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Mathematical Dynamics of Nystagmus in an Oculomotor Neural Network Model

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Introduction

- **Cranial Nerve III (CN III)** directly influences extrinsic oculomotor muscle, eyelid, **lens movement**, and gaze fixing.
- **Nystagmus** is the **involuntary motion** of one or both eyes resulting in horizontal, vertical, or rotary movements

Damage to CN III → Abnormal Eye movements → Nystagmus

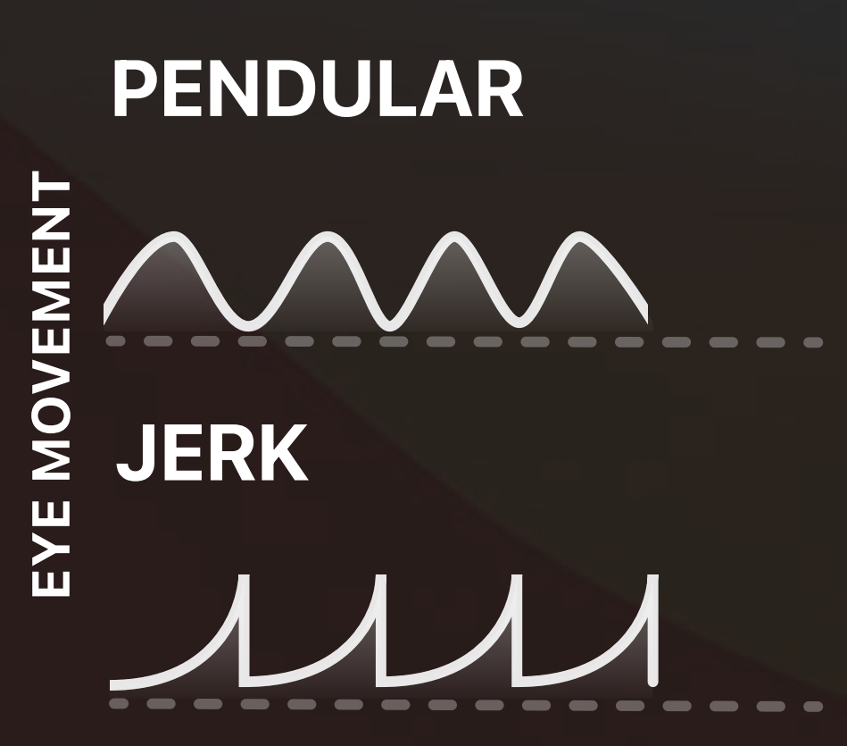


Figure 1: Pendular vs Jerk Nystagmus
(Papageorgiou, E.; McLean, R. J.; Gottlob, I.)

www.sciencedirect.com/science/article/pii/S187595721400103X

- Previous studies have produced theoretical models of the oculomotor nerve and cortex
  - Little effort has been made to focus these networks towards nystagmus
- We created simplified models **replicating the oculomotor cortex** in Python
- The core of our model is a network of **lateral and self-inhibiting neurons** that act as a **temporal integrator**
  - Prior studies show neurons in the oculomotor system have firing patterns for both **eye velocity and position**
  - This integrator generates eye-position commands from eye-velocity signals
- We model nystagmus as a series of **lesions** and **weakened connections** between neurons

Figure 2: Cranial Nerve III
Oculomotor Nerve shares connections with CN IV, CN VI, and CN VIII

www.biologycorner.com/anatomy/nervous/cranial_nerves_coloring.html

Results

The graph below represents the paths through condensed space of a 32-neuron network for a range of input spike sizes, applying dimensionality reduction to simplify a 32-dimensional trajectory while preserving its most significant features.
This enables visualization of the integrator network's line attractor dynamics: attraction towards a 32-dimensional 'line' of coordinates on which every trajectory ends, where each coordinate is a weighted sum of neuron outputs at that time step.

The shift from red to blue represents the shift from smaller to larger firing rate spike ratios in the inputs.

Figure 5: Trajectories of 32-neuron networks in dimension-reduced space for varied spike sizes

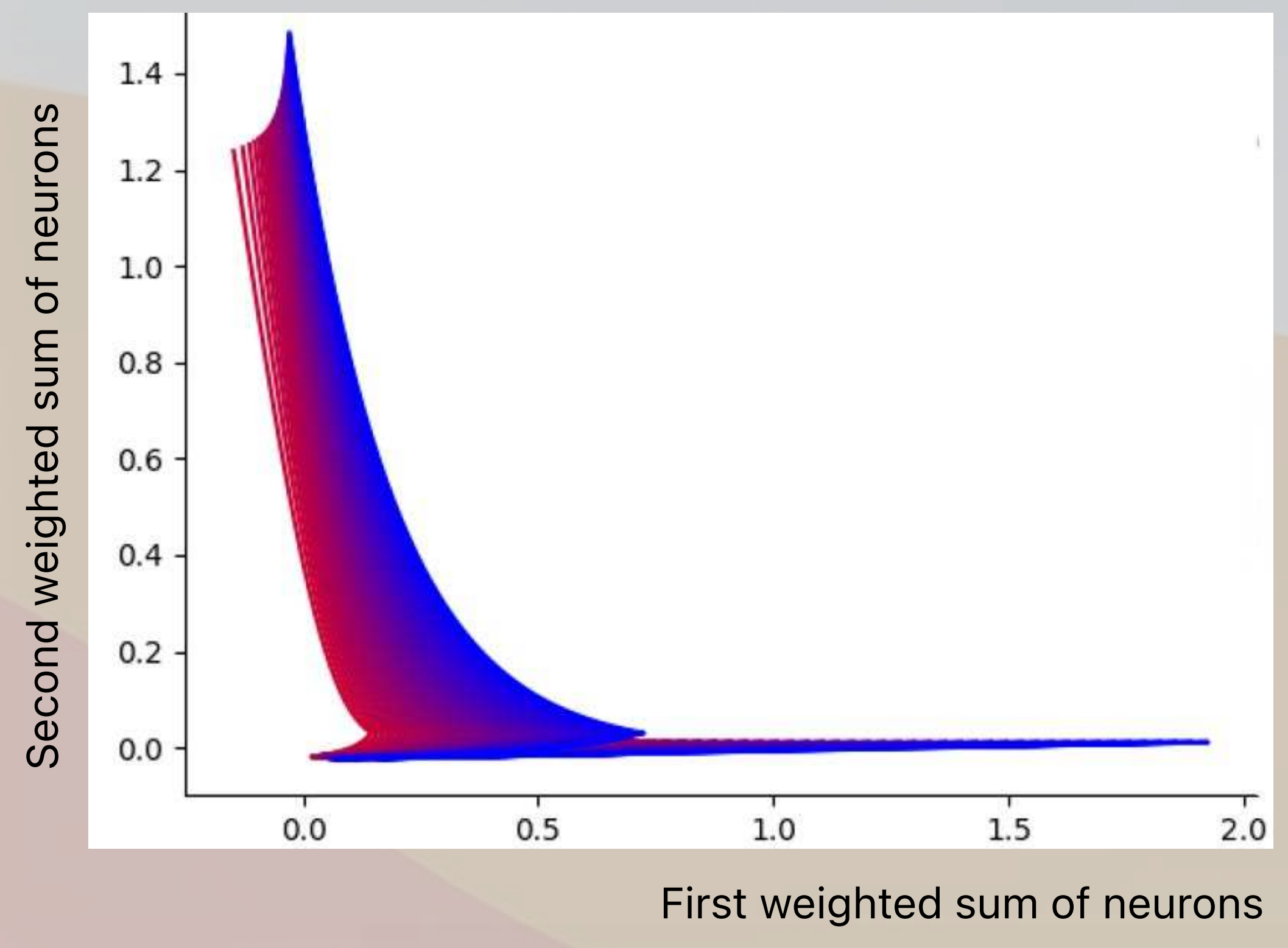
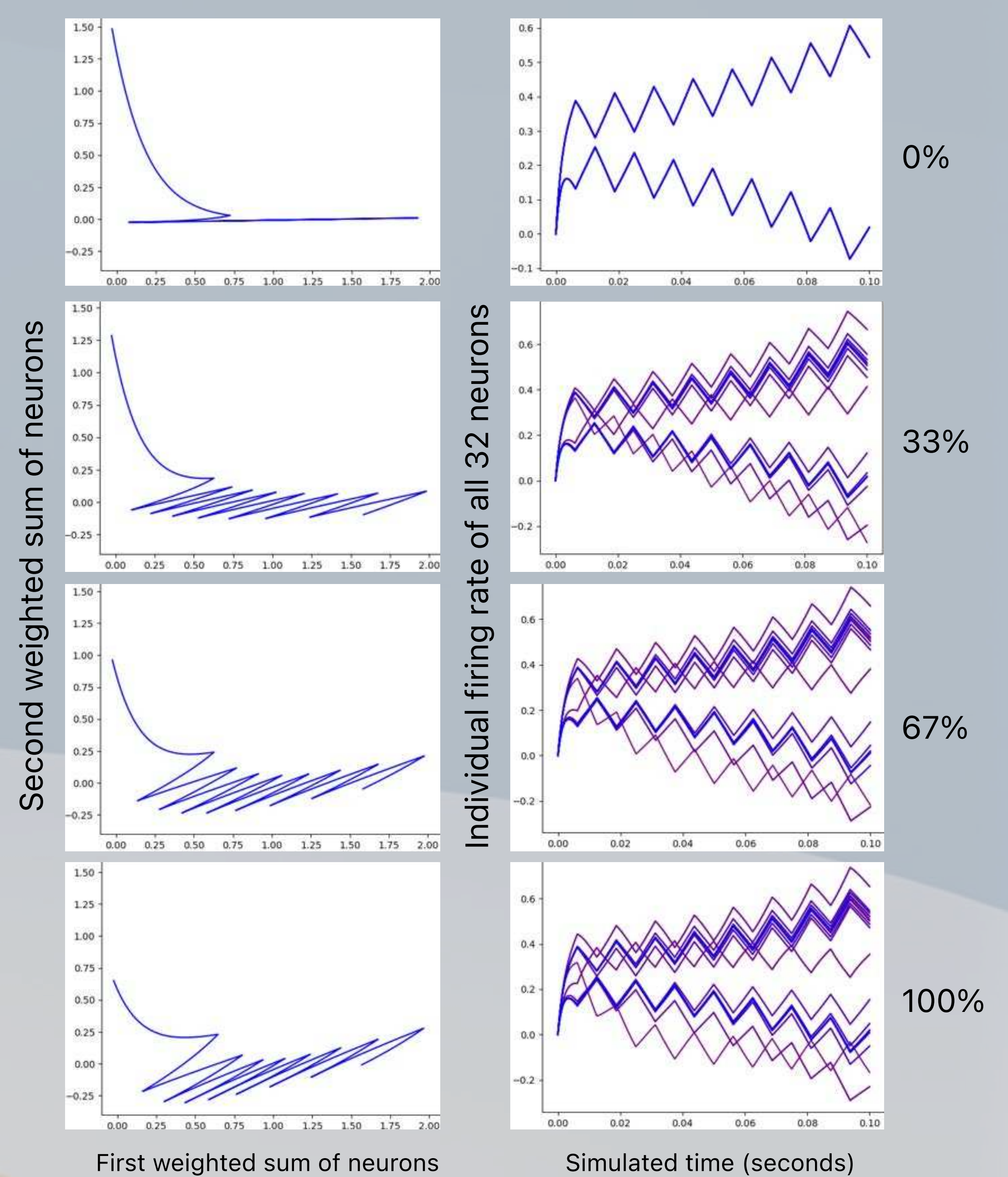


Figure 6: Lesioned network trajectories and firing rates



The graphs above use the same methodology to explore the dynamics of a specific network under lesions. As damage — the percent by which a targeted neuron's connections are weakened — increases from top to bottom, the network's trajectory (left) becomes less 'smooth' but never deviates from the original line, illustrating an abstraction of jerk nystagmus. The firing rates (right) of individual neurons, color-coded to differentiate their position in the network, mirror this destabilization.

Note: Although not shown here, plots in phase space prove all networks below exhibit consistent line attractor dynamics regardless of lesions.

Conclusion

Done

Summary

- Developed **simplified oculomotor cortex** to study healthy and nystagmus-affected individuals
- Modeled **32 neurons**, with potential for more
- More than lens movement, the oculomotor system integrates **complex inputs** like head/neck position, hearing, and smell

Future Growth

- Did not assume self-inhibition and neighbor inhibition less than distant neuron inhibition
- Second layer for more **biophysical accuracy** (feedback/cluster neurons to split inputs)
- Dimensionality reduction (PCA) simplifies data, limiting practical application

Future Research

- Incorporating visual inputs, testing motion stimuli
- Model adaptation for precise robotic control movements
- Predicting drug effects on nystagmus treatment
- Using more advanced processing beyond dimensionality reduction

Creating the model

The Model (Cannon et. al.)

- The response of the i^{th} neuron depends on:
  - **Firing rate** of j^{th} input (u_j)
  - **Weight of input** on neuron (v_{ij})
  - Weight of lateral inhibition by the j^{th} neuron (w_{ij}) or of self inhibition (w_s)
- Inhibition **weakens with distance**
- The **time constant** (τ) of an individual neuron is ~5ms, based on empirical data
  - **Integrator's time constant** (T_n) ~20s

Inputs

- **Spatial frequency** (P) represents diversity of input arrangements across neurons
- $P = 0$: Neurons receive identical inputs from one source
- $P_{max} = 0.5$ cycles/neuron: Half the neurons get input from one vestibular nucleus, and the rest from the second nucleus

Lesions

- Lateral inhibition network (**LIN**) modeled by **removing lateral inhibition** from one neuron
  - Tested integrator's response by combining neuron responses
  - Network's response dropped **rapidly** after input, then decayed like a healthy network
  - Neurons near the lesion lost positive feedback, distant neurons ones less affected

Figure 3: LIN of two neurons
u_1 & u_2 are input firing rates, and u_1 & u_2 are output firing rates. (Cannon et. al., 1983)

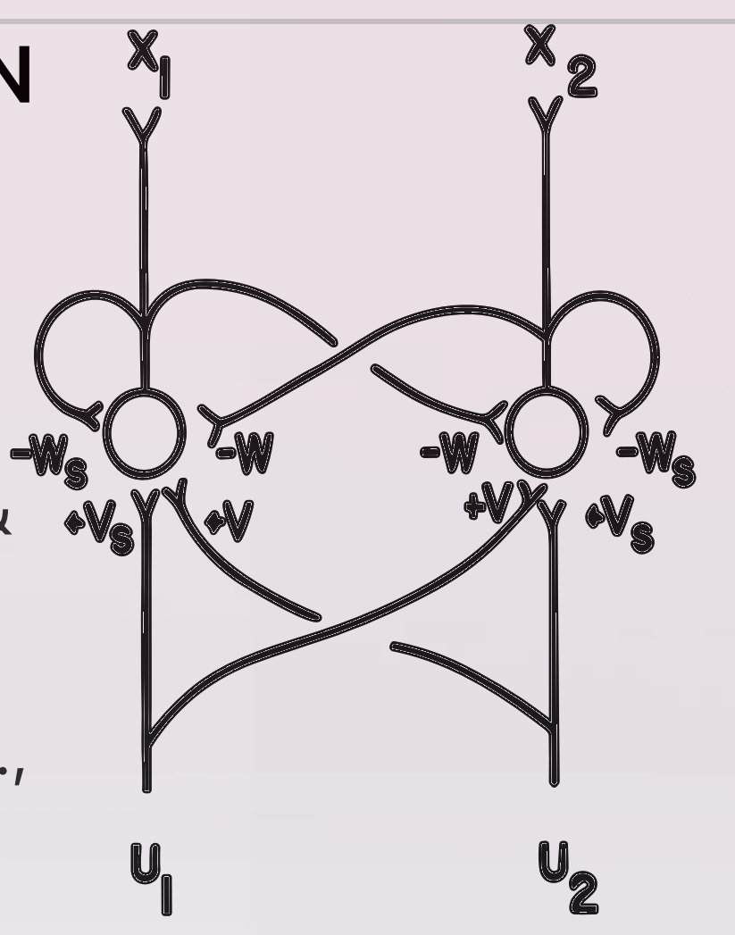


Figure 4: Lateral inhibition profile visualization

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