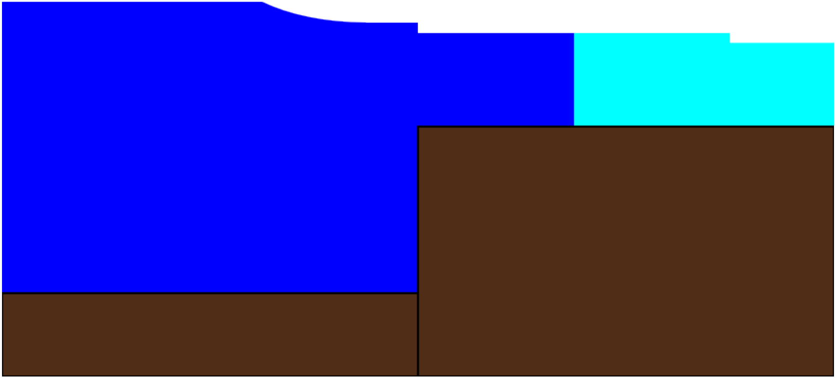

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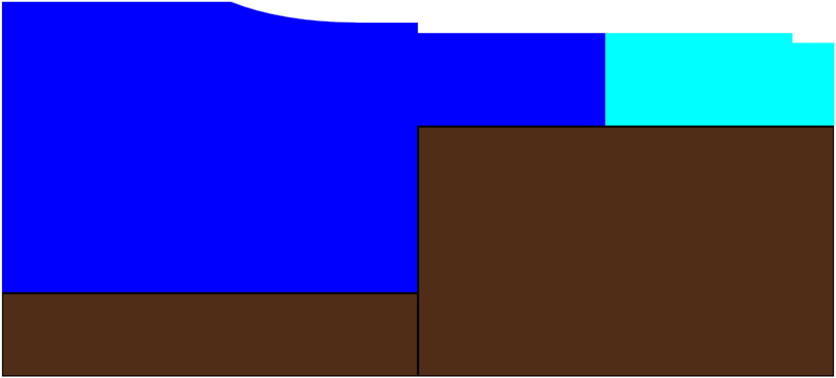

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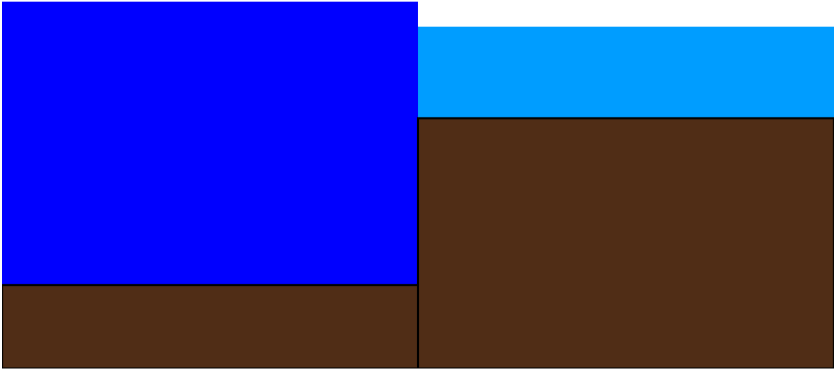


Shorelines and inundation

- shallow water equations present difficulties where $h \to 0$
- determining the motion of the shoreline can be difficult
- often ad hoc approaches are used
- this is prone to numerical instabilities

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0,$$
$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}(hu^2 + \frac{1}{2}gh^2) = -gh\frac{\partial b}{\partial x},$$

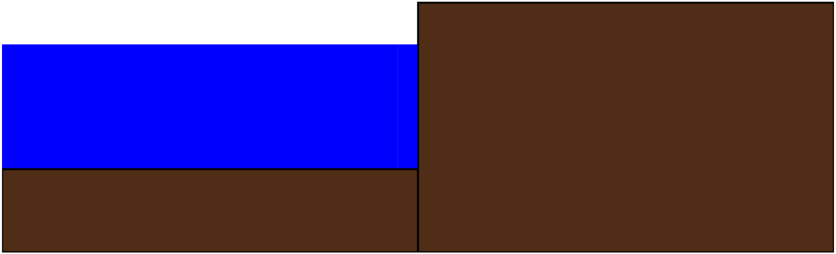
Shorelines and inundation

- the equations are only valid where $h > 0$
- the Riemann problem between wet and dry states can be exactly solved
- $\rightarrow$ only for the homogeneous problem (no bathymetry!)

$$\frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} = 0,$$

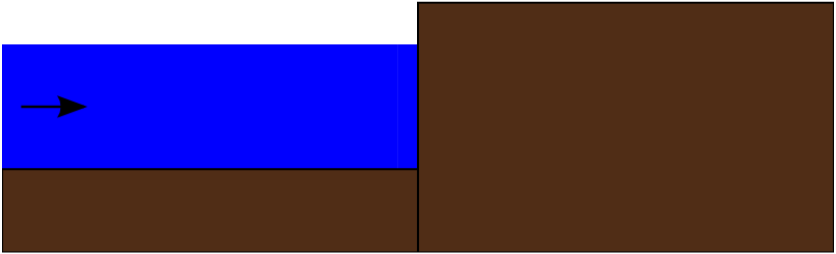
$$\frac{\partial (hu)}{\partial t} + \frac{\partial}{\partial x}(hu^2 + \frac{1}{2}gh^2) = -gh\frac{\partial b}{\partial x},$$

motionless steady state at the shoreline: no waves.



The Riemann problem at the shoreline.

flow at the shoreline: velocity insufficient for inundation.



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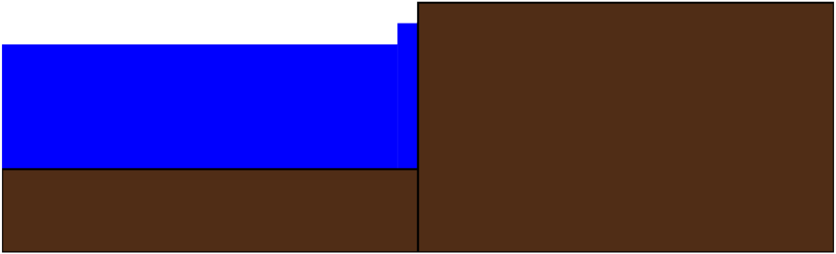

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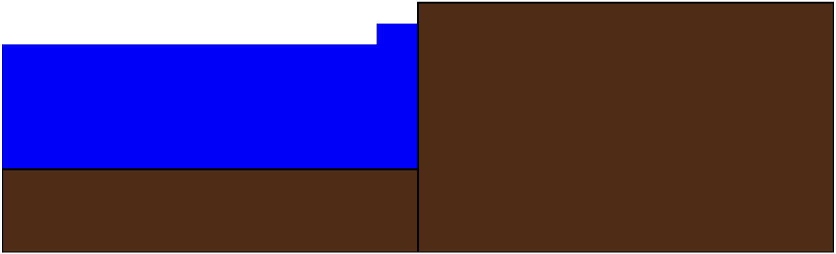

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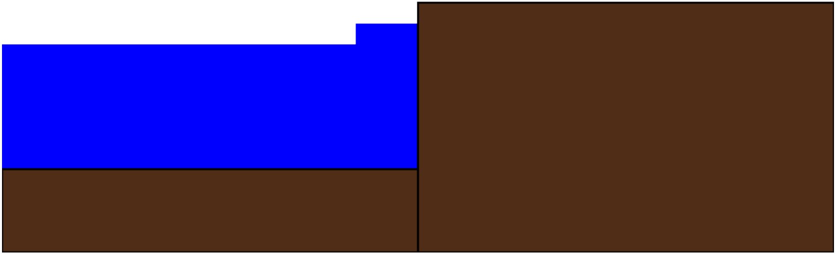

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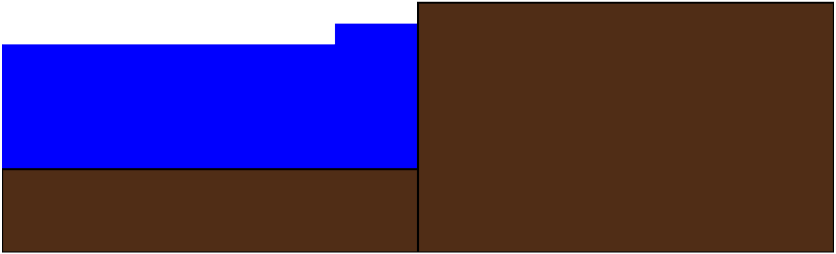

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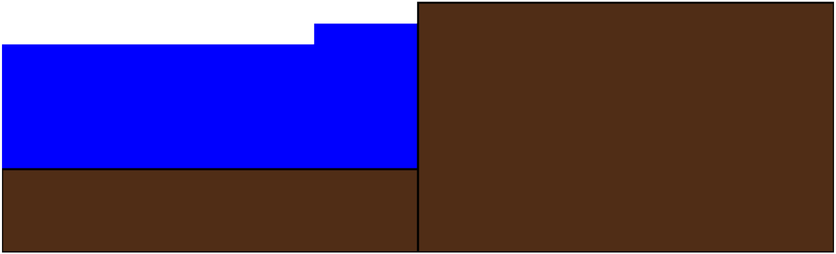

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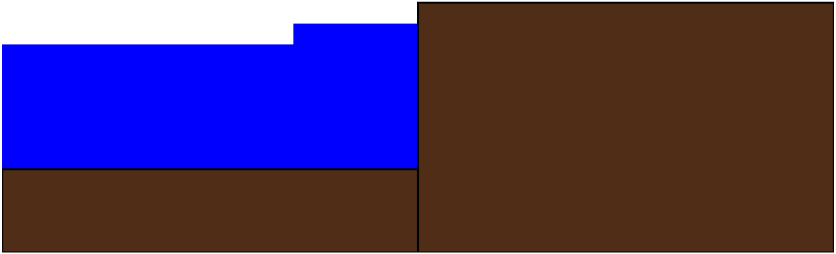

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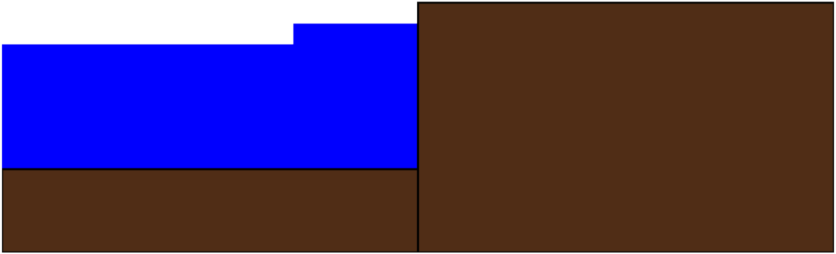

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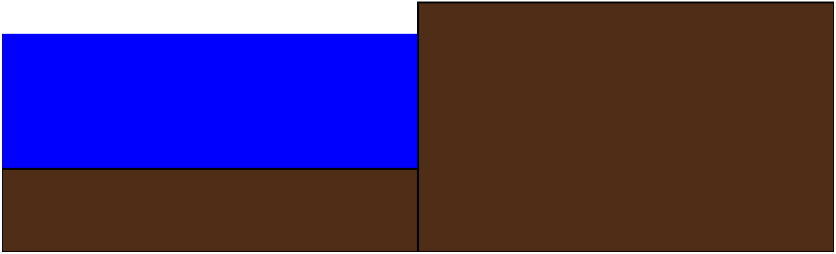

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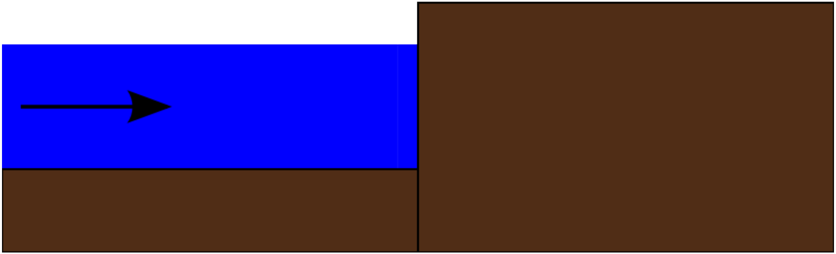

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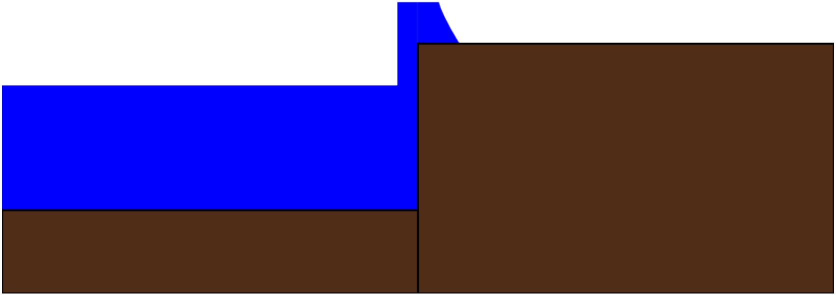

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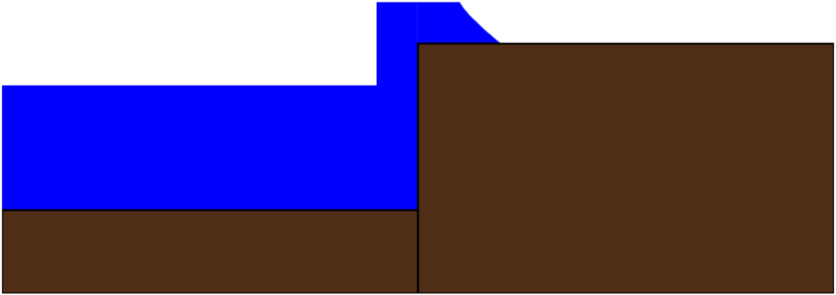

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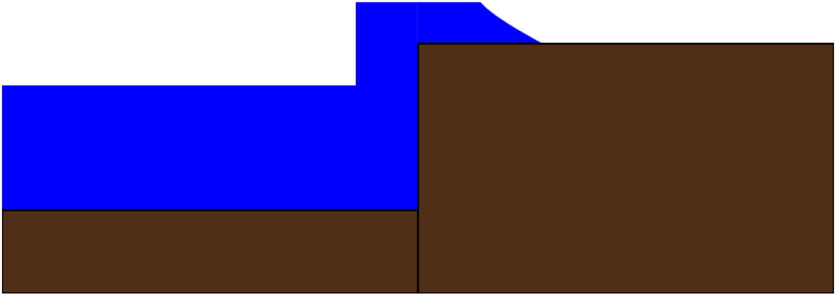

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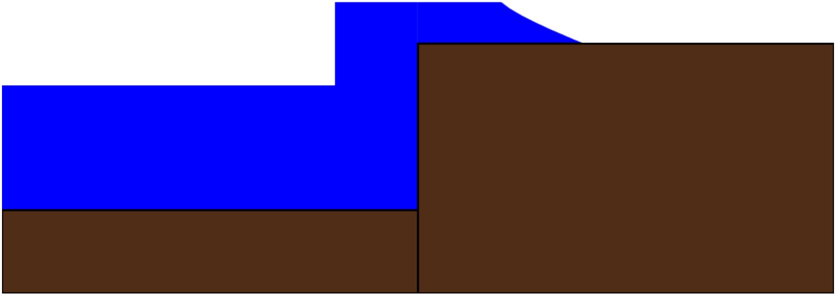
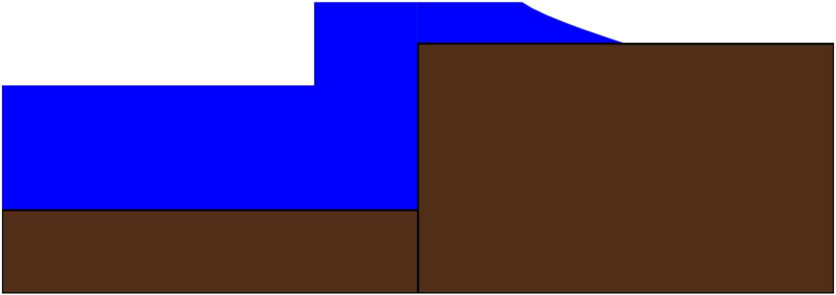


The Riemann problem at the shoreline.

flow at the shoreline: inundation.



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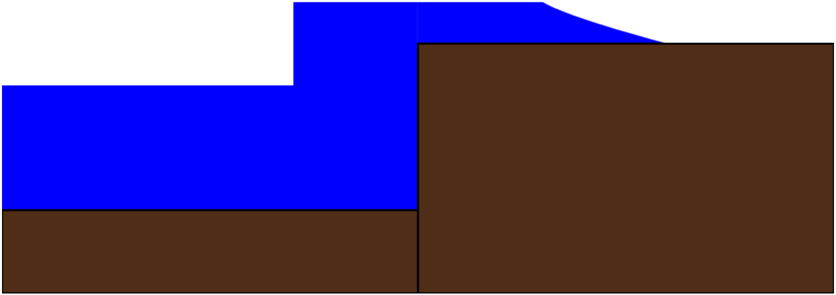

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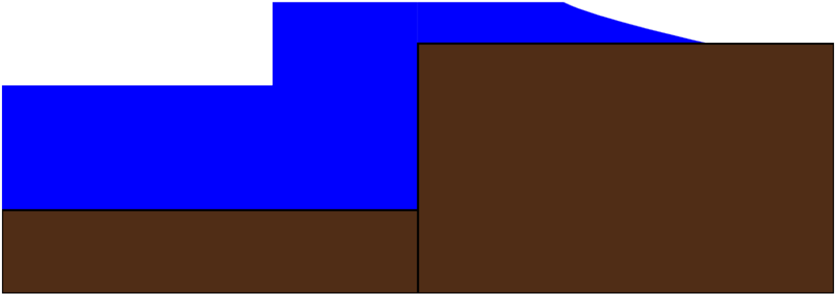

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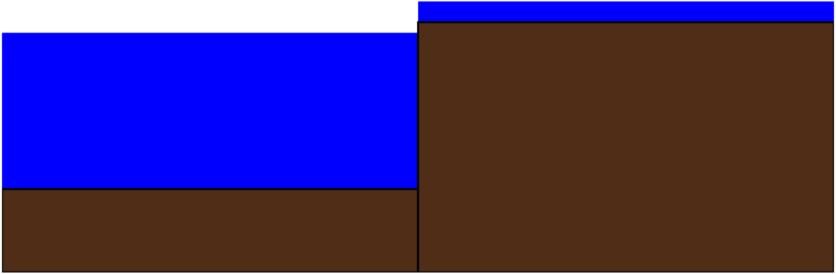

Overland flooding: Malpasset dam, France 1959

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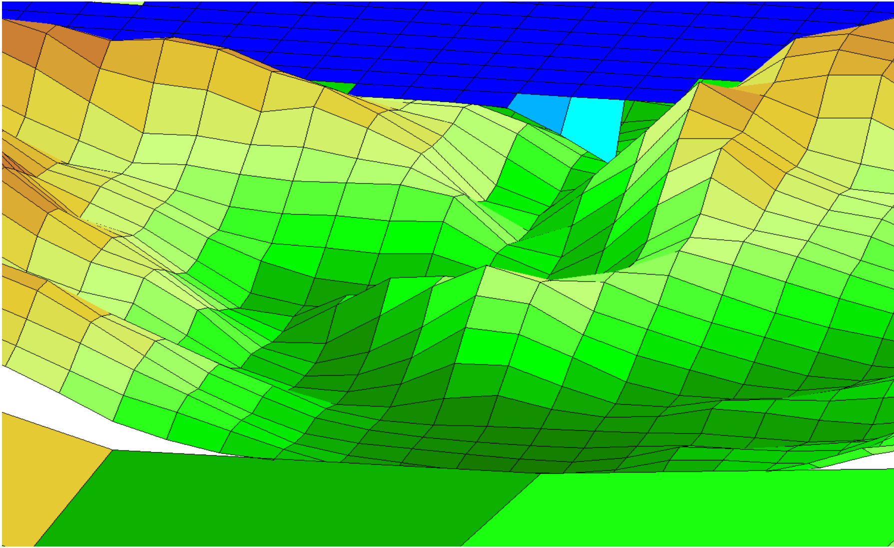
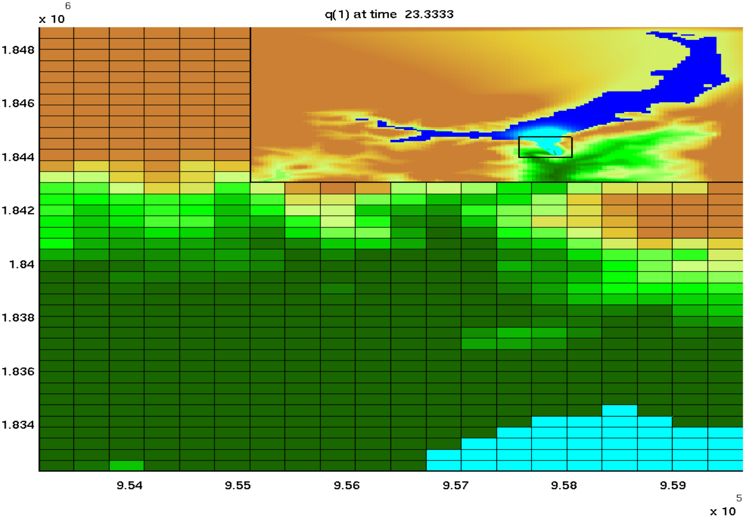


Overland flooding: Malpasset dam, France 1959



Landslides and Debris Flows

Debris flows, landslides etc.: liquified masses of soil and rock.

Indonesian Lahar Movie   Ritigraben Switzerland Debris Flow Movie

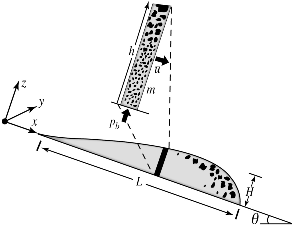
Debris-flow mobilization

Mobilization: $q_t + f(q)_x + \mathcal{W}(q)q_x = \psi(q,x)$

- failure occurs when the driving forces slightly exceed the shear strength at any one point: a small perturbation to a balanced steady state: $f(q)_x + \mathcal{W}(q)q_x \approx \psi(q,x)$
- failure: an equilibrium is perturbed...what happens next?
  1. rapid temporary instability: (shear contraction $\rightarrow$ increased pore pressure $\rightarrow$ decreased shear strength $\rightarrow$ landslide.)
  2. quick stabilization: (shear dilation $\rightarrow$ decreased pore pressure $\rightarrow$ shear strength reestablished $\rightarrow$ localized slump.)
  3. anything intermediate...*e.g.*, stick-slip.
- modeling the outcome at least requires well-balanced methods...if it can be done at all. Numerical conservation must be maintained for $f(q)_x$ while simultaneously being well-balanced.

Mathematical model

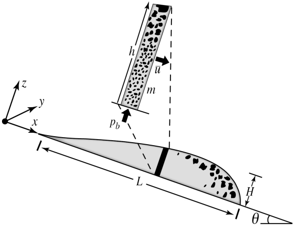
Model equations are depth-averaged:



- Integrating out the vertical component gives 2D system for h, u, v, m, p_b
- degree of accuracy lost upon depth-averaging depends on shallowness;
- computationally tractable for large-scale problems.

Mathematical model

Model equations are a hyperbolic system:



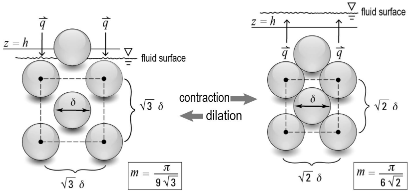
- similar in form to the shallow water equations;
- strictly hyperbolic system (desirable stability properties);

$$q_t + \mathcal{A}(q)q_x + \mathcal{B}(q)q_y = \psi(q),$$

$$\text{where} \quad q = (h, hu, hv, hm, p_b)^{\mathrm{T}}$$

- coupled evolution of pore pressure and solid-volume fraction

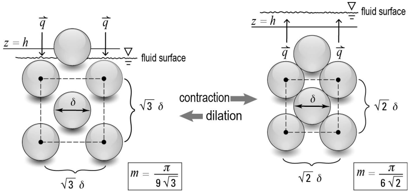
Pore-fluid pressure solid-volume fraction coupling:



- dilation/contraction of solid phase affects pore-pressure
- pore-pressure mediates Coulomb solid stress through buoyancy (effective stress)
- frictional stress determines stability of sediment mass
- dilation/contraction based on dilatancy angle
- feedback loop: shearing ⇔ contraction/dilation ⇔ pore-pressure ⇔ frictional resistance

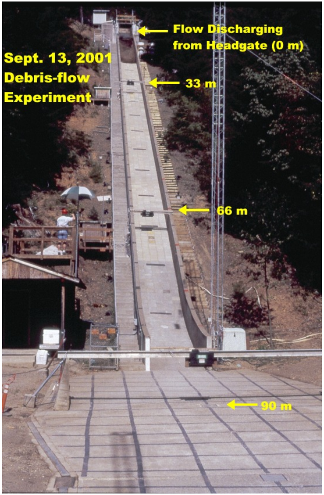
Mathematical model

Modeling mobilization/stability:



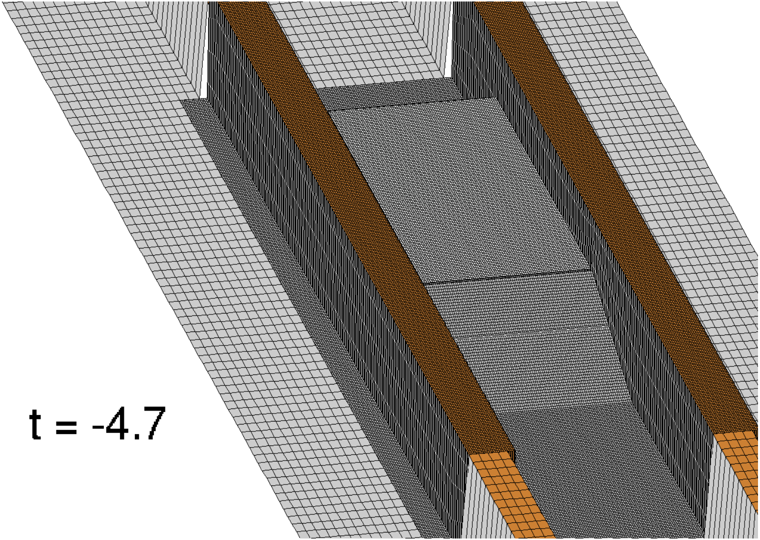
- an initial rise in pore pressure can perturb stability
- evolution and feedback determine whether mass destabilizes into debris flow
- contractive case: shearing leads to higher pressure & less resistance (positive feedback)
- dilative case: shearing leads to lower pressure & higher resistance (negative feedback)

USGS experimental debris-flow flume



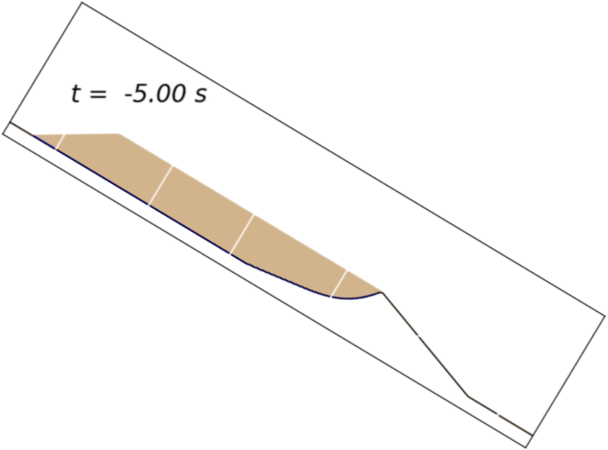
Play Movie

Simulating natural initiation by rising pore-pressure



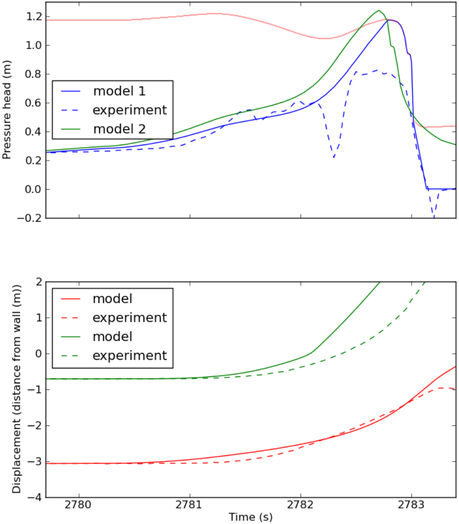
USGS experimental debris-flow flume

Simulating natural initiation by rising pore-pressure

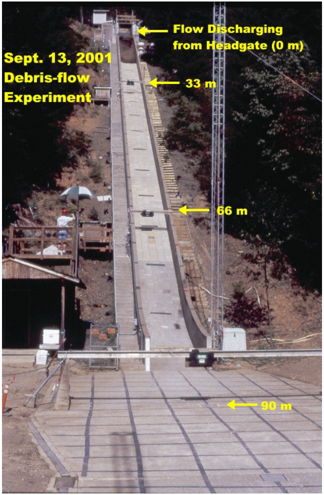


USGS experimental debris-flow flume

Comparison of fluid-pore pressure



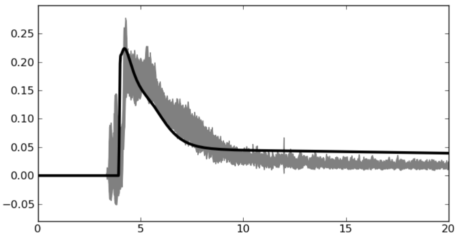
USGS experimental debris-flow flume

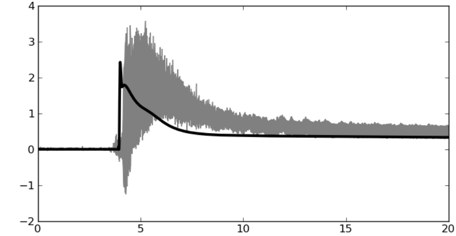


Play Movie

USGS experimental debris-flow flume

Comparison of downstream (32m) depth and fluid-pore pressure





Some future directions

- depth-averaged models can be coupled
  - landslide$\rightarrow$ tsunamis through "dtopo"
  - landslides $\leftrightarrow$ tsunami
  - granular-fluid model near inundation fronts
- always consider the possibility of numerical artifacts and/or instabilities in numerical solutions.
- validate, verify, compare, share.

Thank You!