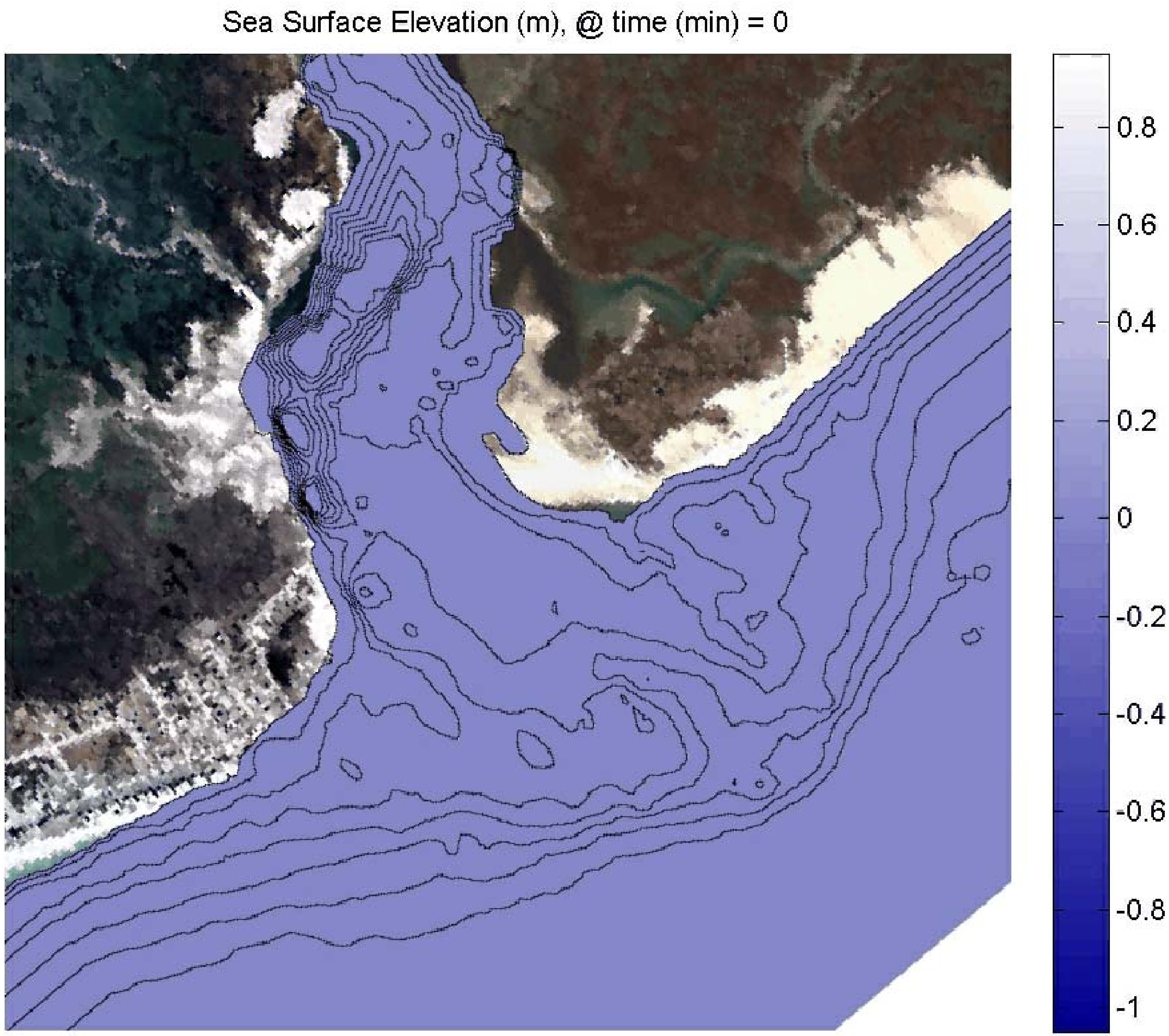
Large-Scale Coastal Modeling: Theory and Benchmarking

Patrick Lynett, University of Southern California, U.S.A


Sea Surface Elevation (m), @ time (min) = 0
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1

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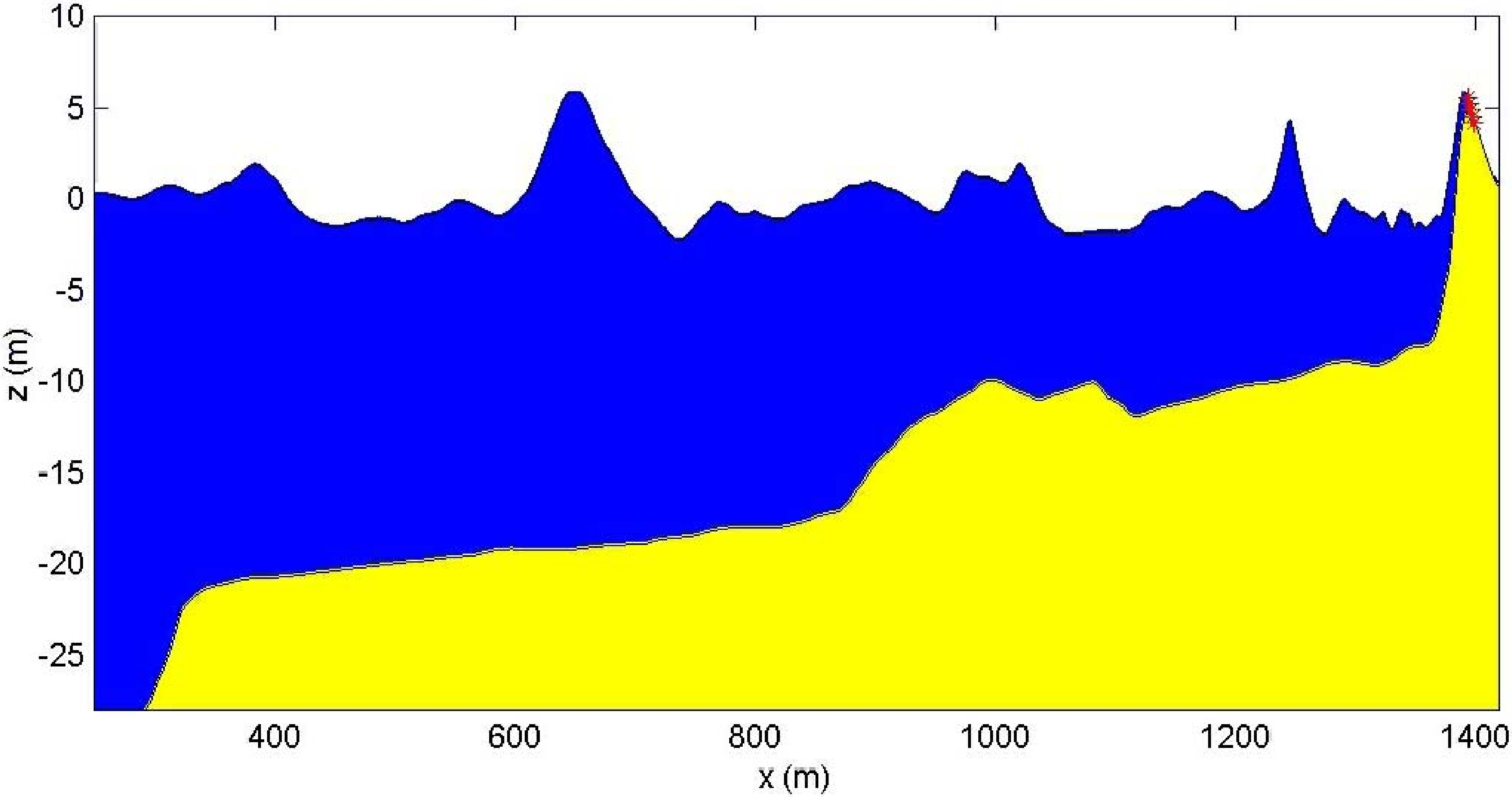
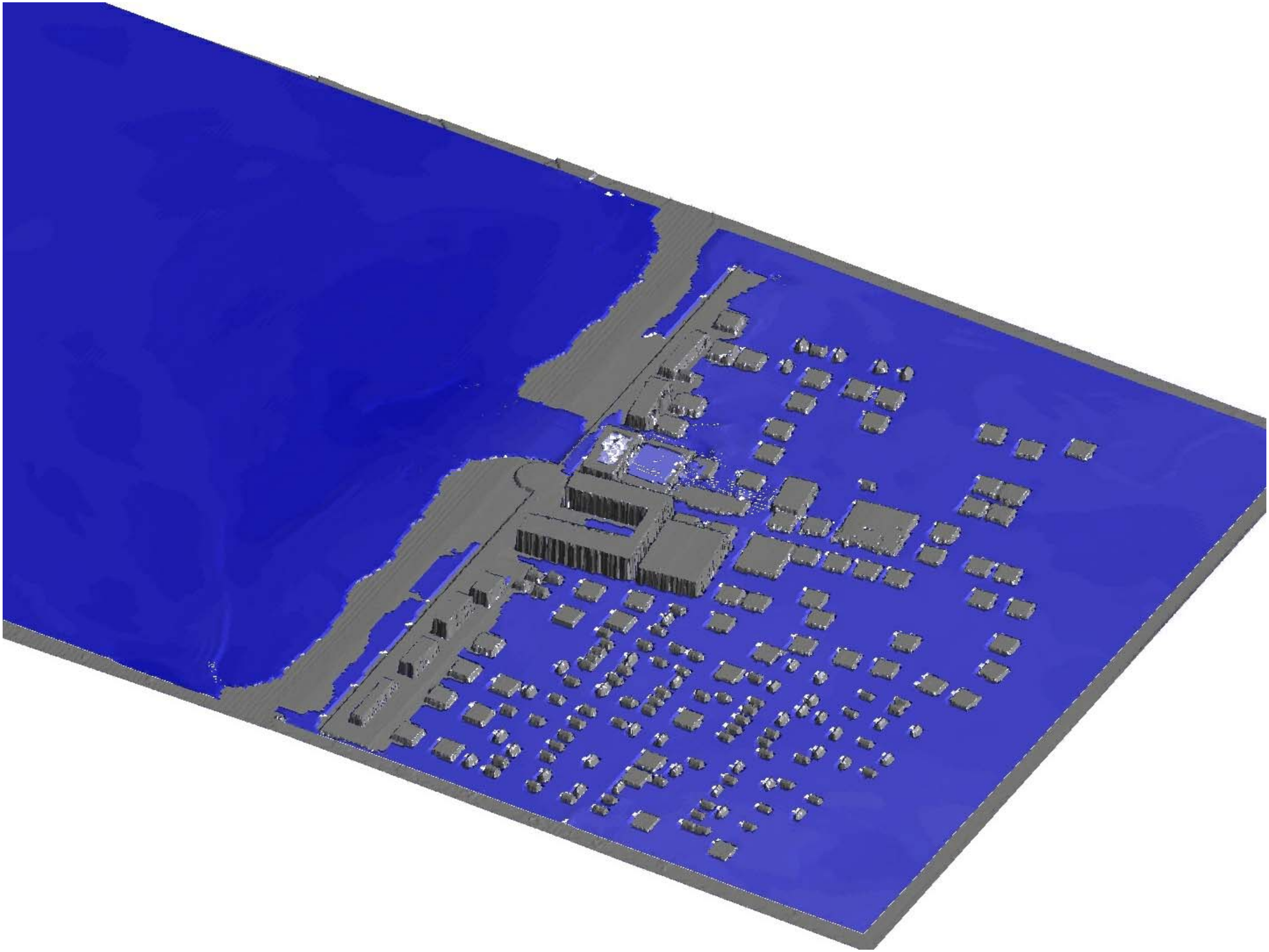
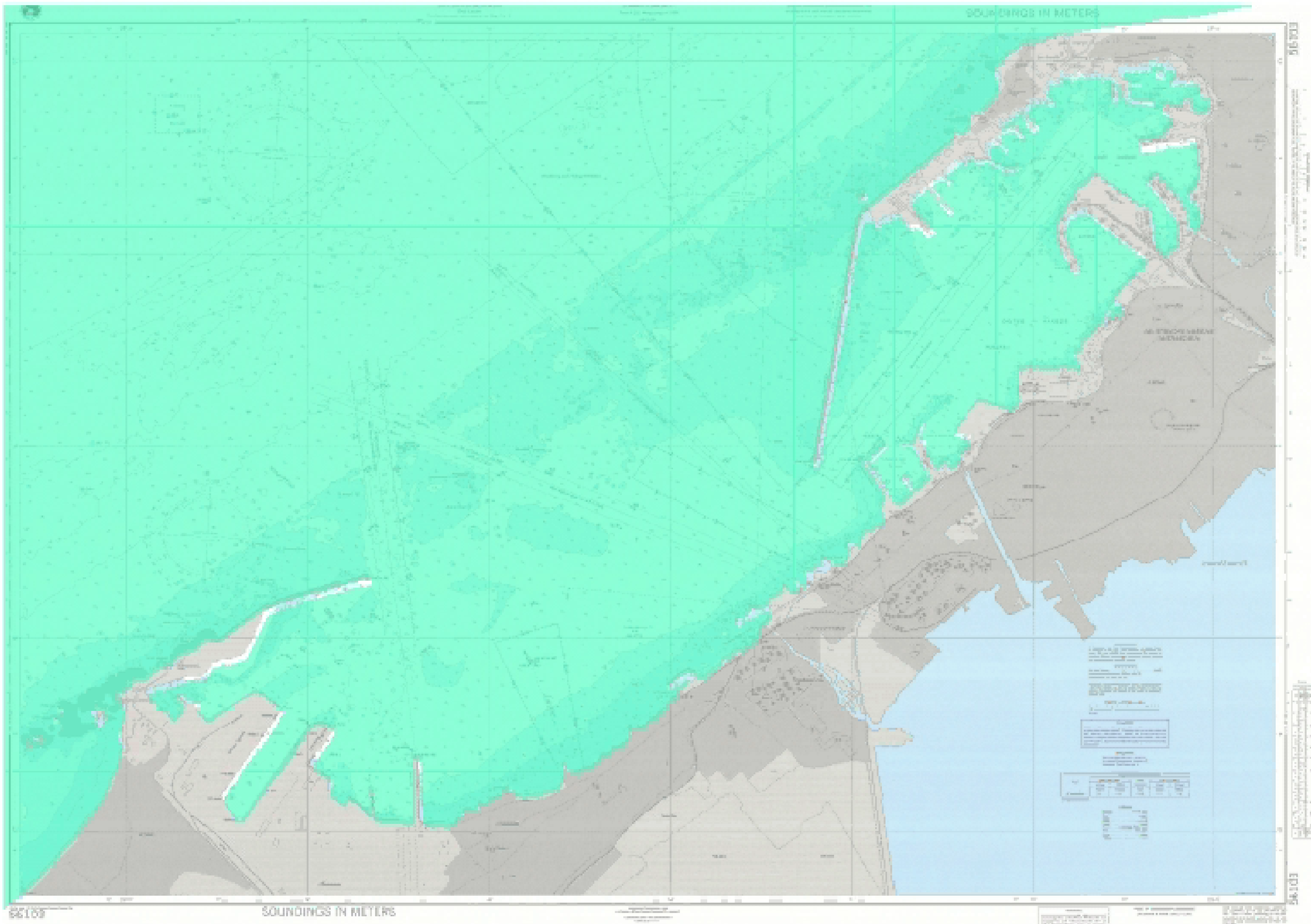
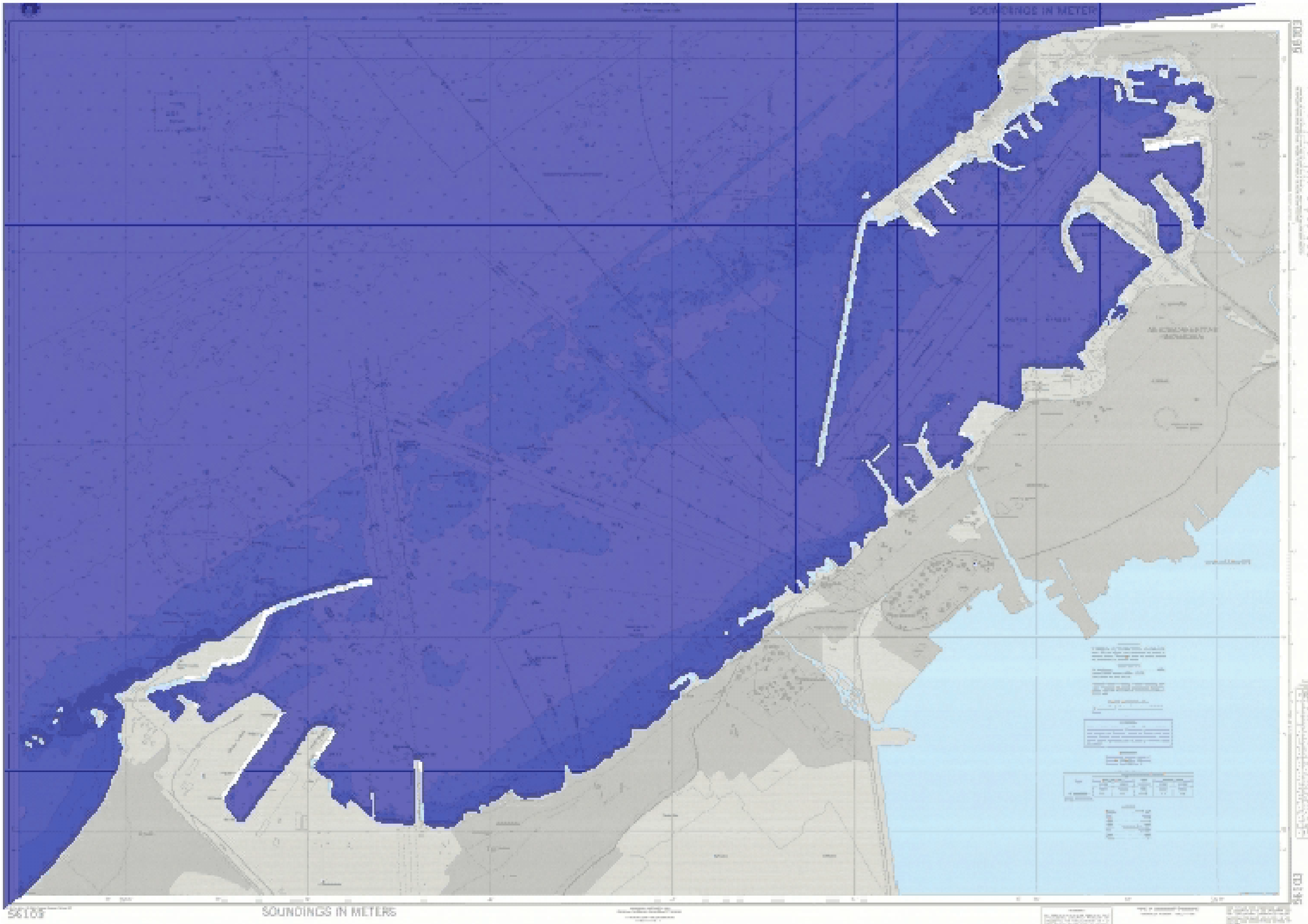




Local Time (min) = 0



Local Time (min) = 0



Presentation Outline

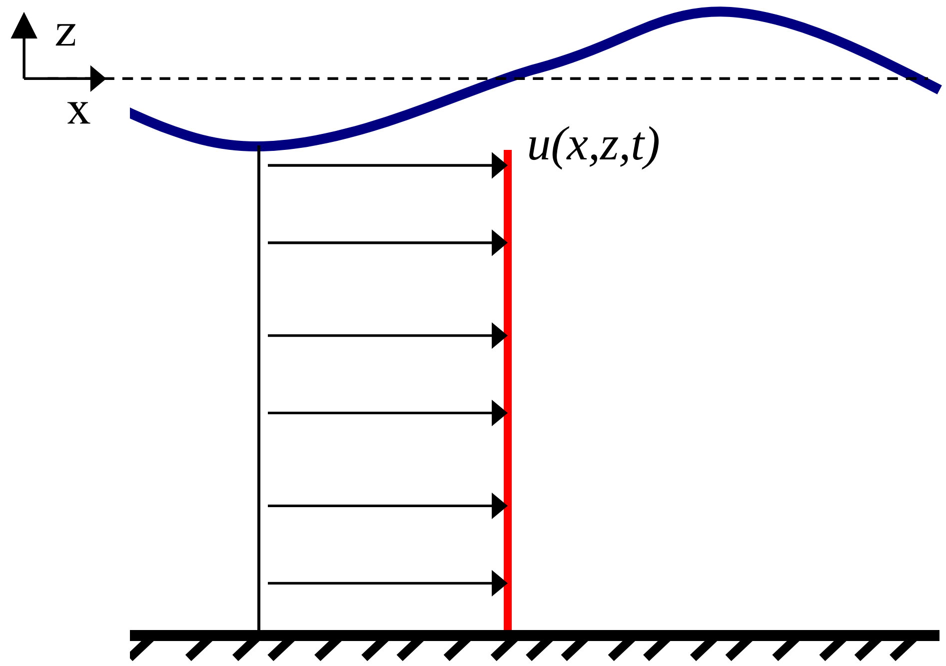
- Overview of water wave modeling approaches
- Review of depth-integrated theories
- Numerical modeling schemes
- Validation and Benchmarking

Available Models - Overview

- Shallow water (2HD, λ>25h, 1CU)
  - Earthquake tsunami source
  - Long Wave, Deep ocean propagation
  - Large-scale (O(1 km)) runup patterns
- Boussinesq (2HD, λ>2h, 50CU)
  - Many landslide tsunami sources
  - Dispersive (short) wave propagation
    - but if we want to model dispersion, we have to be able to resolve dispersive waves, $\Delta x \sim O(h)$
  - Nearshore, nonlinear evolution
    - Empirical, but calibrated breaking models
- Navier-Stokes (3D, 500CU)
  - Anything
    - have to resolve the scale of interest

History of Depth-Integrated Approach

- What is a “depth-integrated” equation?
  - A quick derivation:
  - Shallow water wave equations: $u(x,z,t) = A(x,t)$

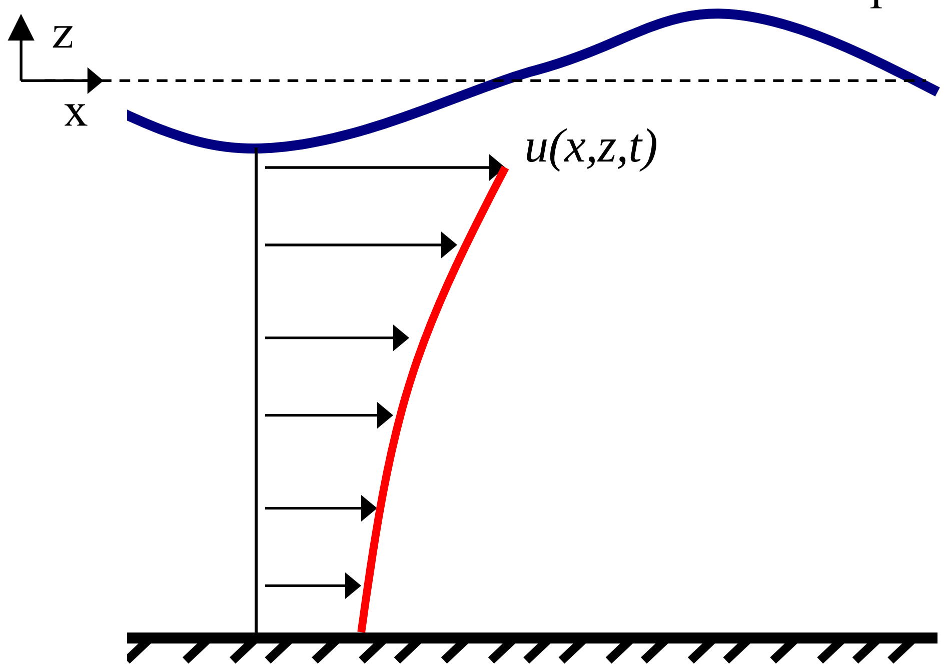


–Accurate only for very long waves, kh<~0.25 (wavelength > ~ 25 water depths)

History of Depth-Integrated Approach

– Boussinesq Equations (Peregrine, 1967; Ngowu, 1993):

$$u(x,z,t) = A(x,t) + \underbrace{\left[z * B(x,t) + z^2 * C(x,t) \right]}_{\text{Should be small compared to A(x,t)}}$$



– Functions *B*, *C* lead to 3rd order spatial derivatives in model (eqns)

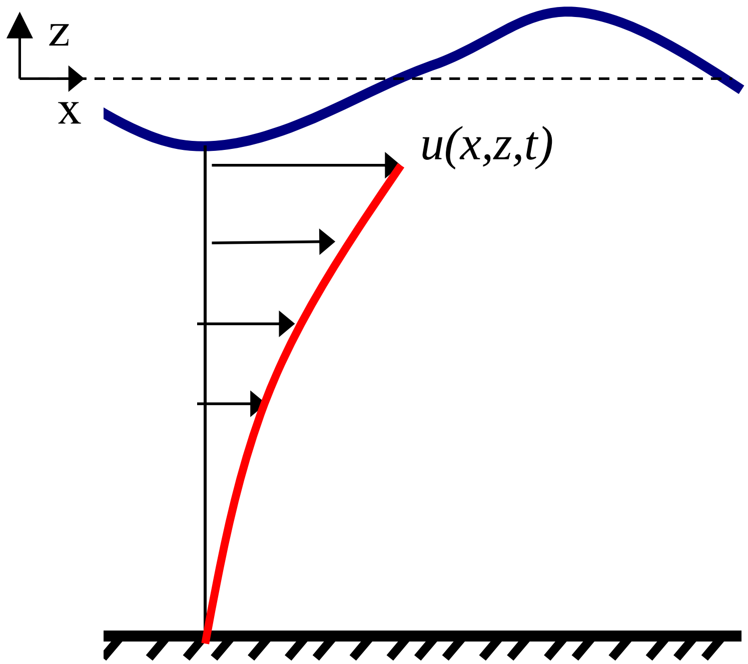
– Accurate for long and intermediate depth waves, kh<~3 (wavelength > ~ 2 water depths)

History of Depth-Integrated Approach

– High-Order Boussinesq Equations (Gobbi *et al*., 2000):

$$u(x,z,t) = A(x,t) + \left[z * B(x,t) + z^2 * C(x,t) \right]$$

$$+ \left[\underbrace{z^3 * D(x,t) + z^4 * E(x,t)}_{\text{Should be small compared to } B,C \text{ group}} \right]$$



– Accurate for long, intermediate, and moderately deep waves, kh<~6 (wavelength > ~ 1 water depth)

– Functions *D*, *E* lead to 5th order spatial derivatives in model

History of Depth-Integrated Approach

- Difficult to solve the high-order model
  - Momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \ldots + C_1\frac{\partial^5 u}{\partial x^5} = 0$$

  - To solve consistently, numerical truncation error (Taylor series error) for leading term must be less important than included terms.
    - For example: 2^{nd} order in space finite difference:

$$\frac{\partial u(x_o,t)}{\partial x} = \frac{u(x_o+\Delta x,t) - u(x_o-\Delta x,t)}{2\Delta x} - \frac{\Delta x^2}{6}\frac{\partial^3 u(x_o,t)}{\partial x^3}$$

    - High-order model requires use of 6-point difference formulas (Δx^6 accuracy)
    - Additionally, time integration would require a Δt^6 accurate scheme

…but if we want a practical nearshore model, what about mixing, rotation, and turbulence??

- Fundamentally, the perturbation-type Boussinesq derivation is a small-parameter (h/L) expansion of potential flow
- To derive the analytic vertical profile of velocity, some assumption of the vertical structure must be made
  - Horizontal vorticity at the order of analysis=0
- Typical Boussinesq-type models include only the expansion terms to n=1
- High-order models include to n=2
  - **In the limit as n -> infinity $\omega_x = \omega_y \equiv 0$.**

$$\frac{\partial U_0}{\partial z} = \nabla W_0 = 0$$

$$\frac{\partial U_1}{\partial z} = \nabla W_1$$

$$\frac{\partial U_2}{\partial z} = \nabla W_2$$

….

$$\frac{\partial U_n}{\partial z} = \nabla W_n$$

$$\omega = \omega_o + \mu^2 \omega_1 + \mu^4 \omega_2 + \cdots$$

$$= \frac{\partial U_o}{\partial z} + \mu^2 \left(\frac{\partial U_1}{\partial z} - \frac{\partial W_o}{\partial x} \right) + \cdots$$

$$\frac{D\omega_x}{Dt} = \omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z} + \nu\left(\frac{\partial^2 \omega_x}{\partial x \partial x} + \frac{\partial^2 \omega_x}{\partial y \partial y} + \frac{\partial^2 \omega_x}{\partial z \partial z}\right)$$

$$\frac{D\omega_y}{Dt} = \omega_x \frac{\partial v}{\partial x} + \omega_y \frac{\partial v}{\partial y} + \omega_z \frac{\partial v}{\partial z} + \nu\left(\frac{\partial^2 \omega_y}{\partial x \partial x} + \frac{\partial^2 \omega_y}{\partial y \partial y} + \frac{\partial^2 \omega_y}{\partial z \partial z}\right)$$

$$\frac{D\omega_z}{Dt} = \omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y} + \omega_z \frac{\partial w}{\partial z} + \nu\left(\frac{\partial^2 \omega_z}{\partial x \partial x} + \frac{\partial^2 \omega_z}{\partial y \partial y} + \frac{\partial^2 \omega_z}{\partial z \partial z}\right)$$

\+

$$(\omega_x, \omega_y) = 0$$

↓

$$0 = \omega_z \frac{\partial u}{\partial z} \qquad \frac{\partial u}{\partial z} \neq 0$$

\+

$$0 = \omega_z \frac{\partial v}{\partial z} \qquad \frac{\partial v}{\partial z} \neq 0$$

→ $$\boxed{\omega_z = 0}$$

If any two components of vorticity are zero, then the full vorticity transport equations show that, if the flow has ANY vertical structure, the remaining vorticity component must also be zero

$$\cancel{\frac{D\omega_x}{Dt}} = \cancel{\omega_x \frac{\partial u}{\partial x}} + \cancel{\omega_y \frac{\partial u}{\partial y}} + \omega_z \frac{\partial u}{\partial z} + \nu\left(\cancel{\frac{\partial^2 \omega_x}{\partial x \partial x}} + \cancel{\frac{\partial^2 \omega_x}{\partial y \partial y}} + \cancel{\frac{\partial^2 \omega_x}{\partial z \partial z}}\right)$$

$$\cancel{\frac{D\omega_y}{Dt}} = \cancel{\omega_x \frac{\partial v}{\partial x}} + \cancel{\omega_y \frac{\partial v}{\partial y}} + \omega_z \frac{\partial v}{\partial z} + \nu\left(\cancel{\frac{\partial^2 \omega_y}{\partial x \partial x}} + \cancel{\frac{\partial^2 \omega_y}{\partial y \partial y}} + \cancel{\frac{\partial^2 \omega_y}{\partial z \partial z}}\right)$$

$$\frac{D\omega_z}{Dt} = \omega_x \frac{\partial w}{\partial x} + \omega_y \frac{\partial w}{\partial y} + \omega_z \frac{\partial w}{\partial z} + \nu\left(\frac{\partial^2 \omega_z}{\partial x \partial x} + \frac{\partial^2 \omega_z}{\partial y \partial y} + \frac{\partial^2 \omega_z}{\partial z \partial z}\right)$$

+

$$(\omega_x, \omega_y) = 0$$

↓

$$0 = \omega_z \frac{\partial u}{\partial z}$$

$$0 = \omega_z \frac{\partial v}{\partial z}$$

+

$$\frac{\partial u}{\partial z} \neq 0$$

$$\frac{\partial v}{\partial z} \neq 0$$

→

$$\boxed{\omega_z = 0}$$

If any two components of vorticity are zero, then the full vorticity transport equations show that, if the flow has ANY vertical structure, the remaining vorticity component must also be zero

Under what modeling conditions does the flow have no vertical structure?

For approximate solutions:

$$(\omega_x, \omega_y) = O(\mu^{2n})$$

$$\omega_z = O(\mu^{2n-2})$$

where n is the order of approximation, e.g. n=1 for shallow water, n=2 for Boussinesq, etc.

- As we increase the order of approximation, vertical vorticity should become smaller and smaller
- Regardless of how we start (potential flow or Eulers equations) *by assuming zero horizontal vorticity to the order of derived equations*, full equations tell us we are implicitly making a statement about vertical vorticity. Our physical asymptote is potential flow…

*shallow water (n=1), $\omega_z = O(1)$

Current & Vertical Vorticity Modeling
Can we vs. Should we?

- Boussinesq is an expansion of potential flow
  - In the limit as n -> infinity $\omega_x=\omega_y\equiv 0$. If this is true, then $\omega_z\equiv 0$ as well. Thus at the mathematical limit of the equations, they are irrotational
- Two choices:
  - Think practically (be an engineer) - if it works…
    - Similar arguments made for pushing the dispersion accuracy limit past kh=1
  - Go back to the beginning of the derivation and figure out a way to include horizontal vorticity explicitly in the physics

$$\frac{\partial U_0}{\partial z} = \nabla W_0 = 0$$

$$\frac{\partial U_1}{\partial z} = \nabla W_1$$

$$\frac{\partial U_2}{\partial z} = \nabla W_2$$

....

$$\frac{\partial U_n}{\partial z} = \nabla W_n$$

Including Horizontal Vorticity in the Boussinesq-type Derivation

- First, realize that with a single-fluid layer, the source of horizontal vorticity will be through no-slip boundary shear (forget about breaking…)
  - Need to include viscous effects
  - Start with spatially filtered N-S equations, and then depth-average

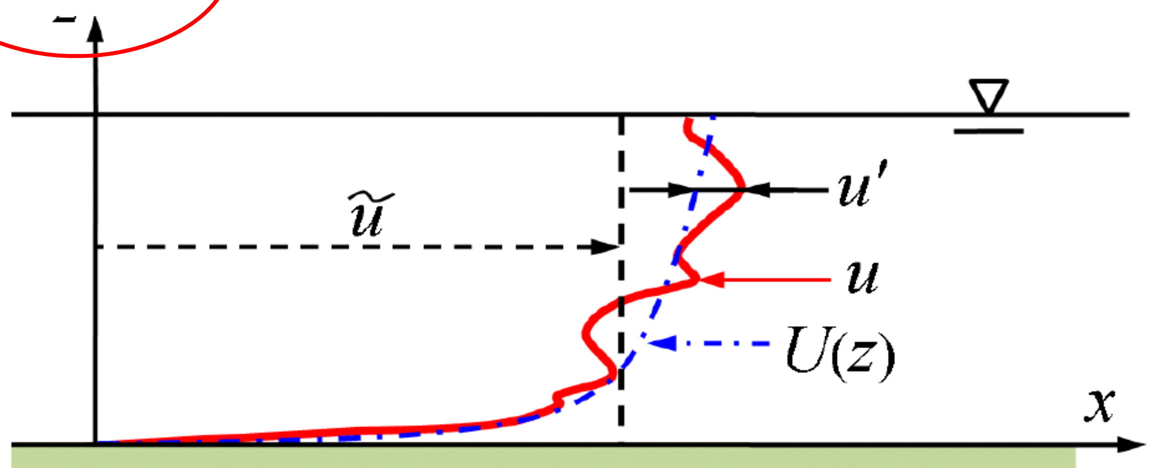
$$\frac{\partial H\widetilde{\overline{u}}_i}{\partial t} + \frac{\partial H\widetilde{\overline{u}}_i\widetilde{\overline{u}}_j}{\partial x_j} + H\widetilde{\frac{\partial \overline{p}}{\partial x_i}}$$

$$= \alpha\mu\frac{\partial}{\partial x_j}\left(2H\nu_t^h\widetilde{\overline{S}}_{ij}\right) + \beta\mu^2 2\nu_t^v\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right) + \beta\mu^2\tau_i^b$$

$$+ \beta\mu^2\frac{\partial}{\partial x_j}\left(H\widetilde{\overline{u_i'u_j'}}\right)$$

$$F_i = C_B\frac{\sqrt{\widetilde{u}^2+\widetilde{v}^2}}{H}\sqrt{\frac{\nu\sqrt{c_f}}{\Delta t}}r_i$$

Stochastic BSM by **Hinterberger, Frohlich, Rodi** (2007)



Horizontal vorticity effects

- $$\omega = \omega_o + \mu^2 \omega_1 + \mu^4 \omega_2 + \cdots$$
$$= \frac{\partial U_o}{\partial z} + \mu^2 \left(\frac{\partial U_1}{\partial z} - \frac{\partial W_o}{\partial x} \right) + \cdots$$

- $$\boldsymbol{U}_1 = -\frac{1}{2} z^2 \nabla S - z \nabla T + \frac{1}{2} h^2 \nabla S - h \nabla T + \int_{-h}^{z} \boldsymbol{\omega}_1 dz$$

- $$\boldsymbol{\omega}_1 = \frac{\partial \boldsymbol{U}_1^r}{\partial z} = \frac{\boldsymbol{\tau}_b}{\nu_t^v} \frac{\zeta - z}{\zeta + h}$$

Linear shear distribution e.g. **Rodi** (1980)

- $$\frac{\partial \nu_t^v \boldsymbol{\omega}_1}{\partial z} = \frac{\partial}{\partial z} \left\{ \nu_t^v \left(\frac{\partial \boldsymbol{U}_1^r}{\partial z} + \frac{\partial \boldsymbol{U}_1^\phi}{\partial z} - \nabla W_o \right) \right\} = \frac{\partial}{\partial z} \left(\nu_t^v \frac{\partial \boldsymbol{U}_1^r}{\partial z} \right) = \frac{\partial \boldsymbol{\tau}}{\partial z}$$

Horizontal vorticity effects

- $$\frac{\partial \boldsymbol{U}_\alpha}{\partial t} + \boldsymbol{U}_\alpha \cdot \nabla \boldsymbol{U}_\alpha + \nabla \zeta + \mu^2 \left(\boldsymbol{D} + \overline{\boldsymbol{\xi}} \right) + \beta\mu \left(\boldsymbol{D}^\nu + \overline{\boldsymbol{\xi}^\nu} \right)$$
  $$- \alpha\mu\nabla \cdot \left(\nu_t^h \nabla \boldsymbol{U}_\alpha \right) + \beta\mu\nu_t^v \nabla S + \beta\mu \frac{\boldsymbol{\tau}_b}{\zeta + h}$$
  $$= O\left(\mu^4, \alpha\mu^3, \beta\mu^3, \beta^2\mu^2 \right)$$

- $$\frac{\partial \zeta}{\partial t} + \nabla \cdot \{ (\zeta + h) \boldsymbol{U}_\alpha \} + \mu^2 \mathcal{M} + \beta\mu\mathcal{M}^\nu = O\left(\mu^4, \beta^2\mu^2 \right)$$

- $\nu_t^v = \frac{\kappa}{6} H u_\tau$: **Elder** (1959)

$$\nu_t^{v\prime} = C_h H' u'_* \qquad u_* = C_* u_b$$

$$\nu_t^{v\prime} = \beta h_o \sqrt{g h_o} H u_b = \beta h_o \sqrt{g h_o} \nu_t^v$$

$$\beta = C_h C_*$$

$$O(\mu^2) = O(\beta\mu) \ll 1$$

$$\frac{\partial p}{\partial z} + 1 = O(\mu^2, \mu\beta)$$

Horizontal vorticity effects

- $$\frac{\partial \boldsymbol{U}_\alpha}{\partial t} + \boldsymbol{U}_\alpha \cdot \nabla \boldsymbol{U}_\alpha + \nabla \zeta + \mu^2 \left(\boldsymbol{D} + \overline{\boldsymbol{\xi}} \right) + \beta\mu \left(\boldsymbol{D}^\nu + \overline{\boldsymbol{\xi}^\nu} \right)$$
$$- \alpha\mu\nabla \cdot \left(\nu_t^h \nabla \boldsymbol{U}_\alpha \right) + \beta\mu\nu_t^v \nabla S + \beta\mu \frac{\boldsymbol{\tau}_b}{\zeta + h}$$
$$= O\left(\mu^4, \alpha\mu^3, \beta\mu^3, \beta^2\mu^2 \right)$$

- $$\frac{\partial \zeta}{\partial t} + \nabla \cdot \{ (\zeta + h) \boldsymbol{U}_\alpha \} + \mu^2 \mathcal{M} + \beta\mu\mathcal{M}^\nu = O\left(\mu^4, \beta^2\mu^2 \right)$$

  - ν_t^h : **Smagorinsky model** (1963)

$$\nu_t^{h\prime} = C_s^2 \Delta^2 h_o \sqrt{g h_o} \sqrt{ \left(\frac{\partial u}{\partial z} \right)^2 + 2\mu^2 \left(\frac{\partial u}{\partial x} \right)^2 + 2\mu^2 \left(\frac{\partial w}{\partial z} \right)^2 + \cdots\cdots }$$

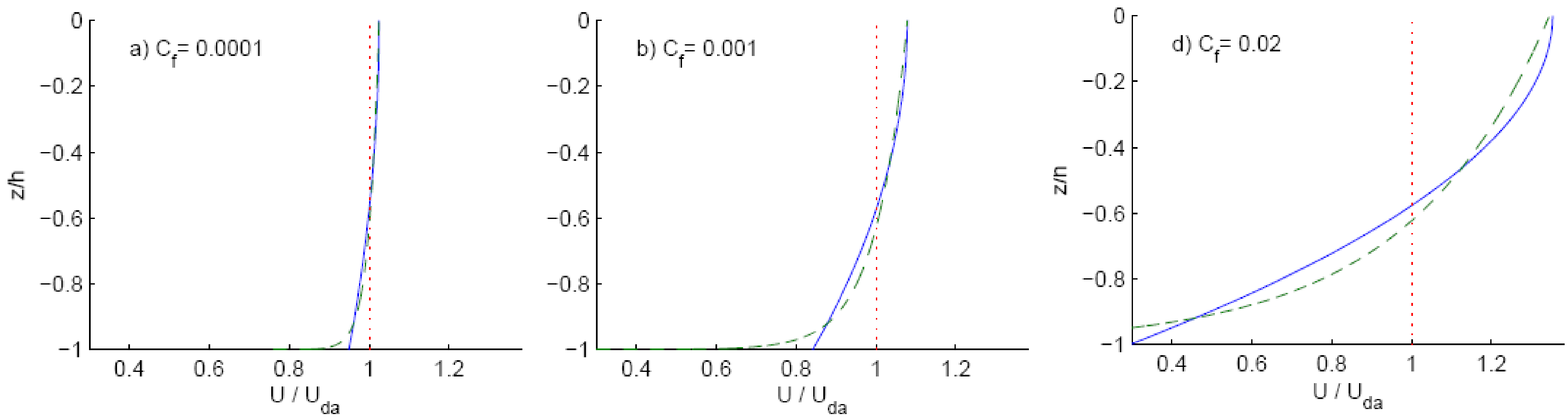
$$\nu_t^{h\prime} = \alpha h_o \sqrt{g h_o} \nu_t^h$$

$$\alpha = C_s^2 \Delta^2 \qquad O(\mu^2) = O(\alpha\mu) \ll 1$$

Horizontal vorticity effects

$$\boldsymbol{U} = \boldsymbol{U}_\alpha + \mu^2 \boldsymbol{U}_1^\phi + \beta\mu \boldsymbol{U}_1^r + O\left(\mu^4, \beta^2\mu^2\right)$$

$$\boldsymbol{U}_1^r = \int_{z_\alpha}^{z} \boldsymbol{\omega}_1 dz = \frac{\boldsymbol{\tau}_b}{\nu_t^v\left(\zeta + h\right)}\left\{\frac{1}{2}\left(z_\alpha^2 - z^2\right) + \zeta\left(z - z_\alpha\right)\right\}$$



$$U(z) = U_\alpha + \frac{\tau_b}{2\nu_t^v h}\left(z_\alpha^2 - z^2\right)$$

$$\tau_b = \rho C_f U_{DA}^2 \qquad \nu_t^v = \frac{\kappa}{6} h u_*$$

$$U_{log}(z) = U_{max} + \frac{u_*}{\kappa} ln\left(\frac{z+h}{h}\right)$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = U_{DA}\sqrt{C_f}$$

Inclusion of Rotational & Turbulent Effects in Depth-Integrated Models

- Theory: Kim et al. (2009, Ocean Modelling); Kim & Lynett (2011, Physics of Fluids)

$$\frac{\partial HU_i}{\partial t} + \frac{\partial HU_iU_j}{\partial x_j} + gH\frac{\partial \zeta}{\partial x_i} + H\left(D_i + \overline{\xi_i} + D_i^{\nu} + \overline{\xi_i^{\nu}}\right) + U_i\left(\mathcal{M} + \mathcal{M}^{\nu}\right)$$

$$- H\frac{\partial}{\partial x_j}\left(2\nu_t^h S_{ij}\right) + 2H\frac{\partial}{\partial x_i}\left(\nu_t^v \frac{\partial U_j}{\partial x_j}\right) + \frac{\tau_i^b}{\rho} - HF_i = 0$$

$$\frac{\partial HU_{\alpha i}}{\partial t^*}+\frac{\partial HU_{\alpha i}U_{\alpha j}}{\partial x_j^*}+gH\frac{\partial\zeta}{\partial x_i^*}
+H\left(\beta_i+\gamma_i+\beta_i^\nu+\gamma_i^\nu\right)+U_{\alpha i}\left(\alpha+\alpha^\nu\right)
-H\frac{\partial}{\partial x_j^*}\left(2\nu_t^h S_{ij}\right)+2H\nu_t^v\frac{\partial}{\partial x_i^*}\left(\frac{\partial U_{\alpha j}}{\partial x_j^*}\right)
+\frac{\tau_i^b}{\rho}-HR_i-HF_i=0$$

$$\beta_i=\frac{1}{2}\nabla\left(z_\alpha{}^2\boldsymbol{U}_\alpha\cdot\nabla S\right)+\nabla\left(z_\alpha\boldsymbol{U}_\alpha\cdot\nabla T\right)+\left(T\nabla T\right)
-\frac{1}{2}\nabla\left(\zeta^2\frac{\partial S}{\partial t^*}\right)-\nabla\left(\zeta\frac{\partial T}{\partial t^*}\right)+\left(\frac{1}{2}z_\alpha{}^2\frac{\partial\nabla S}{\partial t^*}+z_\alpha\frac{\partial\nabla T}{\partial t^*}\right)
-\frac{1}{2}\nabla\left(\zeta^2\boldsymbol{U}_\alpha\cdot\nabla S\right)-\nabla\left(\zeta\boldsymbol{U}_\alpha\cdot\nabla T\right)+\nabla\left(\frac{1}{2}\zeta^2S^2\right)+\nabla\left(\zeta TS\right)$$

$$\beta_i^\nu=\frac{(\zeta-h)}{2}\frac{\partial\boldsymbol{\psi}\zeta}{\partial t^*}-\frac{(\zeta^2-\zeta h+h^2)}{6}\frac{\partial\boldsymbol{\psi}}{\partial t^*}+\frac{\partial}{\partial t^*}\left\{\boldsymbol{\psi}\left(\frac{z_\alpha^2}{2}-\zeta z_\alpha\right)\right\}
+\frac{(\zeta-h)}{2}\nabla\left\{\boldsymbol{U}_\alpha\cdot(\boldsymbol{\psi}\zeta)\right\}-\frac{(\zeta^2-\zeta h+h^2)}{6}\nabla\left(\boldsymbol{U}_\alpha\cdot\boldsymbol{\psi}\right)
+\nabla\left[\boldsymbol{U}_\alpha\cdot\left\{\boldsymbol{\psi}\left(\frac{z_\alpha^2}{2}-\zeta z_\alpha\right)\right\}\right]
-\boldsymbol{\psi}\left\{\frac{(\zeta^2+\zeta h-2h^2)\,S}{6}+\frac{(\zeta+h)\,T}{2}\right\}$$

$$\gamma_x=-V_\alpha\left\{\frac{\partial z_\alpha}{\partial x^*}\left(z_\alpha\frac{\partial S}{\partial y^*}+\frac{\partial T}{\partial y^*}\right)-\frac{\partial z_\alpha}{\partial y^*}\left(z_\alpha\frac{\partial S}{\partial x^*}+\frac{\partial T}{\partial x^*}\right)\right\}
-\left(\frac{\partial V_\alpha}{\partial x^*}-\frac{\partial U_\alpha}{\partial y^*}\right)\left[\left\{\frac{z_\alpha^2}{2}-\frac{(\zeta^2-\zeta h+h^2)}{6}\right\}\frac{\partial S}{\partial y^*}+\left\{z_\alpha-\frac{(\zeta-h)}{2}\right\}\frac{\partial T}{\partial y^*}\right]$$

$$\gamma_y=U_\alpha\left\{\frac{\partial z_\alpha}{\partial x^*}\left(z_\alpha\frac{\partial S}{\partial y^*}+\frac{\partial T}{\partial y^*}\right)-\frac{\partial z_\alpha}{\partial y^*}\left(z_\alpha\frac{\partial S}{\partial x^*}+\frac{\partial T}{\partial x^*}\right)\right\}
+\left(\frac{\partial V_\alpha}{\partial x^*}-\frac{\partial U_\alpha}{\partial y^*}\right)\left[\left\{\frac{z_\alpha^2}{2}-\frac{(\zeta^2-\zeta h+h^2)}{6}\right\}\frac{\partial S}{\partial x^*}+\left\{z_\alpha-\frac{(\zeta-h)}{2}\right\}\frac{\partial T}{\partial x}\right. \quad (5$$

$$\gamma_y^\nu=-V_\alpha\left[\frac{\partial}{\partial x^*}\left\{\psi_y\left(\frac{1}{2}z_\alpha^2-z_\alpha\zeta\right)\right\}-\frac{(\zeta^2-\zeta h+h^2)}{6}\frac{\partial\psi_y}{\partial x^*}+\frac{(\zeta-h)}{2}\frac{\partial\psi_y\zeta}{\partial x^*}\right.
\left.-\frac{\partial}{\partial y^*}\left\{\psi_x\left(\frac{1}{2}z_\alpha^2-z_\alpha\zeta\right)\right\}+\frac{(\zeta^2-\zeta h+h^2)}{6}\frac{\partial\psi_x}{\partial y^*}-\frac{(\zeta-h)}{2}\frac{\partial\psi_x\zeta}{\partial y^*}\right]
-\left(\frac{\partial V_\alpha}{\partial x^*}-\frac{\partial U_\alpha}{\partial y^*}\right)\psi_y\left\{\frac{z_\alpha^2}{2}-z_\alpha\zeta+\frac{(2\zeta^2-2\zeta h-h^2)}{6}\right\} \quad (5$$

$$\gamma_y^\nu=U_\alpha\left[\frac{\partial}{\partial x^*}\left\{\psi_y\left(\frac{1}{2}z_\alpha^2-z_\alpha\zeta\right)\right\}-\frac{(\zeta^2-\zeta h+h^2)}{6}\frac{\partial\psi_y}{\partial x^*}+\frac{(\zeta-h)}{2}\frac{\partial\psi_y\zeta}{\partial x^*}\right.
\left.-\frac{\partial}{\partial y^*}\left\{\psi_x\left(\frac{1}{2}z_\alpha^2-z_\alpha\zeta\right)\right\}+\frac{(\zeta^2-\zeta h+h^2)}{6}\frac{\partial\psi_x}{\partial y^*}-\frac{(\zeta-h)}{2}\frac{\partial\psi_x\zeta}{\partial y^*}\right]
+\left(\frac{\partial V_\alpha}{\partial x^*}-\frac{\partial U_\alpha}{\partial y^*}\right)\psi_x\left\{\frac{z_\alpha^2}{2}-z_\alpha\zeta+\frac{(2\zeta^2-2\zeta h-h^2)}{6}\right\} \quad (5$$

n which $\nabla=(\partial/\partial x^*,\partial/\partial y^*)$, $T=(\partial hU_\alpha/\partial x^*+\partial hV_\alpha/\partial y^*)$ and $\boldsymbol{\psi}=(\psi_x,\psi_y$

Inclusion of Rotational & Turbulent Effects in Depth-Integrated Models

- Theory: Kim et al. (2009, Ocean Modelling); Kim & Lynett (2011, Physics of Fluids)

O(μ^2) Dispersive Corrections

O($\beta\mu$) Turbulent-Rotational Corrections

O(1) Shallow Water terms

$$\frac{\partial HU_i}{\partial t} + \frac{\partial HU_iU_j}{\partial x_j} + gH\frac{\partial \zeta}{\partial x_i} + H\left(D_i + \overline{\xi_i} + D_i^{\nu} + \overline{\xi_i^{\nu}}\right) + U_i\left(\mathcal{M} + \mathcal{M}^{\nu}\right)$$

$$- H\frac{\partial}{\partial x_j}\left(2\nu_t^h S_{ij}\right) + 2H\frac{\partial}{\partial x_i}\left(\nu_t^v \frac{\partial U_j}{\partial x_j}\right) + \frac{\tau_i^b}{\rho} - HF_i = 0$$

O($\alpha\mu$) Turbulent Mixing in Horizontal Plane. Eddy viscosity closed with Smagorinsky model

O($\beta\mu$) Turbulent Mixing in Vertical Plane. Eddy viscosity closed with Elder's model

O($\beta\mu$) Bottom Stress, closed with Mannings, Moody, etc.

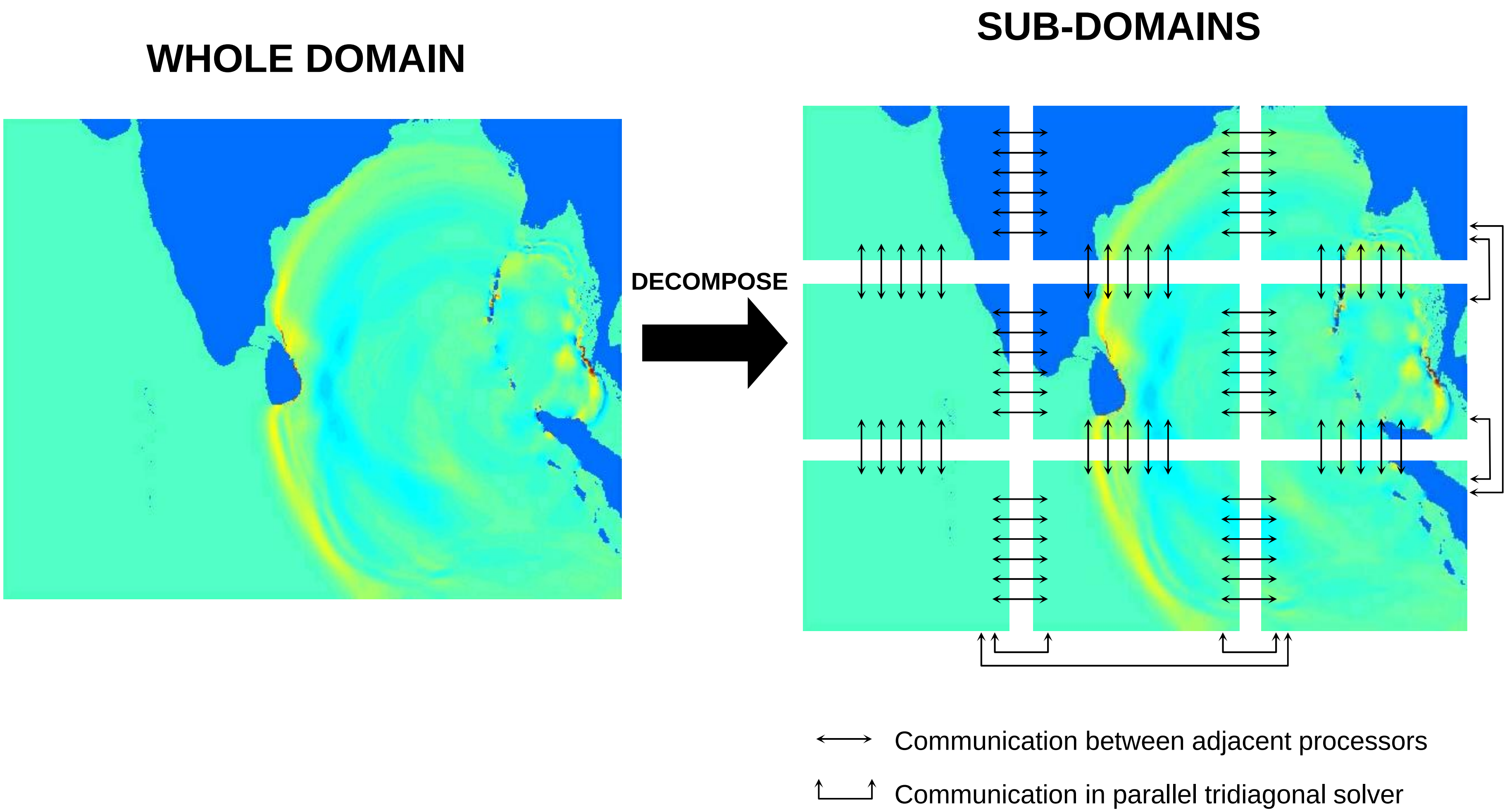
O(γ) Depth-averaging stress terms, closed with BSM

Boussinesq-Numerical Algorithm COULWAVE

- Time integration :
  - 4^{th}-order Predictor–Corrector scheme
- Leading-order term :
  - 4^{th}-order MUSCL-TVD scheme, FVM
  - **Yamamoto & Daiguji** (1993)
- High-order term :
  - FVM discretization by **Lacor et al.**(2004)
  - 4^{th}-order or 2^{nd}-order accuracy

Parallel Boussinesq Approach

The whole domain is divided into several sub-domains, each is processed in a single processor.

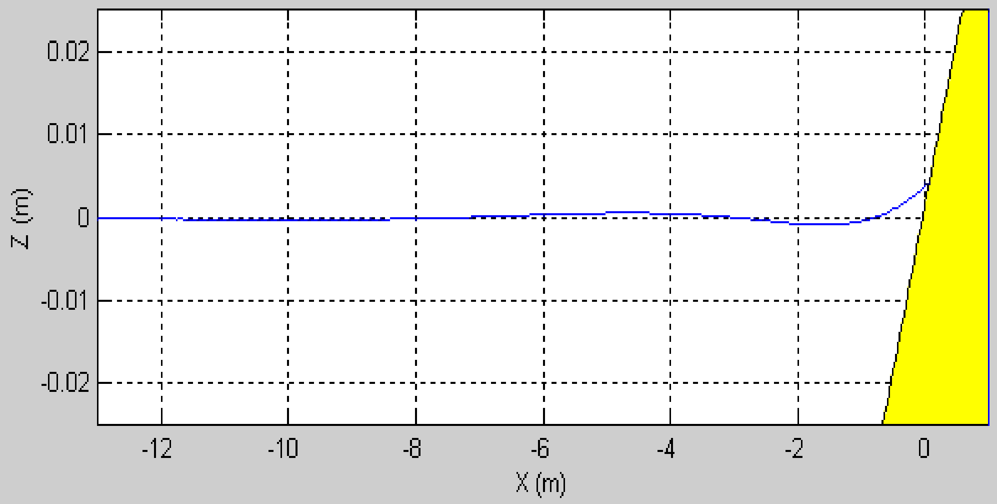


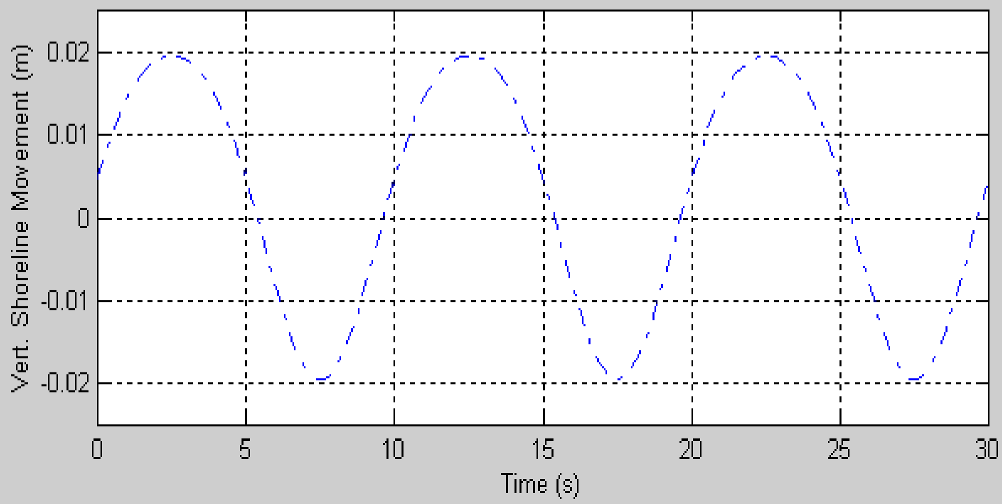
Benchmarking Coastal Wave Models

- Community effort to standardize runup benchmarking of tsunami and wave codes (Long Wave Runup Workshops)
- NOAA has generated a validation procedure, for which all tsunami codes used for NOAA (e.g. National Tsunami Hazard Mitigation Program) purposes must satisfactorily complete
- Rely heavily on highly-controlled laboratory data and analytical solutions
  - Uncertainties and lack of precision in field runup data
- Here, provide an overview of the most common benchmark cases
  - So of these can be simulated during the lab sessions later

Standard Runup Benchmarks – 1HD

- Carrier and Greenspan – analytic solution to nonlinear shallow water equations for single harmonic wave runup and rundown on a plane beach
- Numerical simulation:
  - Wave amplitude = 0.003 m, wave period = 10 s
  - Still water depth = 0.5 m
  - Beach slope = 1:25
  - Δx = 0.02 m

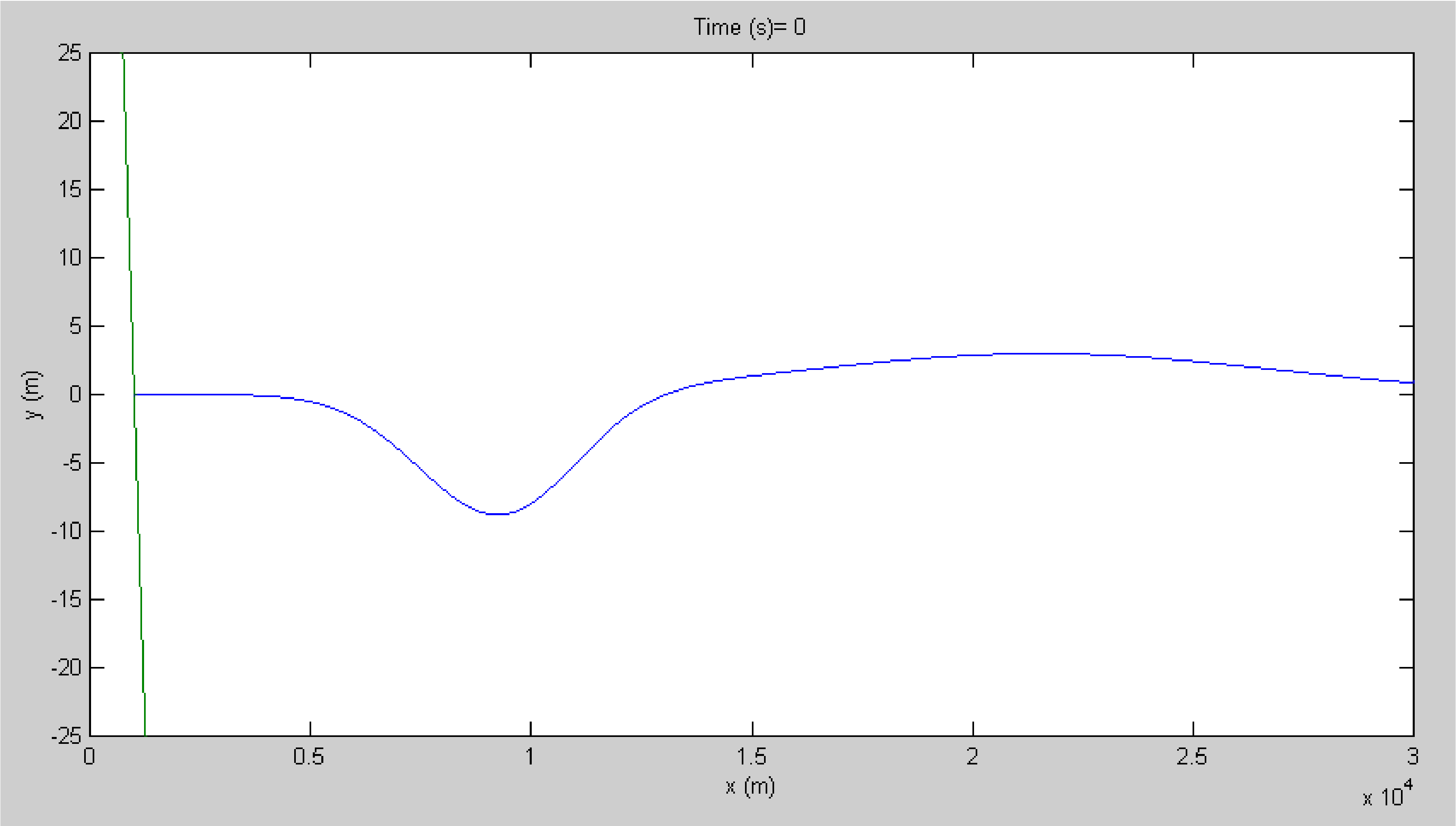




Analytic runup shown by dashed line

Standard Runup Benchmarks – 1HD

- Analytic solution to NLSW for arbitrary initial condition
  - Carrier *et al* (2004)



Standard Runup Benchmarks – 1HD

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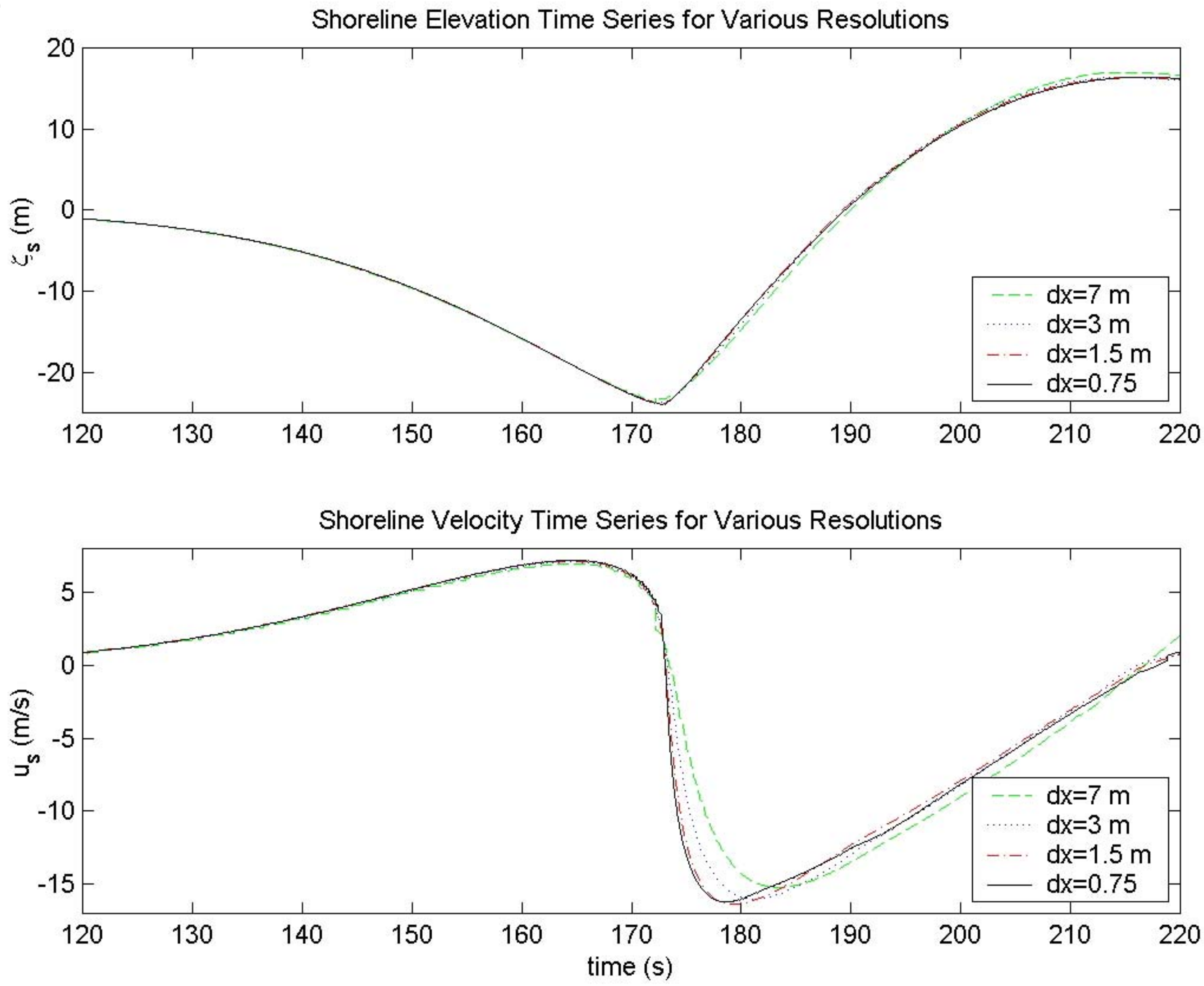
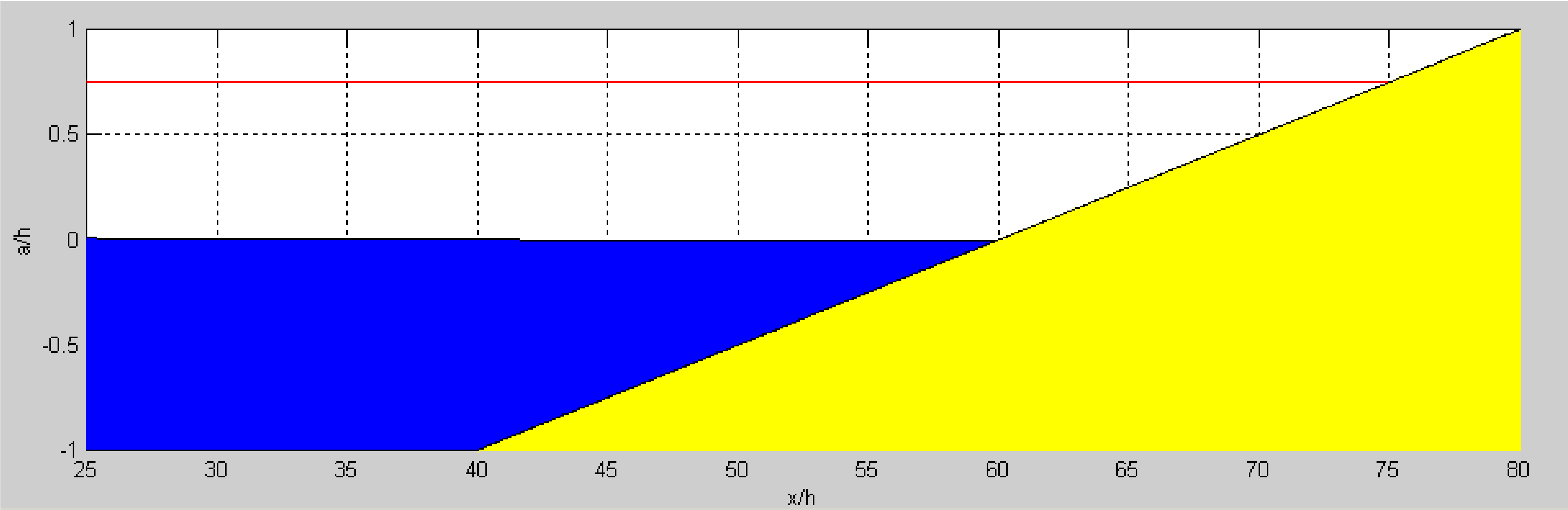


Standard Runup Benchmarks – 1HD

- Runup of solitary waves
  - Comparison with experimental data taken from Synolakis (1987)
- Numerical simulation parameters:
  - Wave height / water depth = 0.4
  - Beach slope = 1:20
  - Wave breaking model – “Eddy-viscosity” model (e.g. Kennedy et al., 2000)



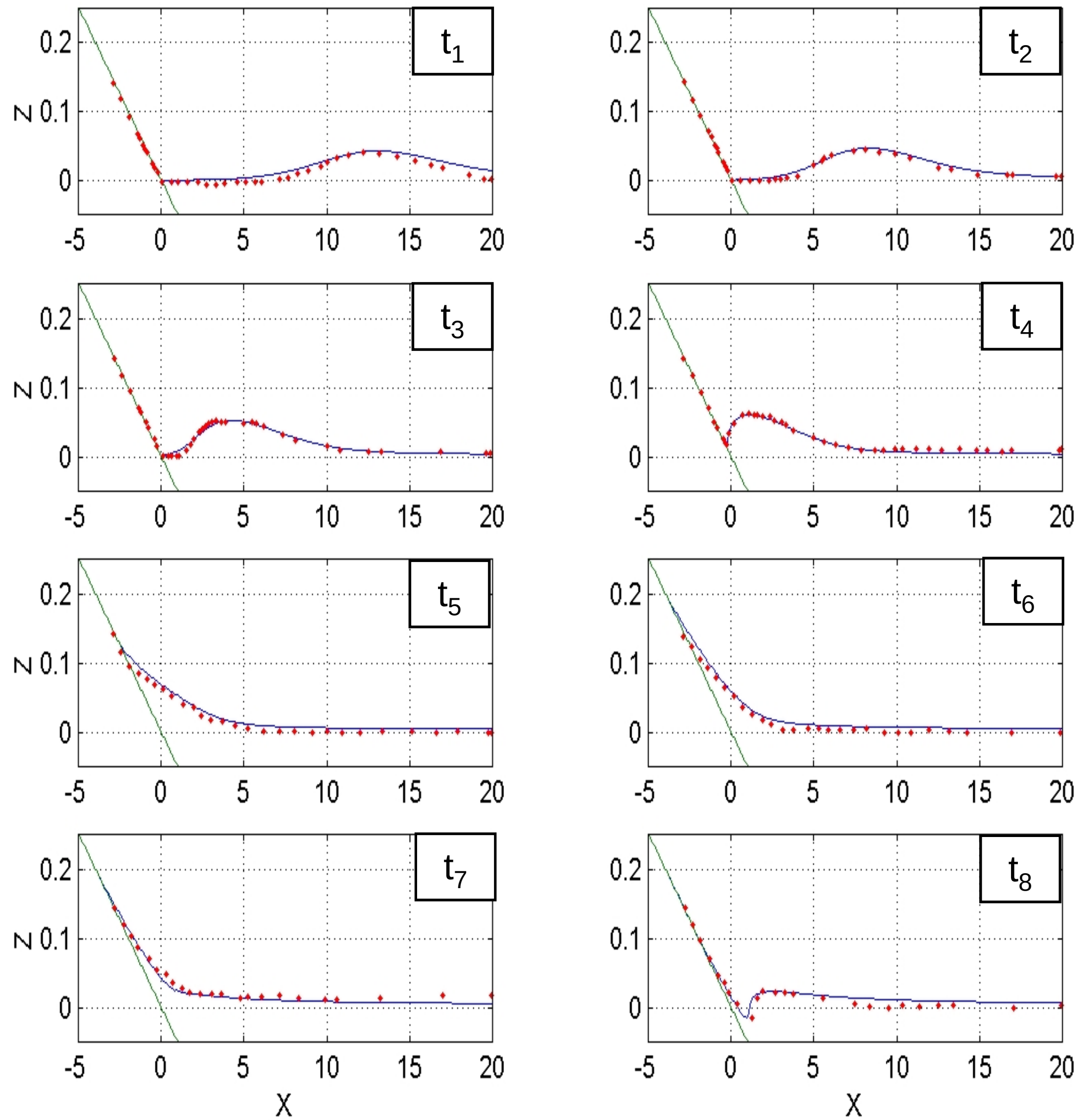
Standard Runup Benchmarks – 1HD

- Runup of solitary waves
  - Comparison with experimental data taken from Synolakis (1987)
- Numerical simulation parameters:
  - Wave height / water depth = 0.04
  - Beach slope = 1:20

Comparison with experimental data:

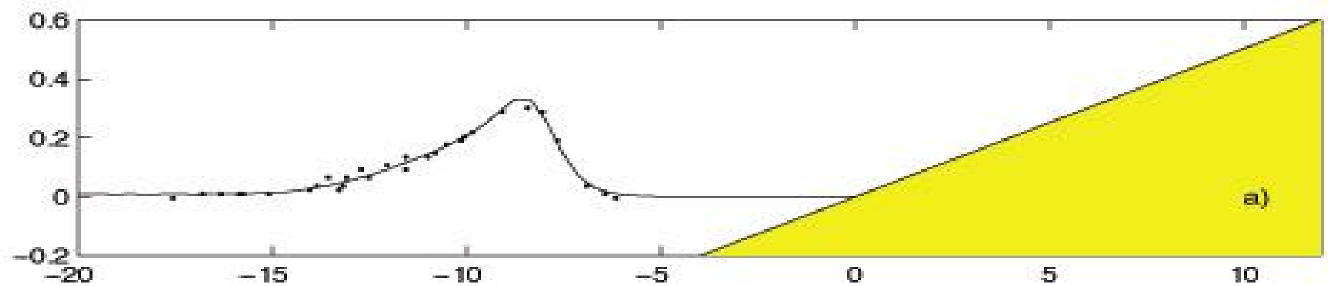
Numerical results

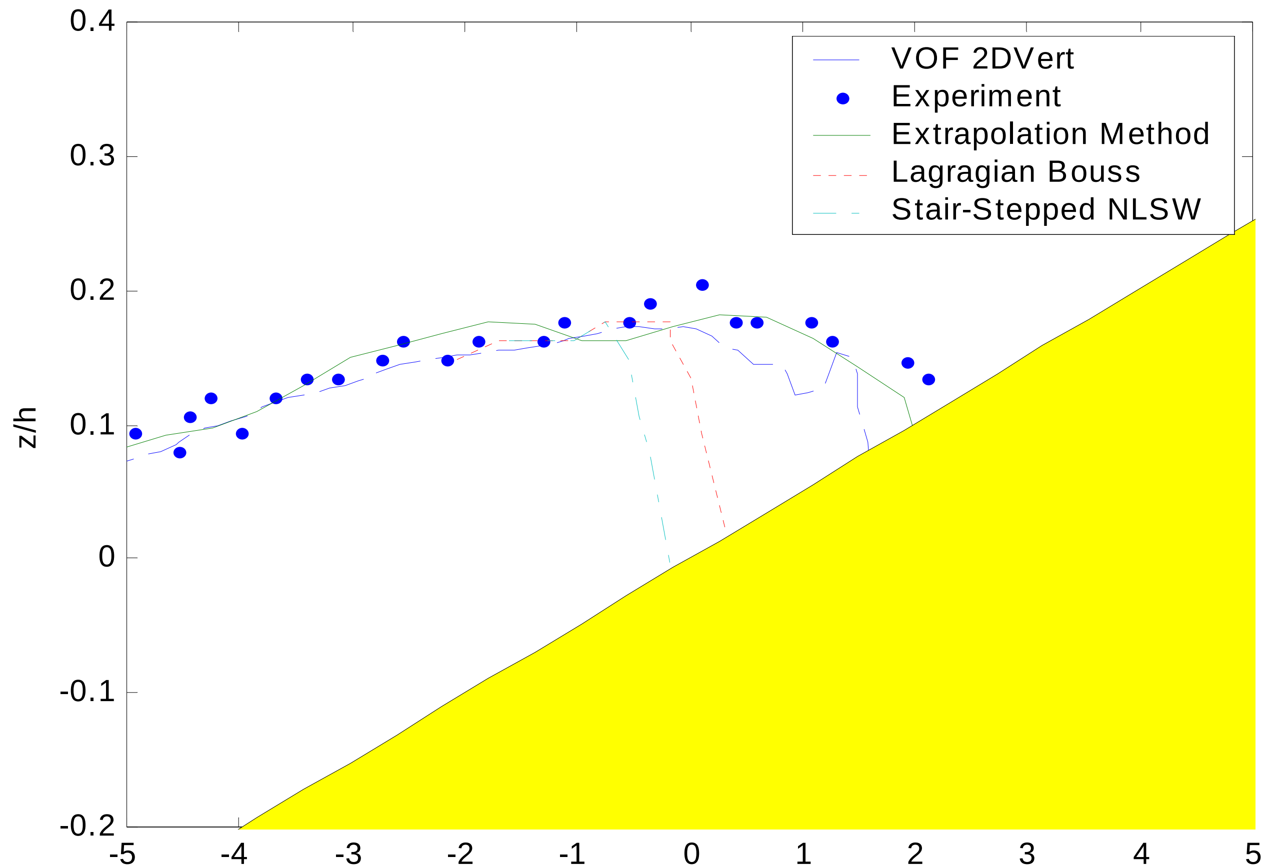
Experimental data



Standard Runup Benchmarks – 1HD

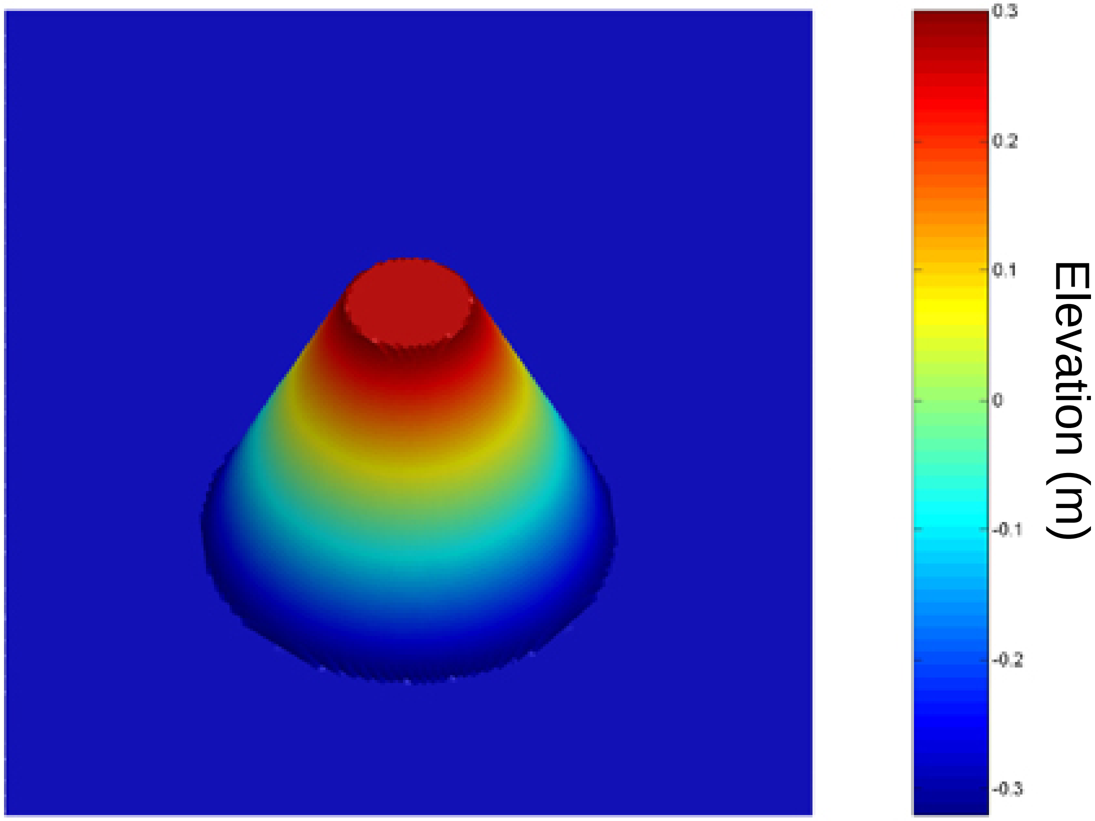
- Runup of breaking solitary waves
  - Wave height / water depth = 0.28
  - Beach slope = 1:20





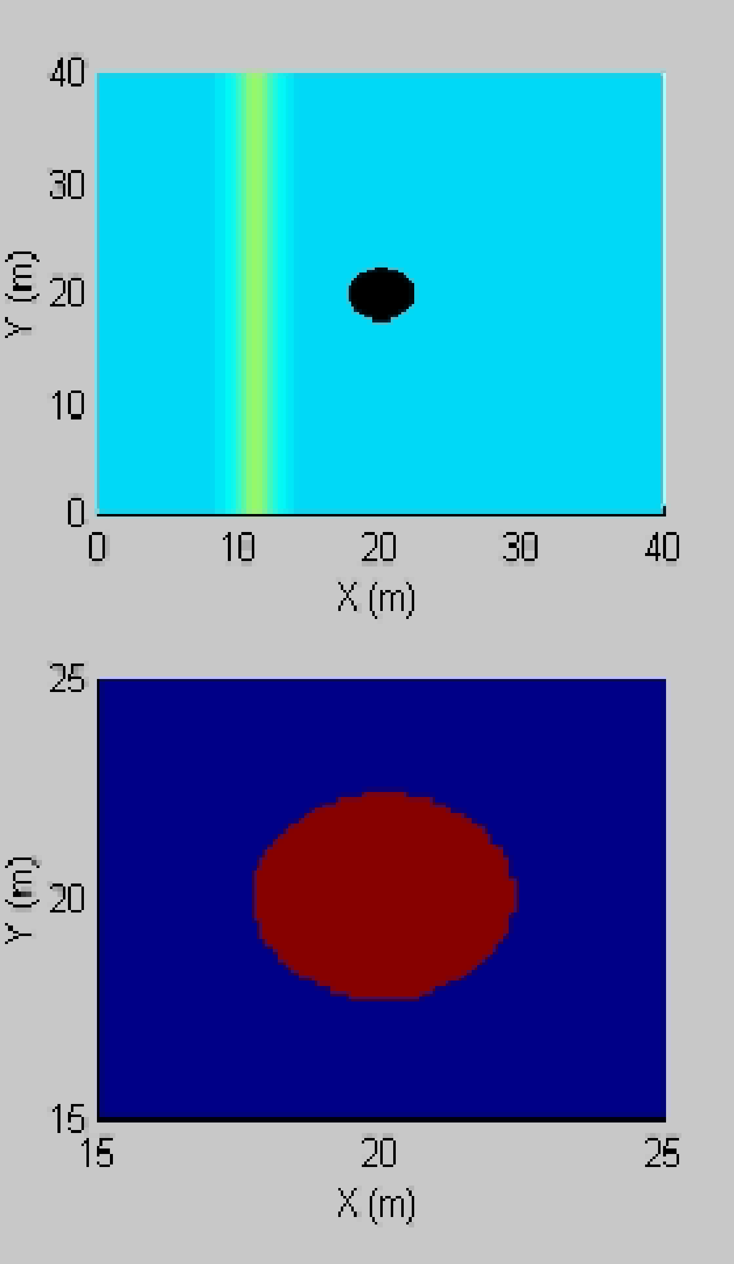
Standard Runup Benchmarks – 2HD

- Runup of solitary wave around a circular island
  - Experimental data taken from Liu et al. (1995)
- Physical setup:
  - Still water depth = 0.32 m
  - Slope of side walls = 1:4
  - Depth profile →



Standard Runup Benchmarks – 2HD

- Numerical simulation of conical island runup:
  - Wave amplitude = 0.028 m
  - Still water depth = 0.32 m
  - Beach slope = 1:4
  - Δx = 0.1 m



Free surface

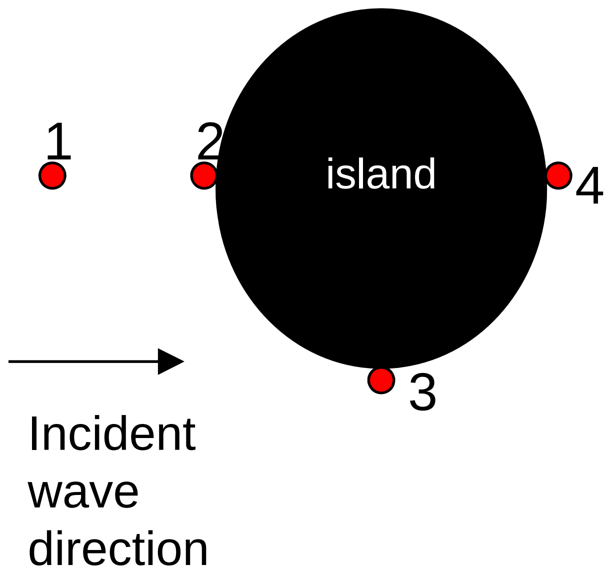
(island is black)

Inundation

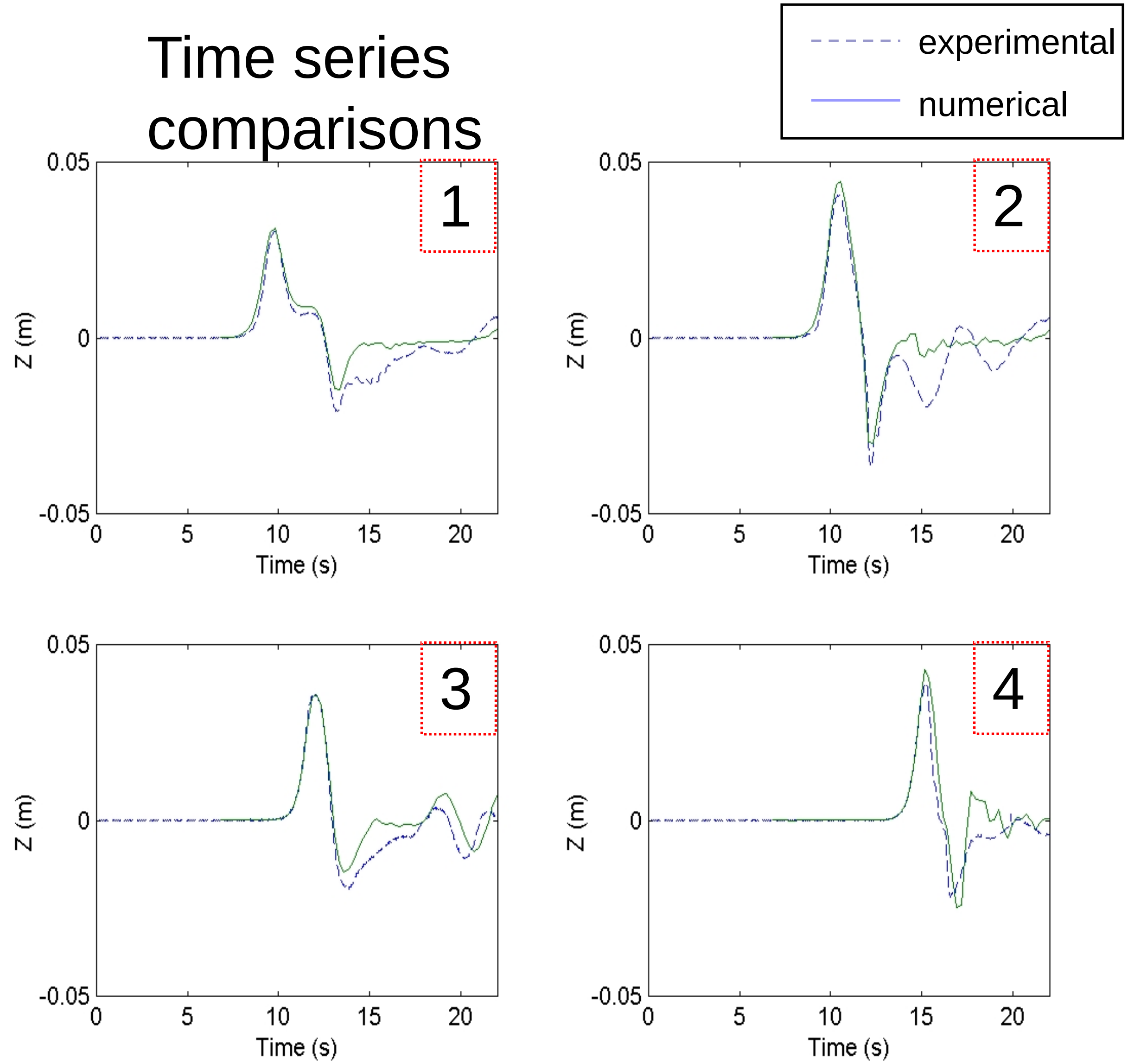
Initial dry land is shown in red – inundation is shown by the green

Standard Runup Benchmarks – 2HD

- Runup of solitary wave around a circular island
  - Experimental data taken from Liu et al. (1995)



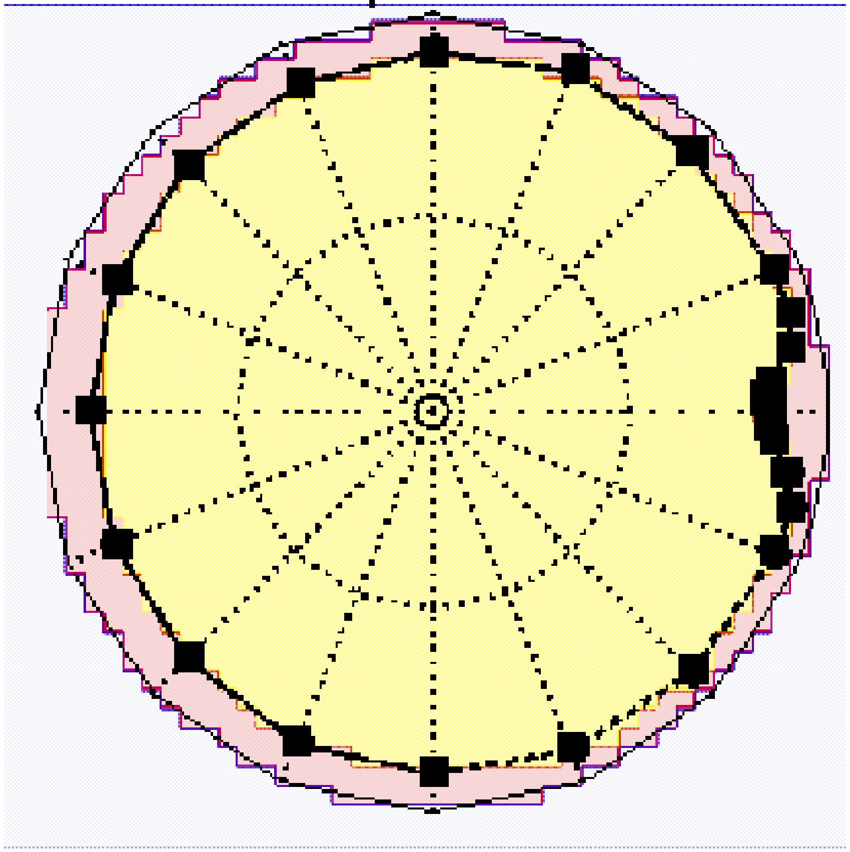
Time series comparisons



Standard Runup Benchmarks – 2HD

- Runup of solitary wave around a circular island
  - Experimental data taken from Liu et al. (1995)

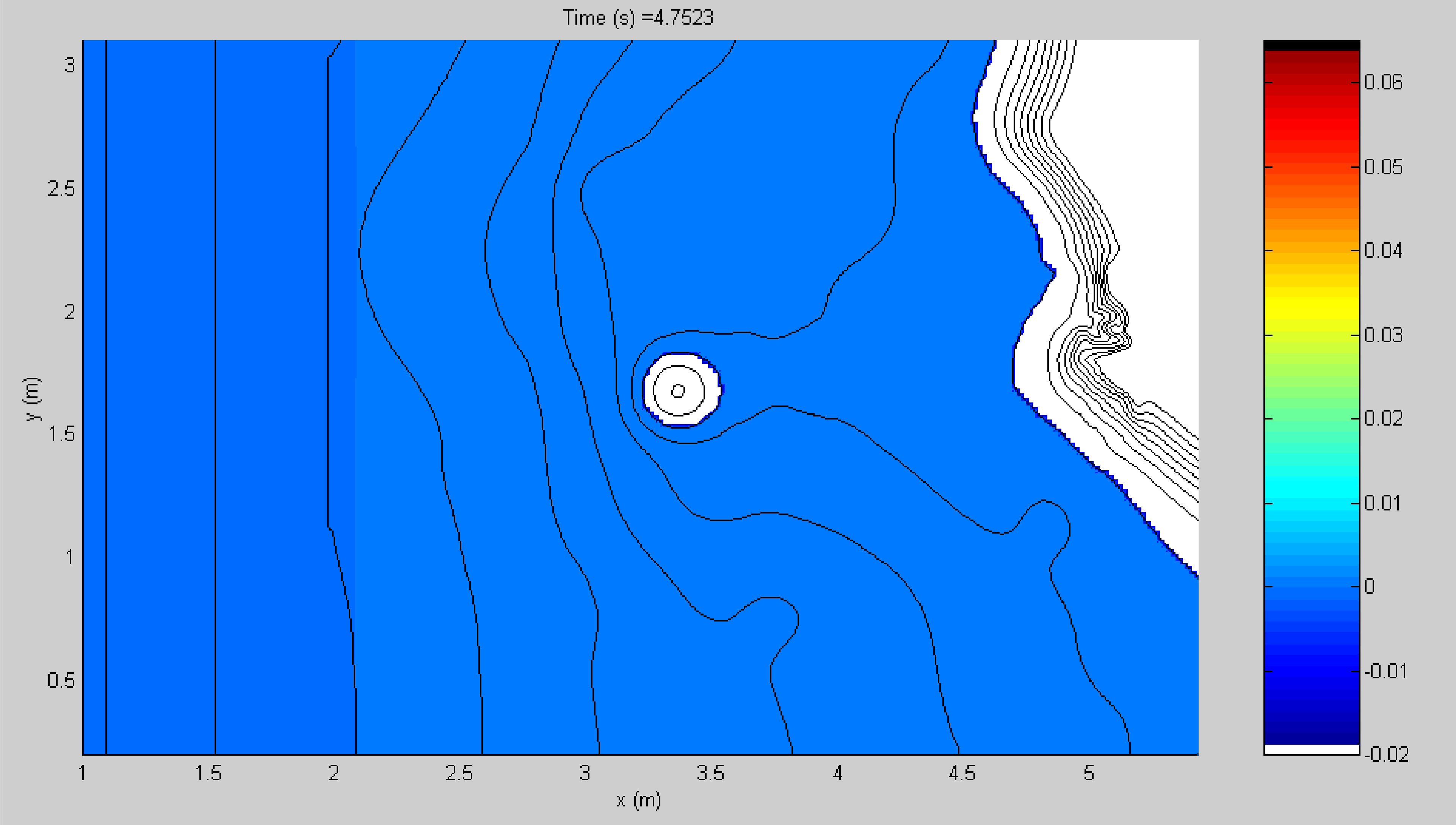
Inundation comparisons



Black dots represent the maximum experimental runup, while the light red shows the inundated area

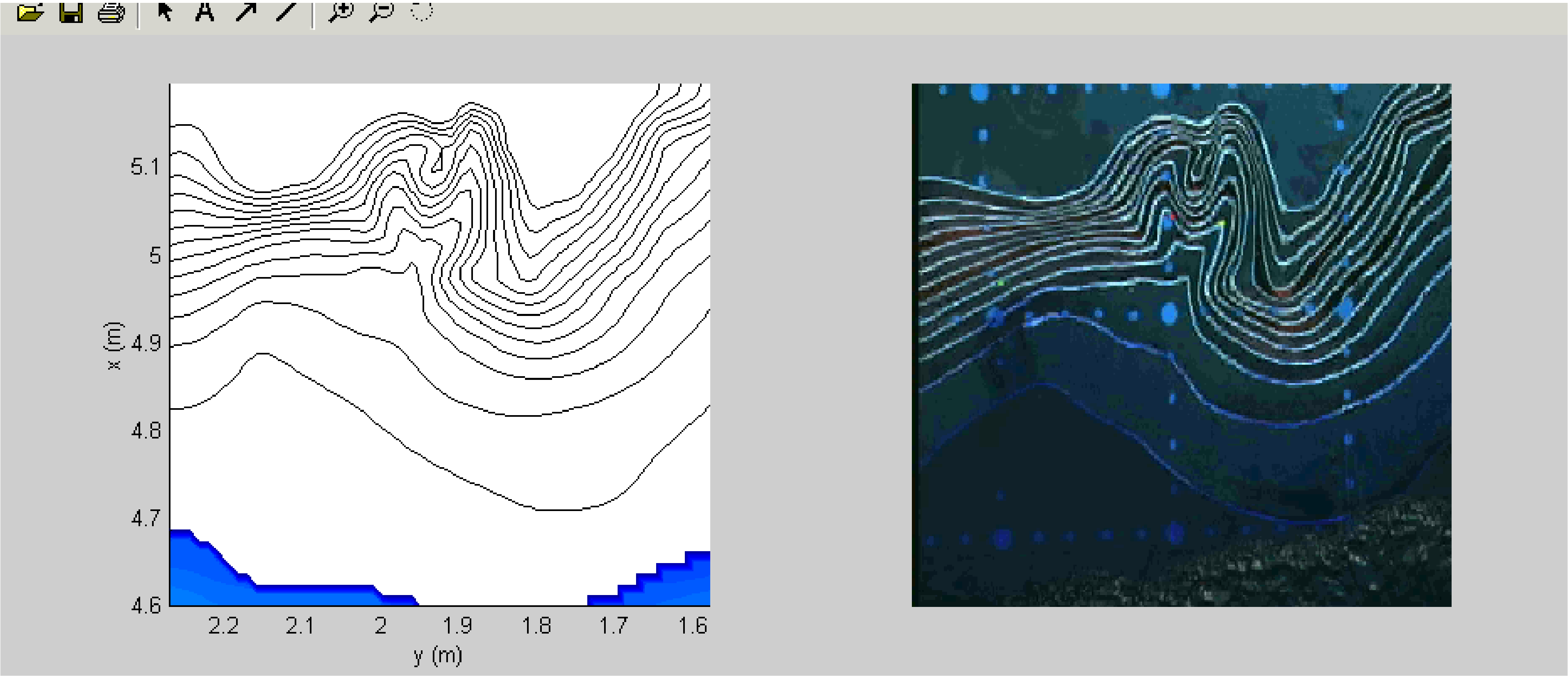
Standard Runup Benchmarks – 2HD

- Tsunami Approach on Complex Bathymetry (lab scale recreation of tsunami flooding near Monai on Okushiri Island due to 1993 tsunami)
  - Plan view



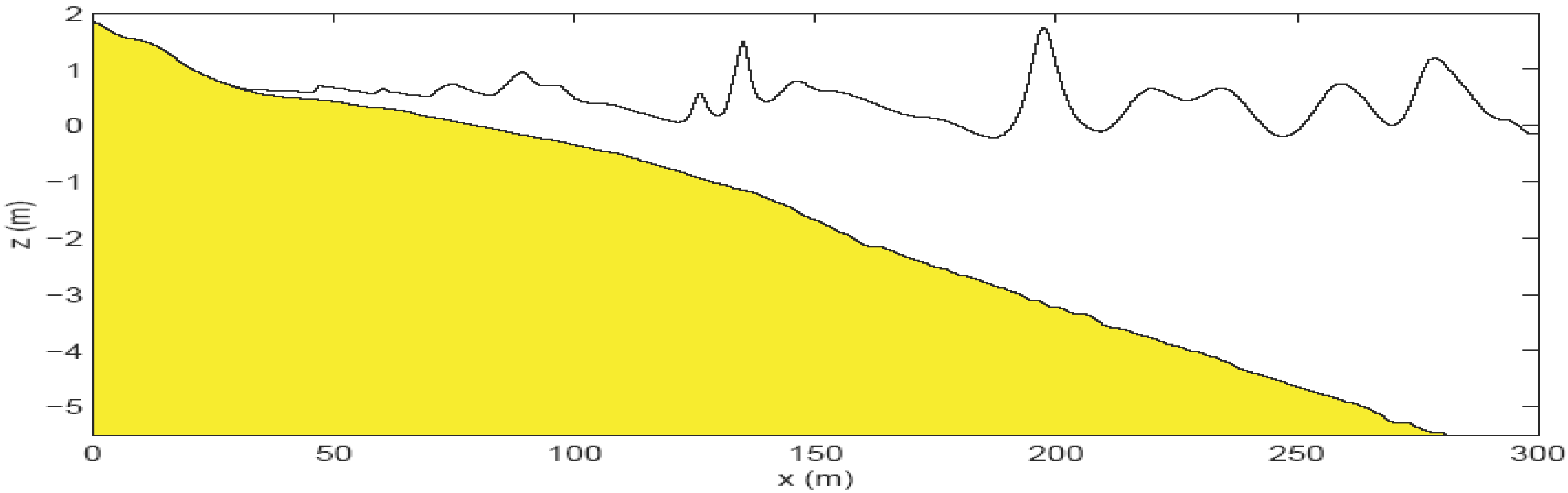
Standard Runup Benchmarks – 2HD

- Tsunami Approach on Complex Bathymetry
  - Comparison w/ video data

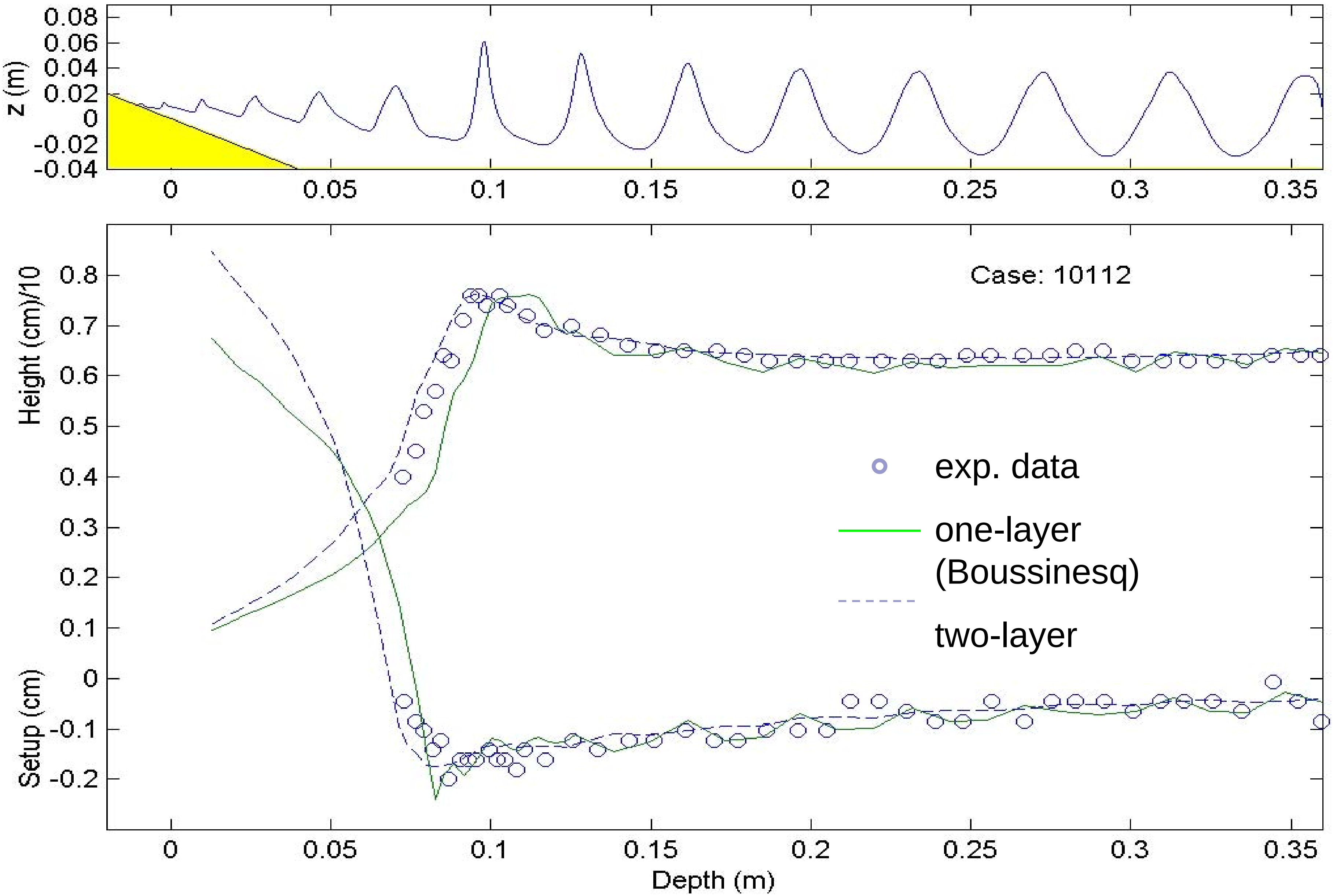


Standard Wave Evolution Benchmarks – 1HD

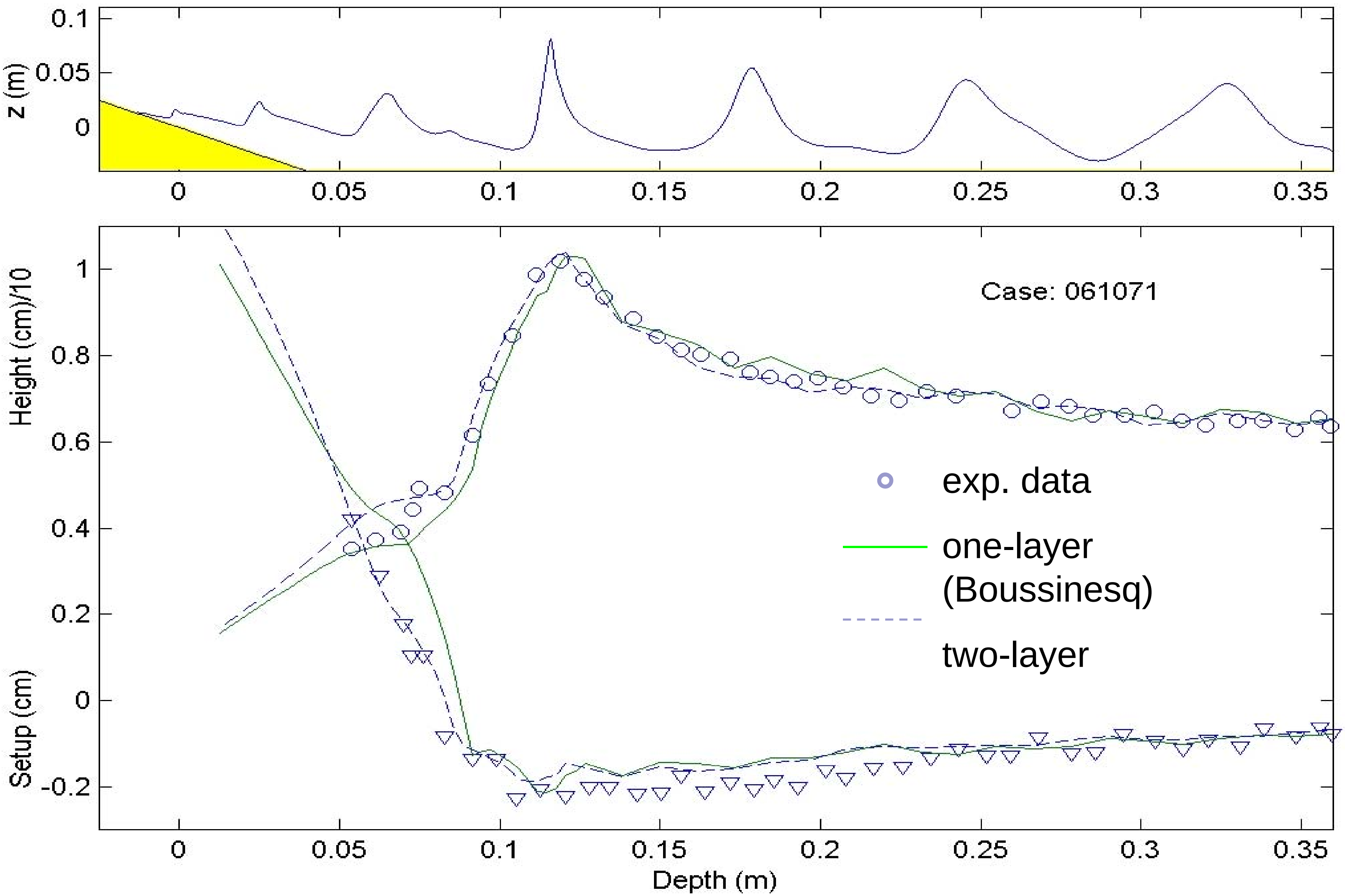
- High-order (two-layer) model and conventional order (highly nonlinear version of Nwogu) model will be compared with experiments
  - All numerical simulation parameters identical
- Experiments to be compared:
  - Regular waves breaking on a planar slope (Hansen & Svendsen, 1979)
  - Cnoidal waves breaking on a planar slope (Ting & Kirby, 1995, 1996)
  - Regular waves breaking over a submerged bar (Dingemans, 1994)
  - Regular and irregular waves breaking onto a shelf (Lee, 2005)
  - Irregular waves breaking over real bathymetry – field data (Raubenhiemer, 2002)



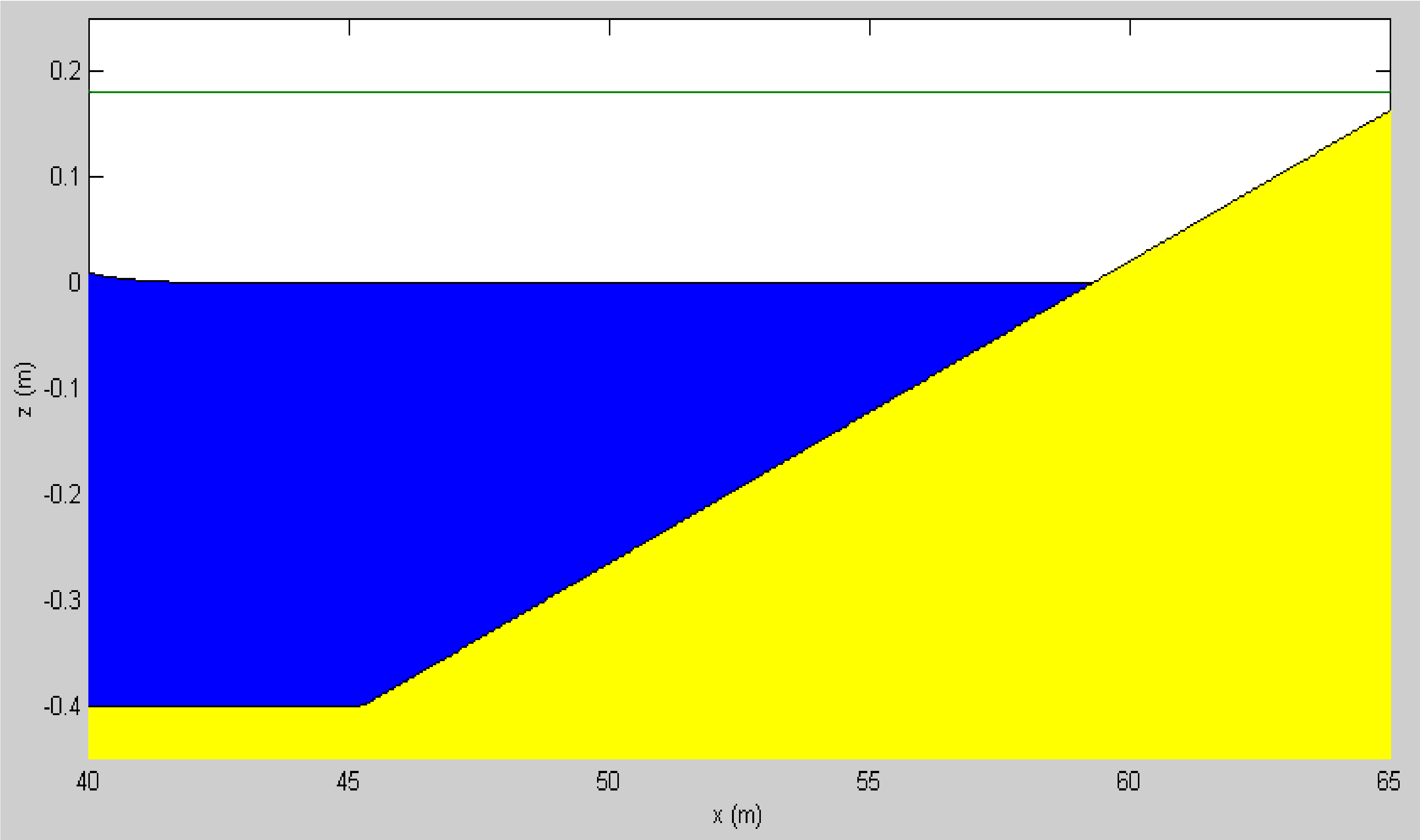
Regular Wave Breaking



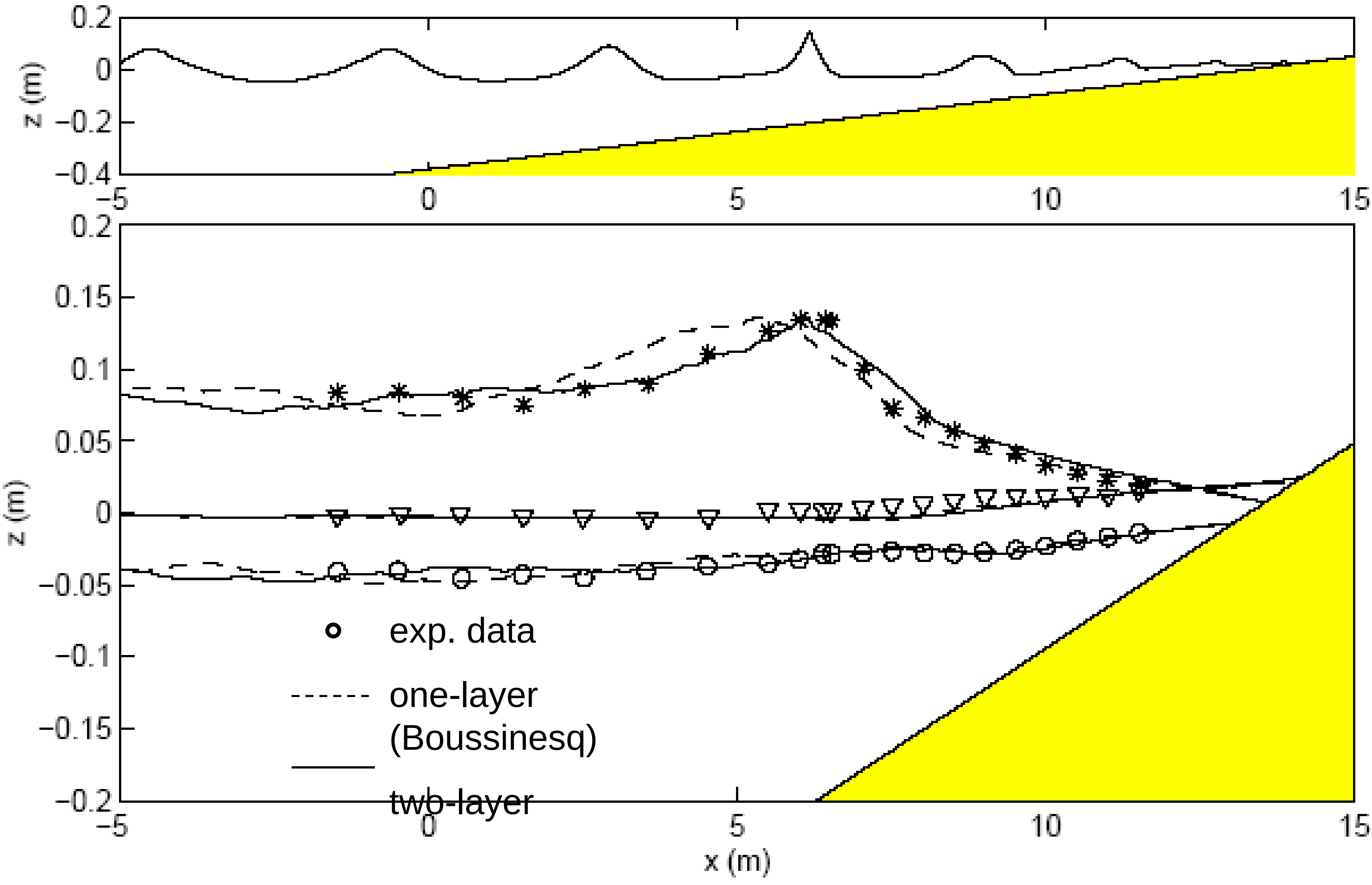
Regular Wave Breaking



Cnoidal Wave Breaking

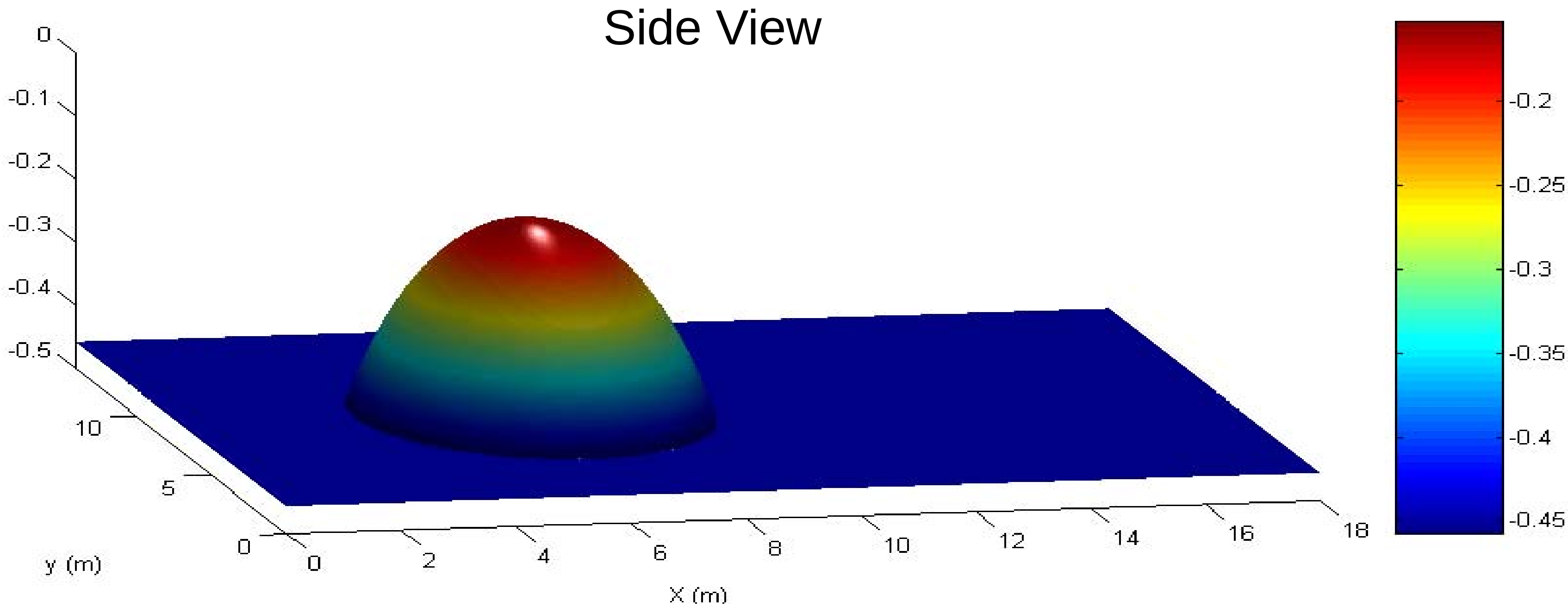


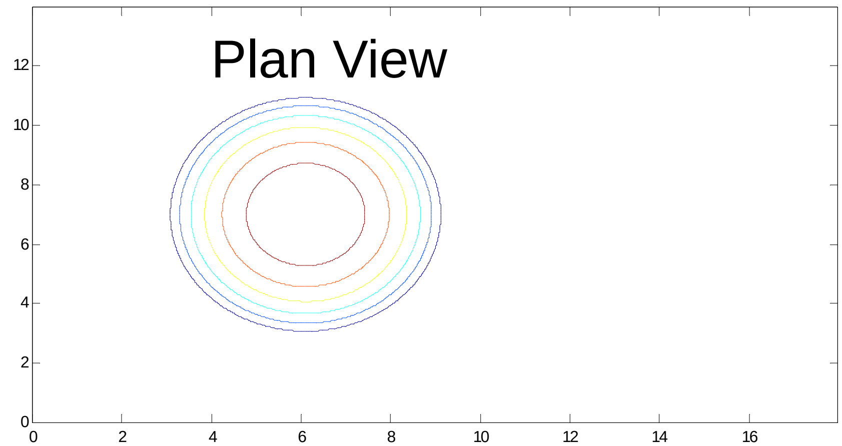
Cnoidal Wave Breaking - Spilling



Standard Wave Evolution Benchmarks – 2HD

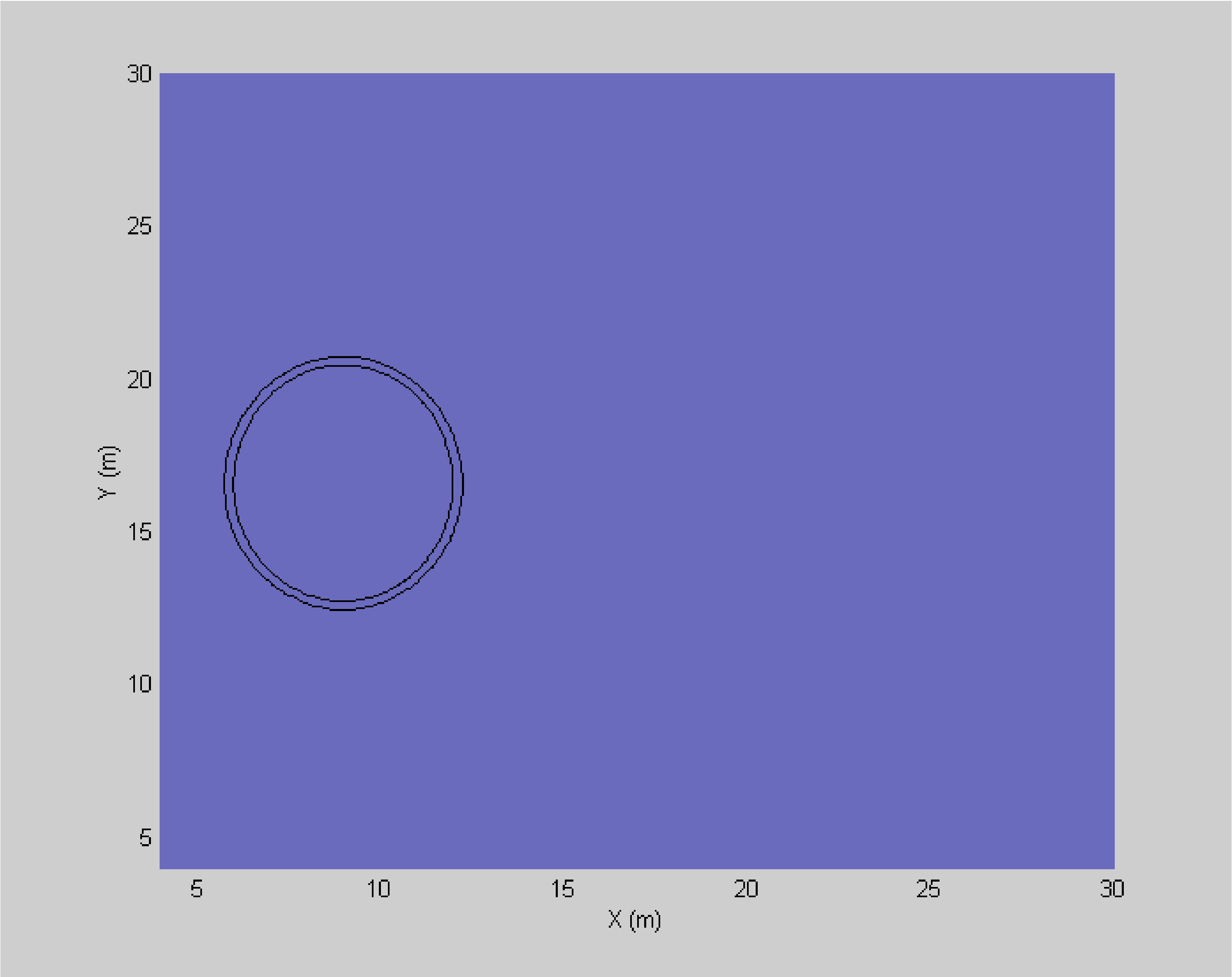
- Vincent & Briggs (1989) experiments





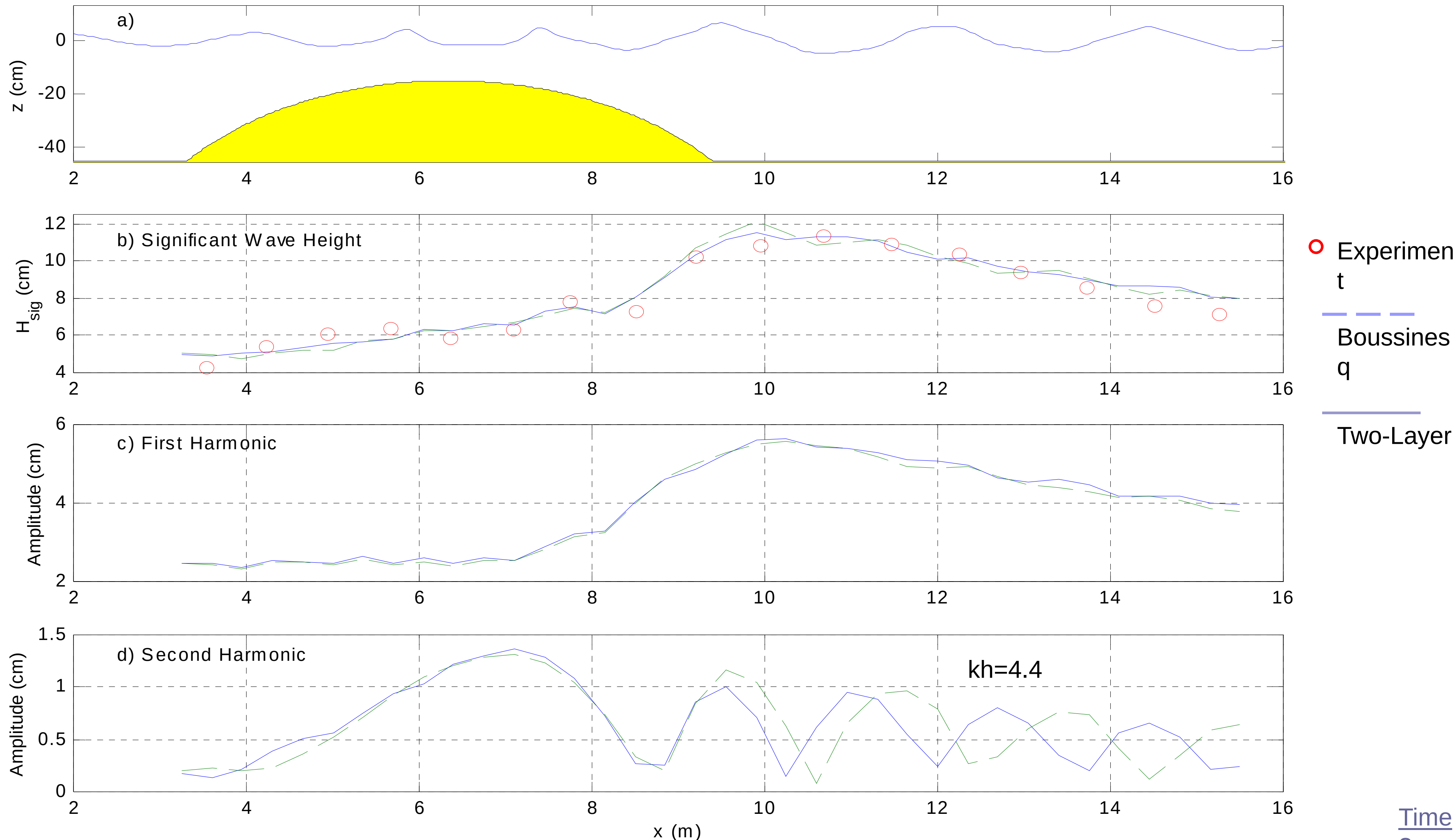
Standard Wave Evolution Benchmarks – 2HD

- Numerical simulation: *a/h*=0.0427, *kh*=1.92



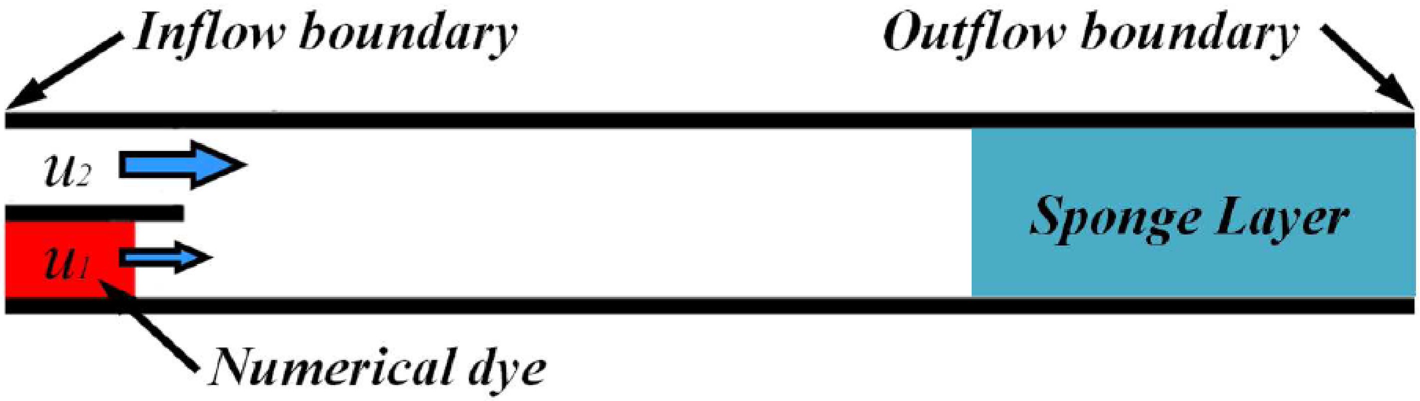
Standard Wave Evolution Benchmarks – 2HD

- Compare One-Layer (Boussinesq), Two-Layer Model, and Experiment along centerline of basin



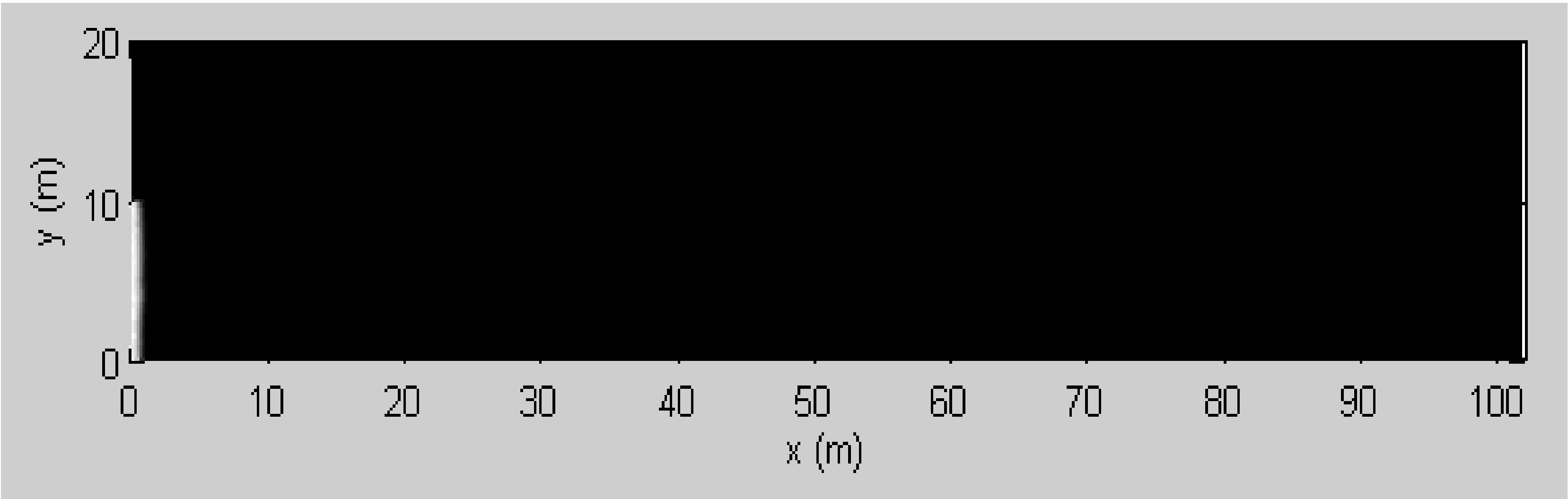
Time ?

Shallow Turbulence Benchmarks



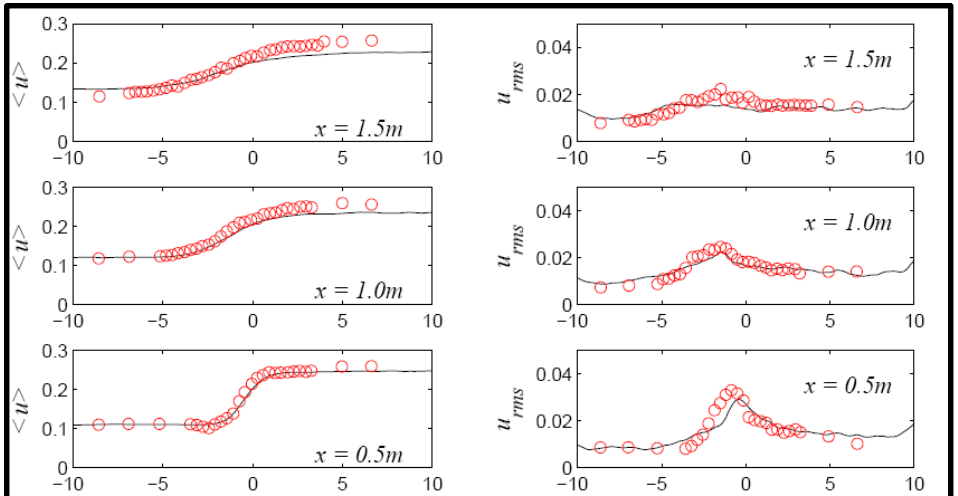
- **u_1** = 0.111m/s, **u_2** = 0.264m/s, Re = 5550

Experiment by **Babarutsi and Chu** (1998)

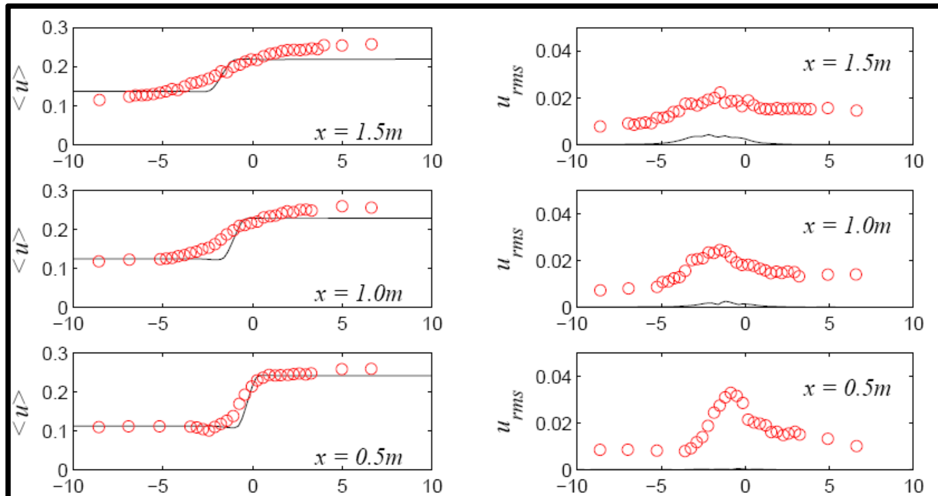


http://ceprofs.tamu.edu/plynett/isec/shear.avi

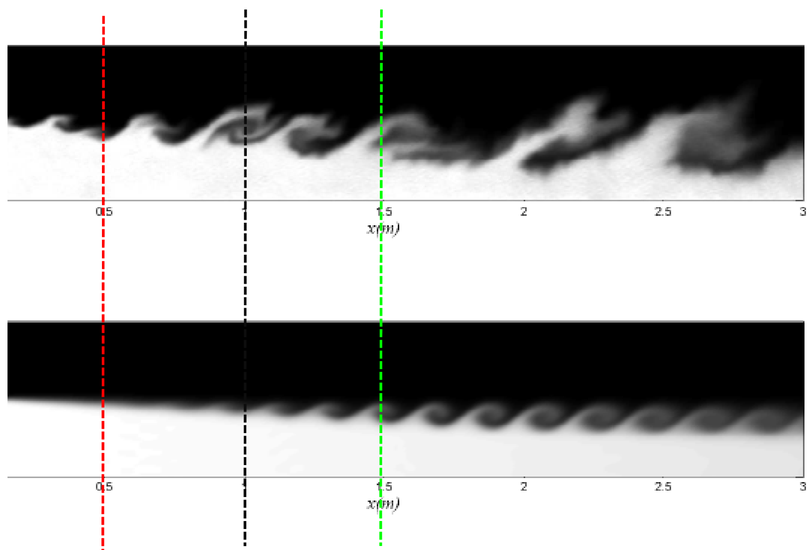
Shallow Turbulence Benchmarks

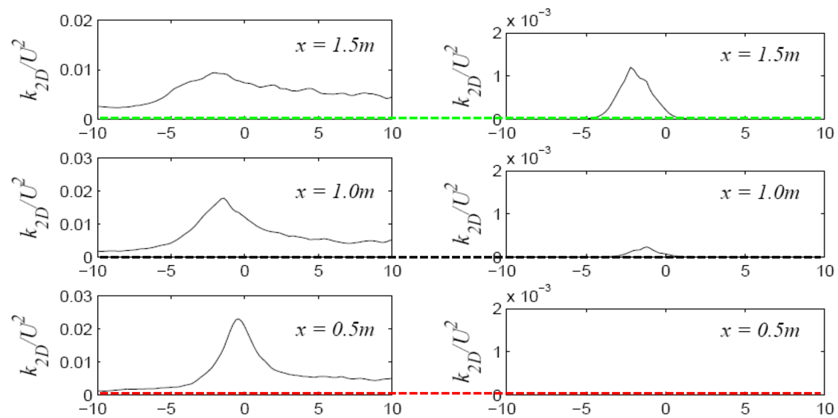


with BSM



without BSM



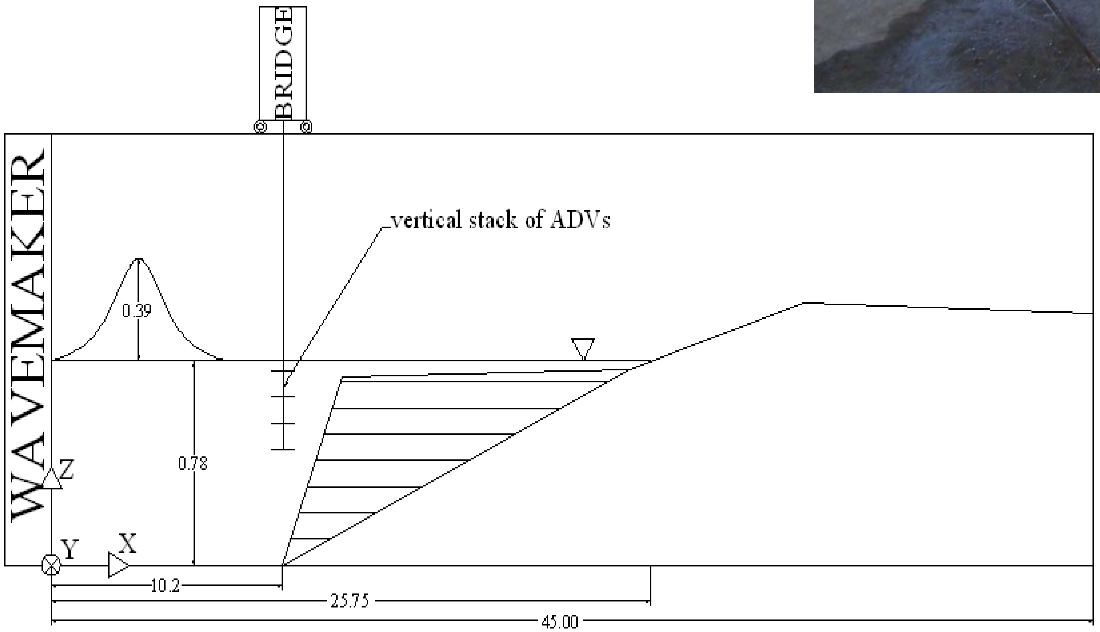


with BSM

without BSM

Wave & Shallow Turbulence Benchmark

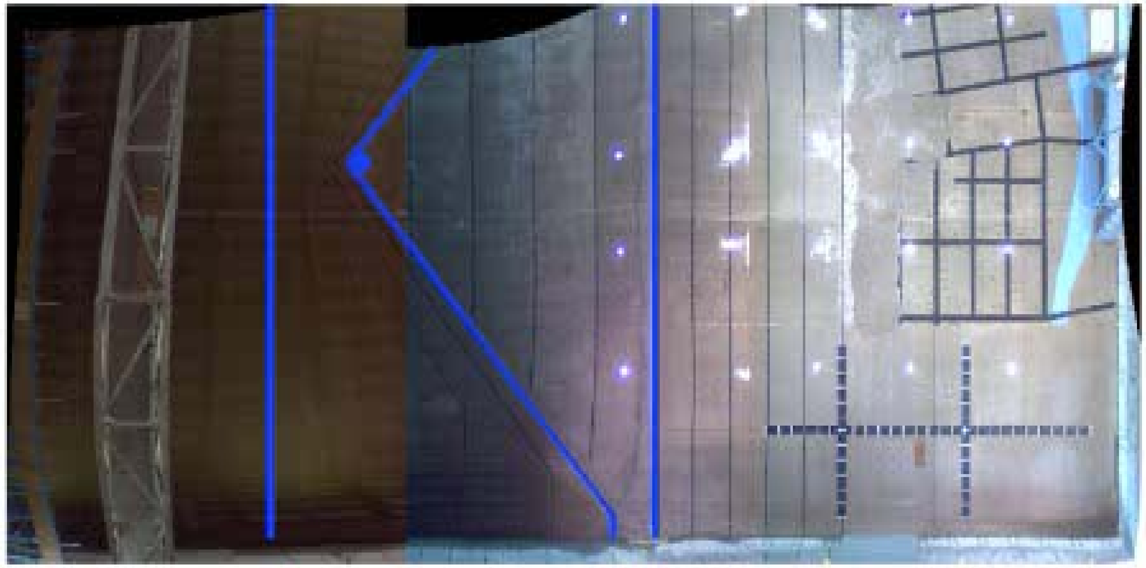
- Basin:
  48.8m x 26.5m x 2.1m
- Piston-type wavemaker
- Bridge spanning width of basin

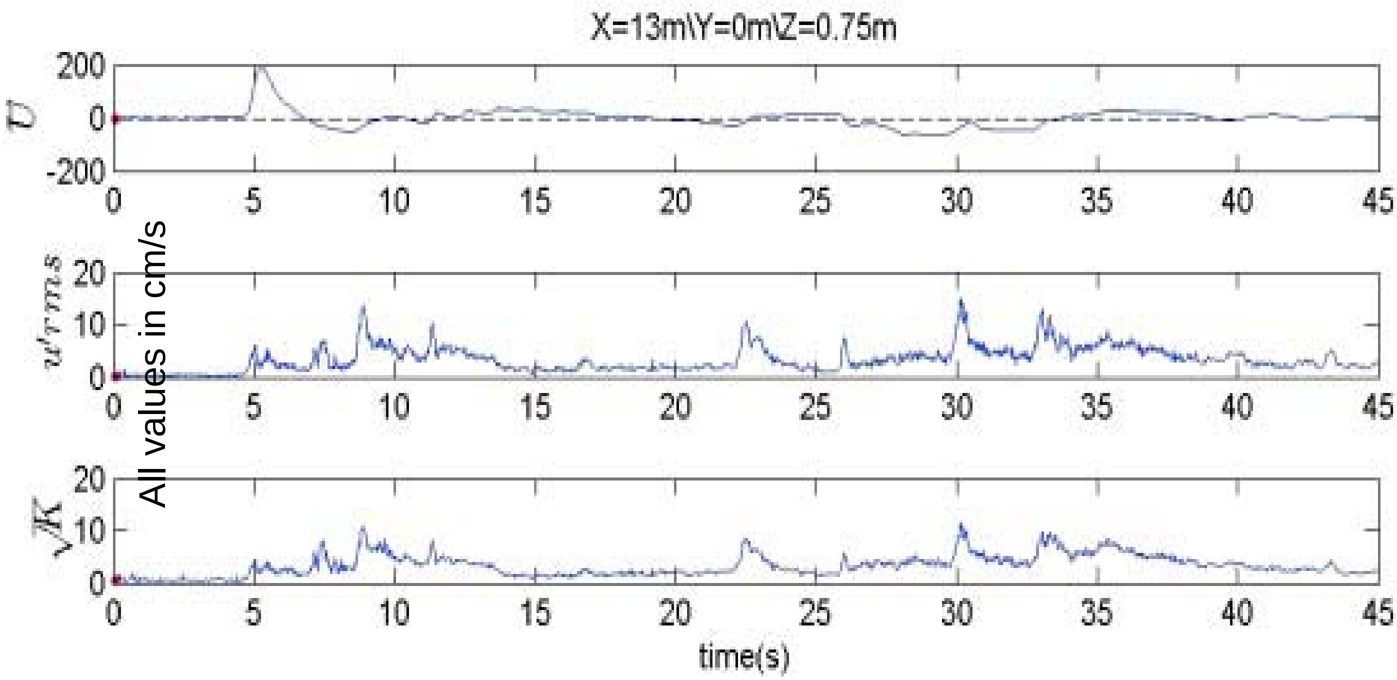


- Complex shelf
- Planar beach
- Water depth: 0.78m

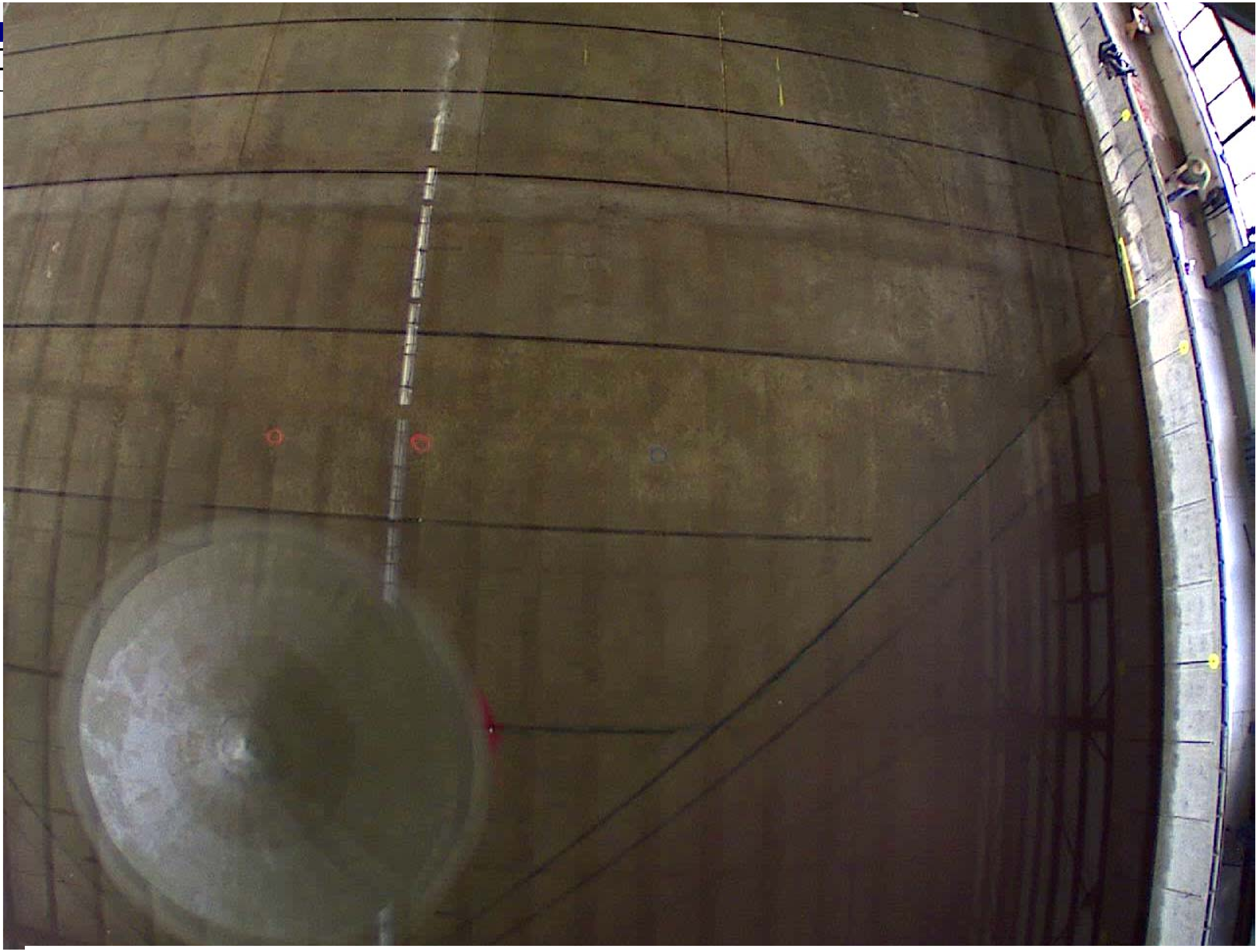
Wave & Shallow Turbulence Benchmark

Wave & Shallow Turbulence Benchmark




X=13m\Y=0m\Z=0.75m
U
u'rms
√K
All values in cm/s
time(s)

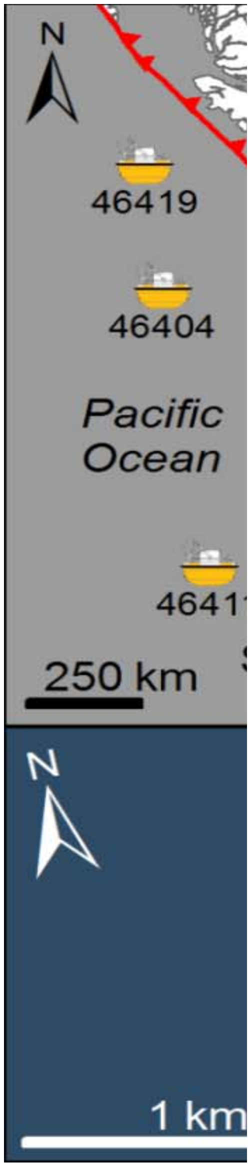
F

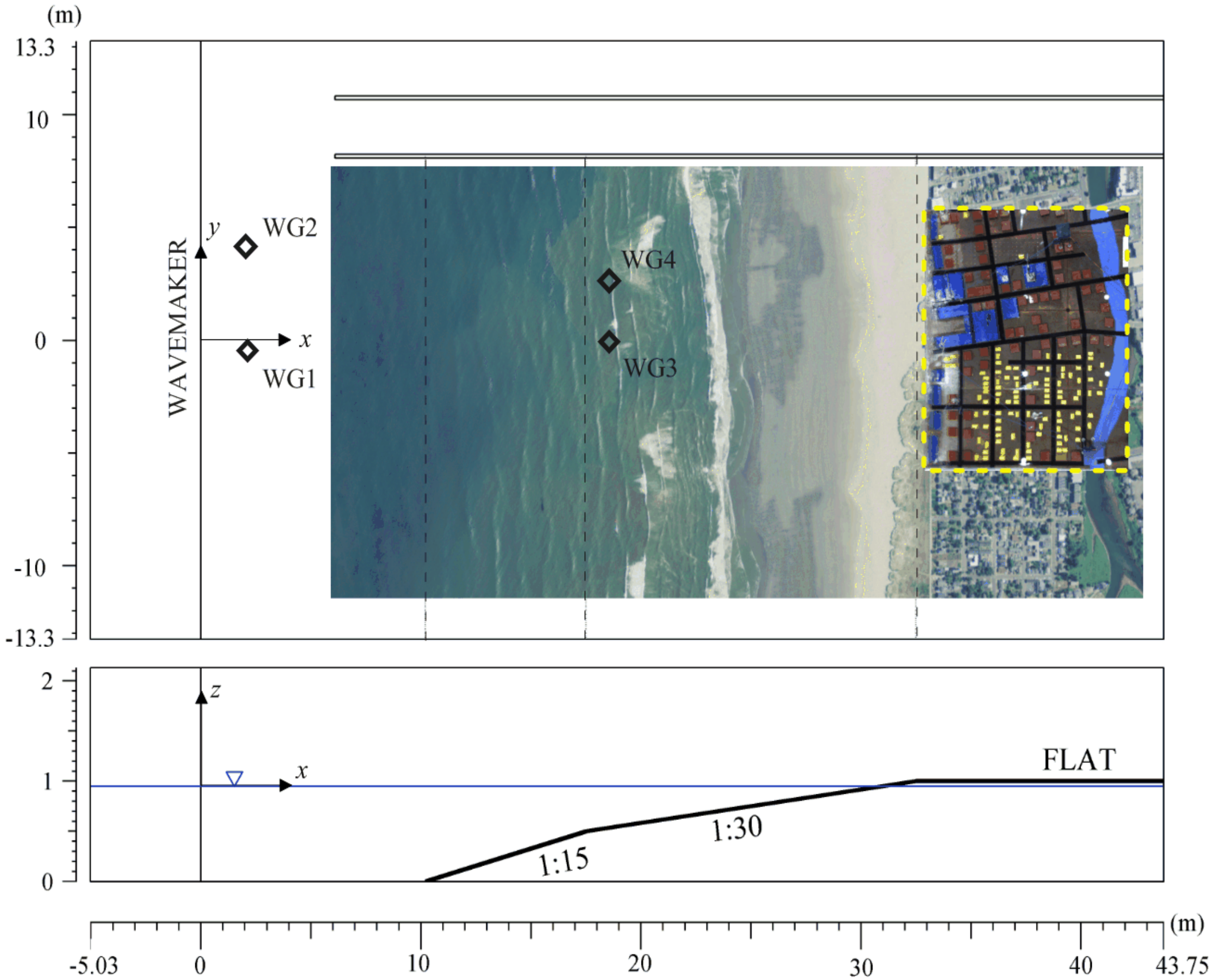


http://ceprofs.tamu.edu/plynett/isec/P_NEEStsunamos3.E_DyeStudy.T_Trial06.cam11.avi

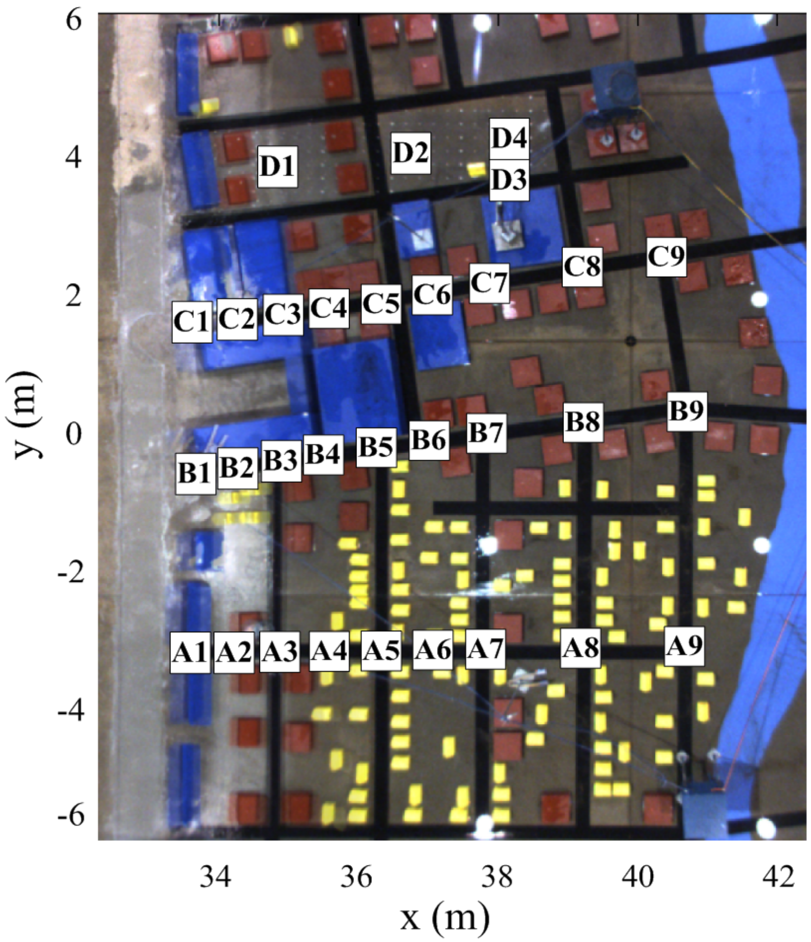
Flow through Complex Topography Benchmark

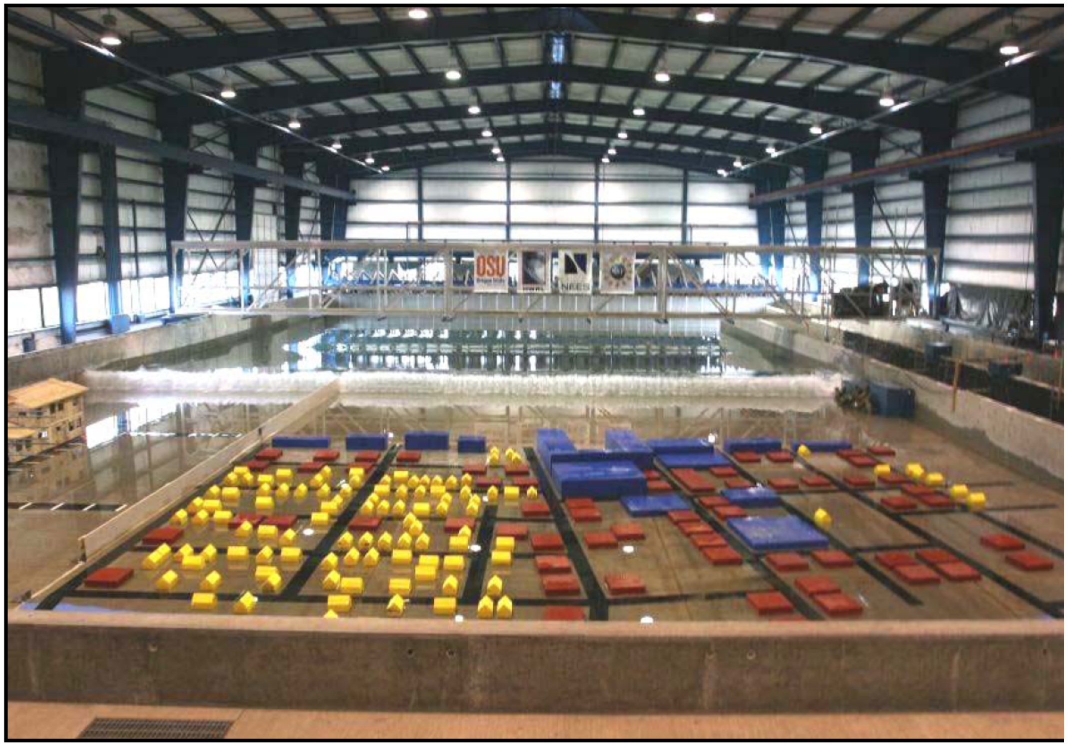
Large-Scale Laboratory Experiments





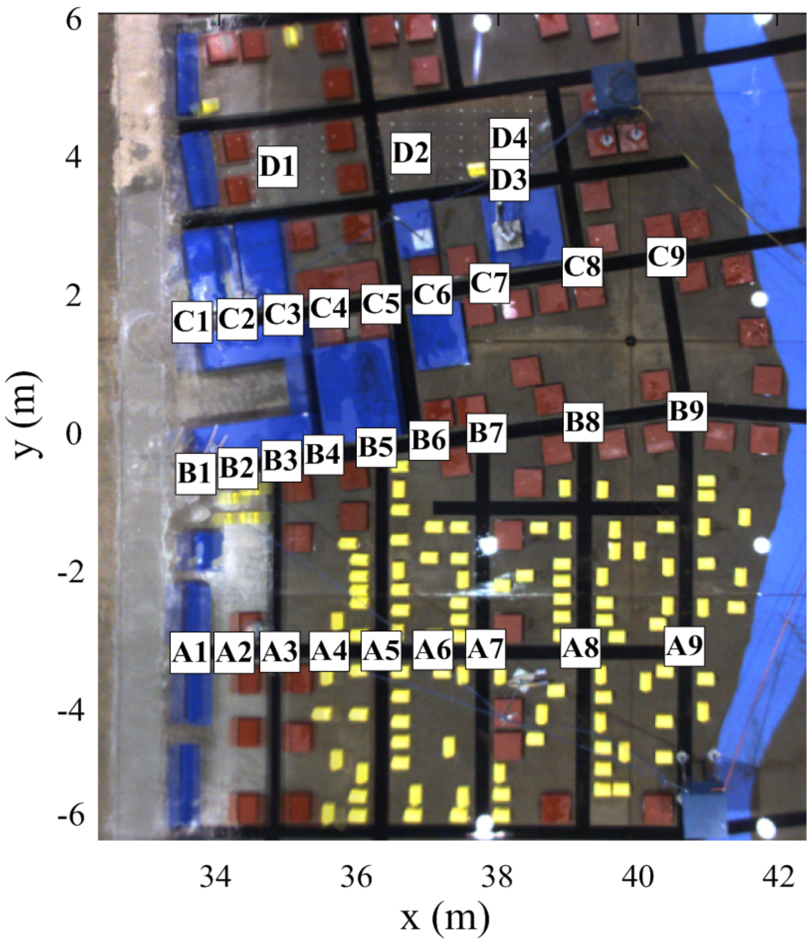
Flow through Complex Topography Benchmark
Large-Scale Laboratory Experiments

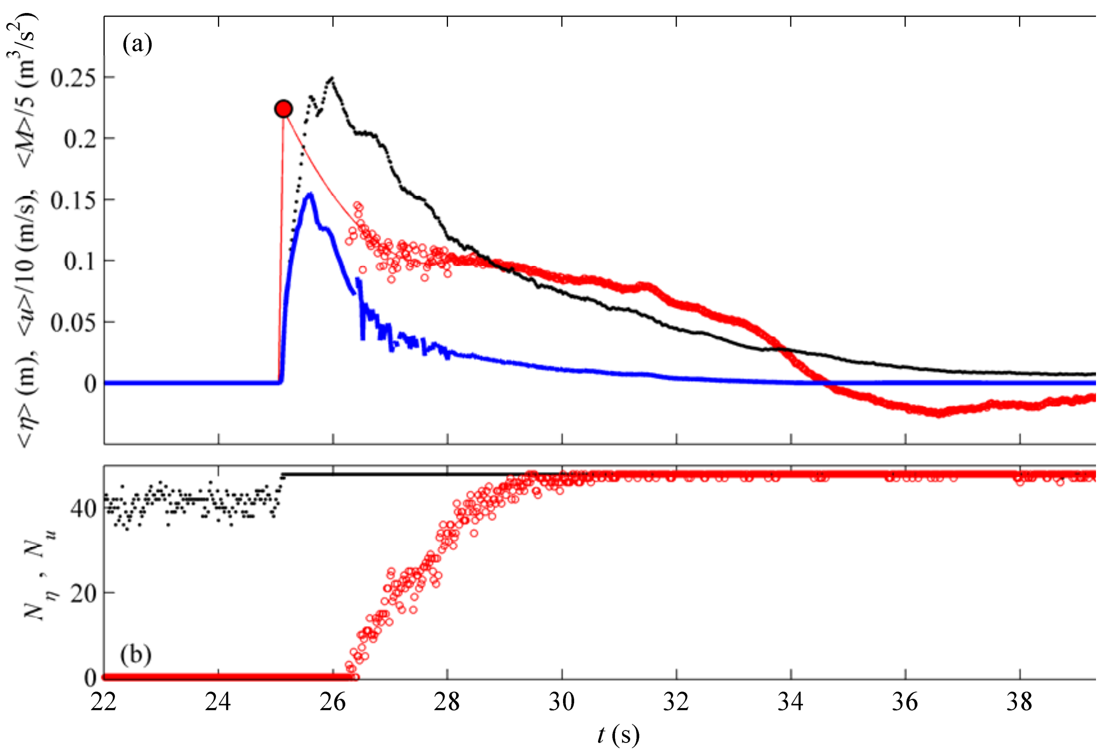




- Co-located wave gage and ADV measurements along transects
- Overhead optical measurements for front tracking and bore speed
- 5-40 trials used to calculate mean properties
- Max stroke wave pulse, ~20 cm, period ~ 5 sec

Flow through Complex Topography Benchmark
Large-Scale Laboratory Experiments





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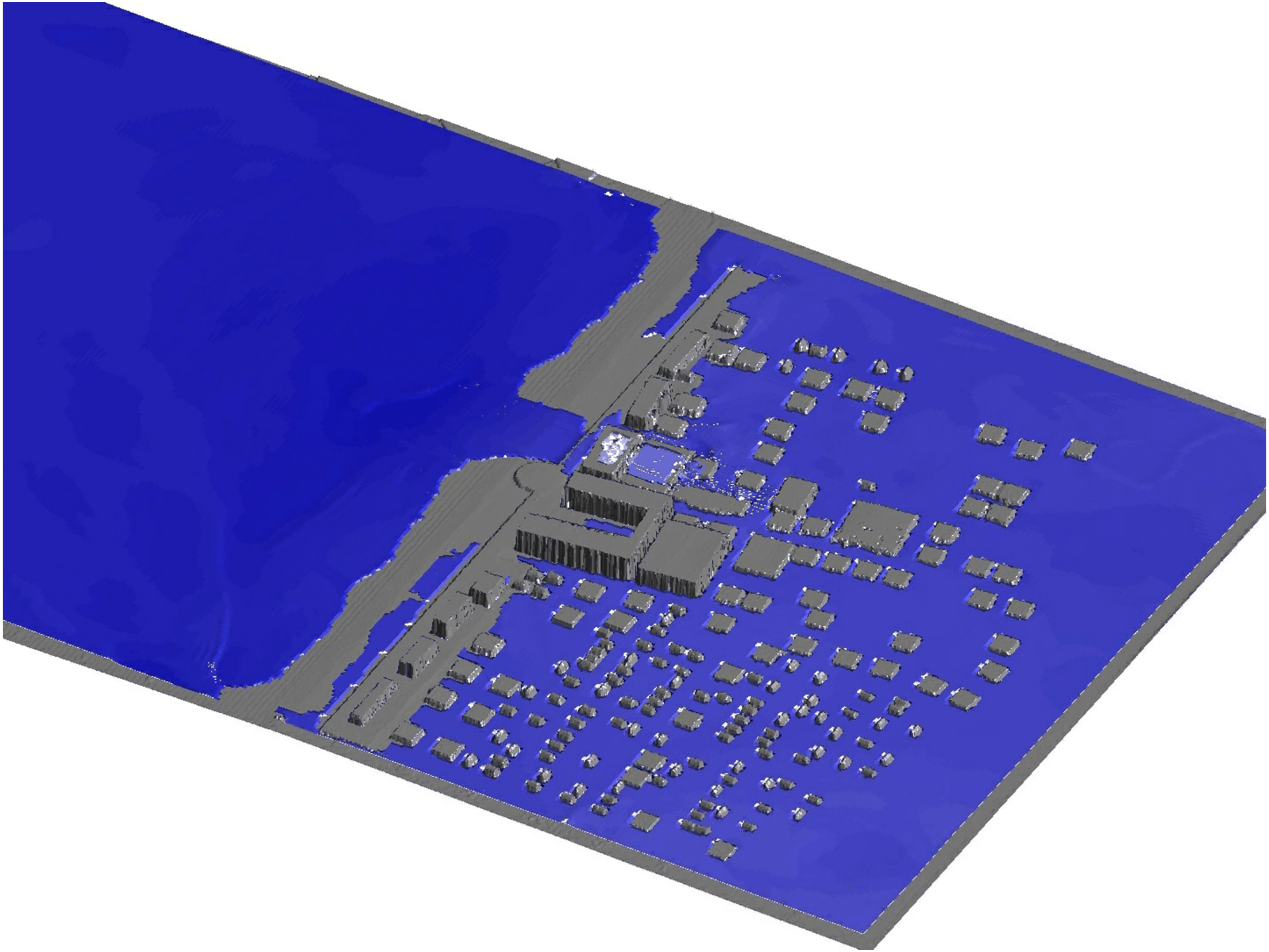
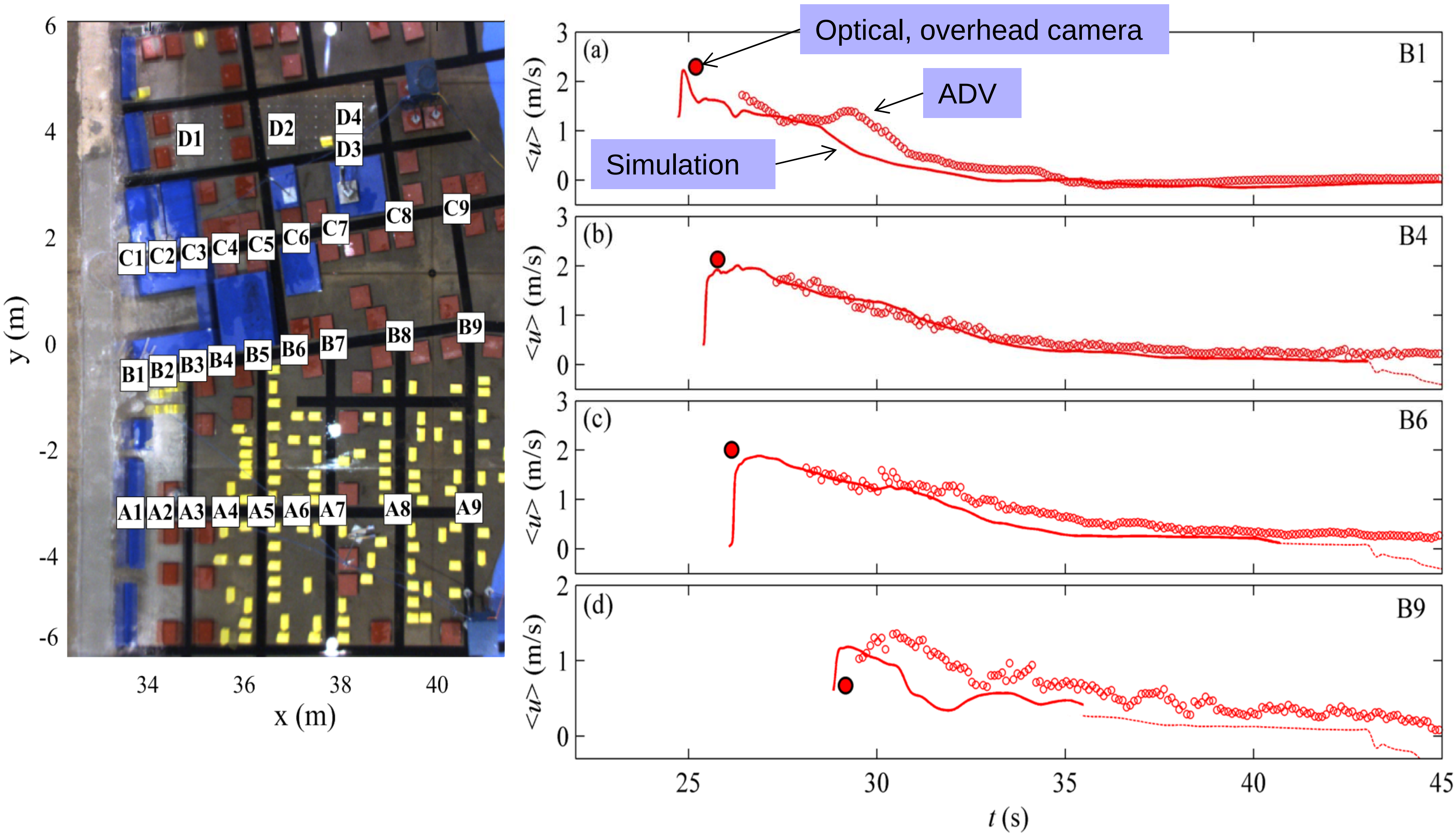
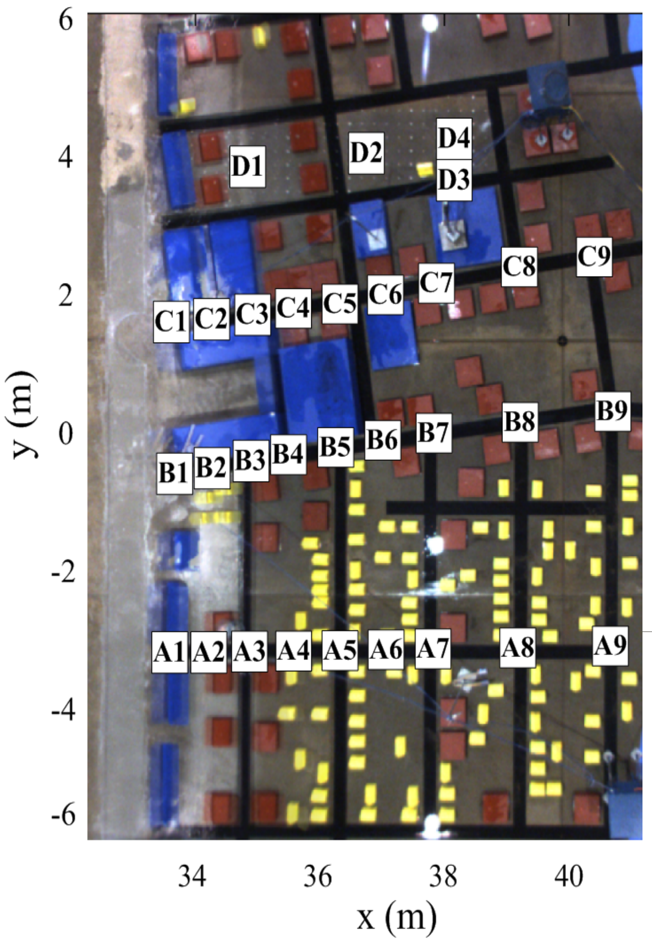


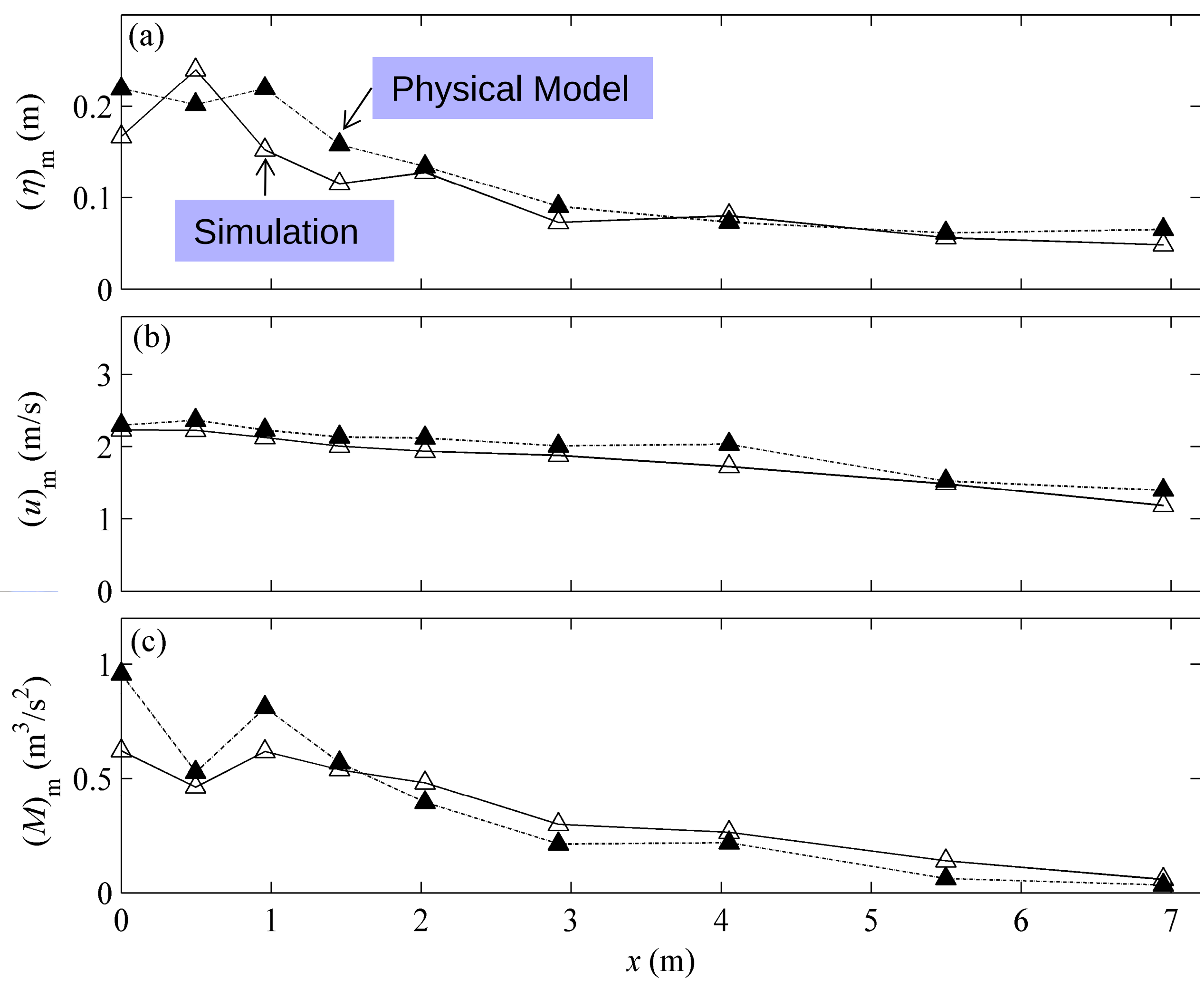
Flow through Complex Topography Benchmark
Experiment – Simulation Comparisons



Flow through Complex Topography Benchmark

Experiment – Simulation Comparisons





Conclusions

- Provide a background on state-of-the-art coastal wave modeling
- Many different “high-order” corrections to the traditional Boussinesq model
  - Better linear (and nonlinear) dispersion
  - O(1) nonlinearity (to order of included dispersion)
  - Rotational and turbulent effects
- All come with additional (and sometimes substantial) computational cost
  - Is the magnitude of the correction greater than uncertainties in specifying the physical problem?
  - Breaking models will always be highly ad-hoc when used with depth-integrated models
  - Science vs Engineering

“Boussinesq” Equations

Continuity Equation

New terms, due to the Boussinesq-type derivation

$$\eta_t + \nabla \cdot (Hu_\alpha) -$$

$$\nabla \cdot \left\{ H\left(\frac{1}{6}\left(\eta^2 - \eta h + h^2\right) - \frac{1}{2} z_\alpha^2 \right) \nabla\left(\nabla \cdot u_\alpha\right) \right\} -$$

$$\nabla \cdot \left\{ H\left(\frac{1}{2}\left(\eta h - h\right) - z_\alpha \right) \nabla\left(\nabla \cdot \left(h u_\alpha\right)\right) \right\} = 0$$

$$H = h + \eta$$

“Boussinesq” Equations

Momentum Equation

New terms, due to the Boussinesq-type derivation

$$u_{\alpha t} + u_\alpha \cdot \nabla u_\alpha + g\nabla\zeta +$$

$$\left\{\frac{z_\alpha^2}{2}\nabla(\nabla\cdot u_{\alpha t}) + z_\alpha \nabla Q_t + z_{\alpha t}\left[z_\alpha\nabla(\nabla\cdot u_\alpha) + z_\alpha\nabla Q\right]\right\} +$$

$$\left[Q\nabla Q - \nabla\eta Q_t + (u_\alpha\cdot\nabla z_\alpha)\nabla Q + z_\alpha\nabla(u_\alpha\cdot\nabla Q)\right] +$$

$$\left\{z_\alpha(u_\alpha\cdot\nabla z_\alpha)\nabla(\nabla\cdot u_\alpha) + \frac{z_\alpha^2}{2}\nabla\left[u_\alpha\cdot\nabla(\nabla\cdot u_\alpha)\right]\right\} +$$

$$\nabla\left\{\frac{\eta^2}{2}\nabla\cdot u_{\alpha t} - \eta u_\alpha\cdot\nabla Q + \eta Q\nabla\cdot u_\alpha\right\} +$$

$$\nabla\left\{\frac{\eta^2}{2}\left[(\nabla\cdot u_\alpha)^2 - u_\alpha\cdot\nabla(\nabla\cdot u_\alpha)\right]\right\} = 0$$

where : $Q = \nabla\cdot(hu_\alpha)$

Back to Main Slide

History of Depth-Integrated Approach

- What is a "depth-integrated" equation??
  - Deriving the shallow water wave equations:
    - Assumption or scaling gives:

$$u(x,z,t) \sim u(x,t)$$

$$\frac{w}{u} \sim 0$$

    - Integrate the continuity equation over the entire depth:

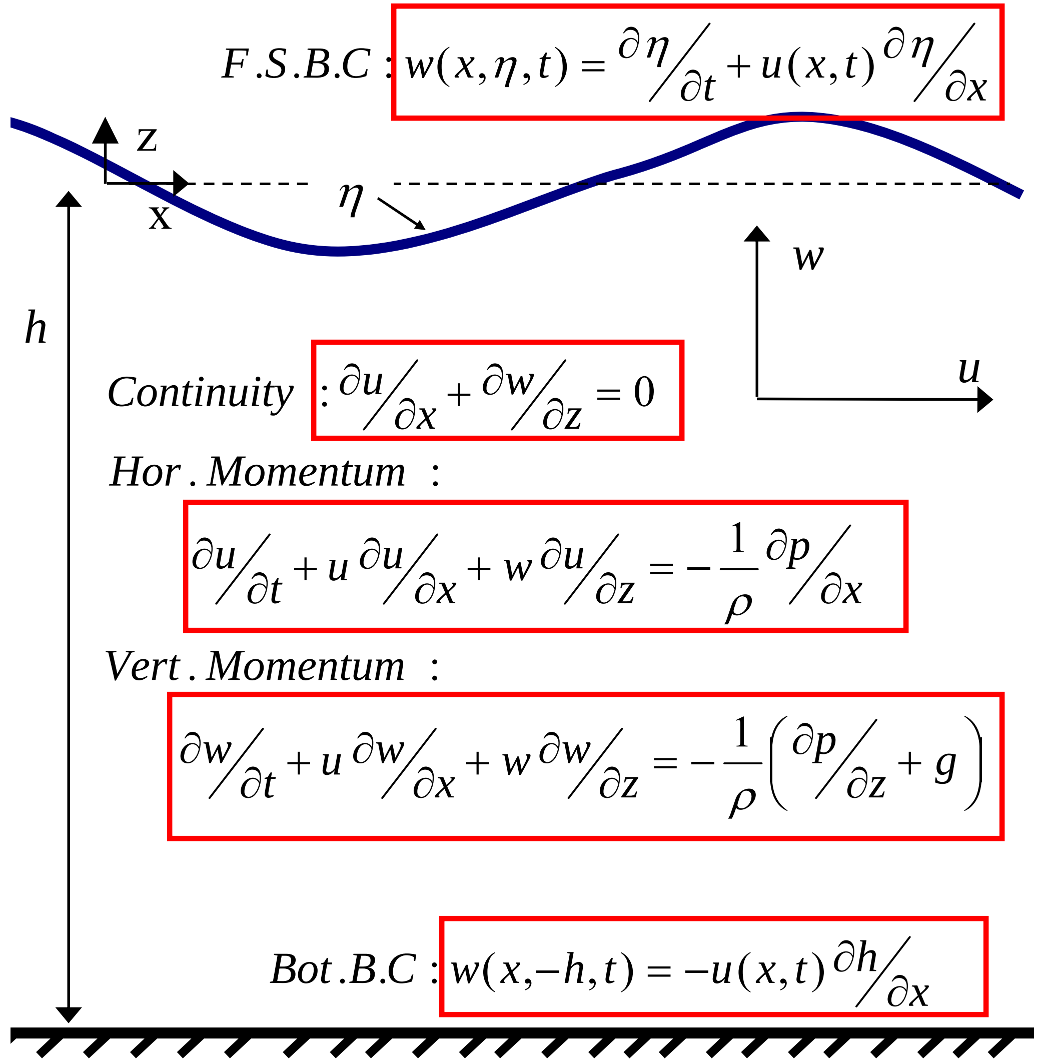
$$\int_{-h}^{\eta} \left(\partial u \big/ \partial x + \partial w \big/ \partial z \right) dz = 0$$

with the F.S.B.C, the Bot.B.C, and some calculus, we have:

$$\partial \eta \big/ \partial t + \partial \left[(h+\eta) u \right] \big/ \partial x = 0$$

    - Integrate the vertical momentum equation over the entire depth to find pressure, p, then substitute expression for p into horizontal momentum equation, giving:

$$\partial u \big/ \partial t + u \, \partial u \big/ \partial x + g \eta = 0$$



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