Towards Design Guidance of Reservoir Computing — An Empirical Study

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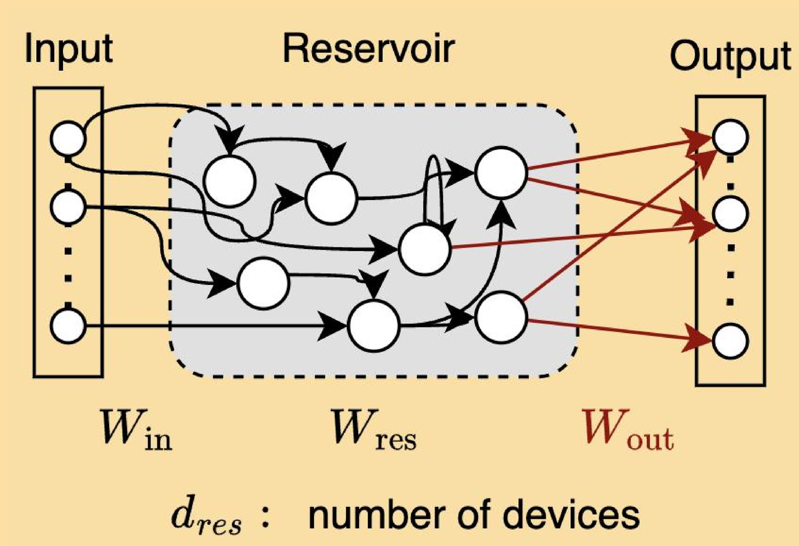
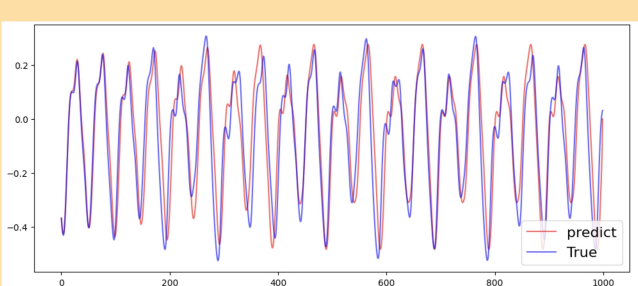
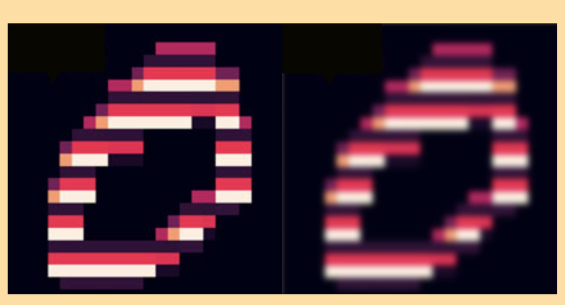
Backgroud

- **Reservoir Computing (RC)** has emerged as a versatile machine learning technique.
  - Recurrent Neural Network (RNN)
  - Reservoir Setup: input layer, reservoir, and output layer.
  - Reservoir Dynamics: The reservoir neurons create complex temporal representations of the input.
  - Training: Set the reservoir's weights, only the weights connecting the reservoir to the output layer are trained.
  - Testing: Task-specific learning



- Finding applications in a wide range of domains,
  - e.g., Time Series Prediction, Pattern Recognition and Image Processing, Wireless Network, Brain-Computer Interfaces (BCI)...





Motivation

Nowadays algorithm designers DON'T care about .... Design

While Reservoir Computing has gained widespread popularity for its versatility in diverse applications, there remains a lack of

- Limited understanding of the algorithm's setting
- The intricate interplay between various model components.
- Comprehensive investigations into the impact of key design choices on model performance.
- Design Factors for more conprehensive analysis, including **activation functions**, **initialization techniques**, **parameter settings**, and **data types**. Our aim is to provide valuable guidance to algorithm designers who are considering the adoption of RC in their applications.

Experiment

To facilitate our study, we create a robust testing environment capable of evaluating RC performance under diverse parameter configurations. Specifically, data is generated from a compact **Echo State Network (ESN)** model to provide an in-depth examination of how different activation functions affect the reservoir's capabilities.

- ESN_0: small scale linear reservoir model; data generator
- x_in: random input signal
- y_out: target signal
- Study how error change with different parameters

Activation Function   Randomly Given

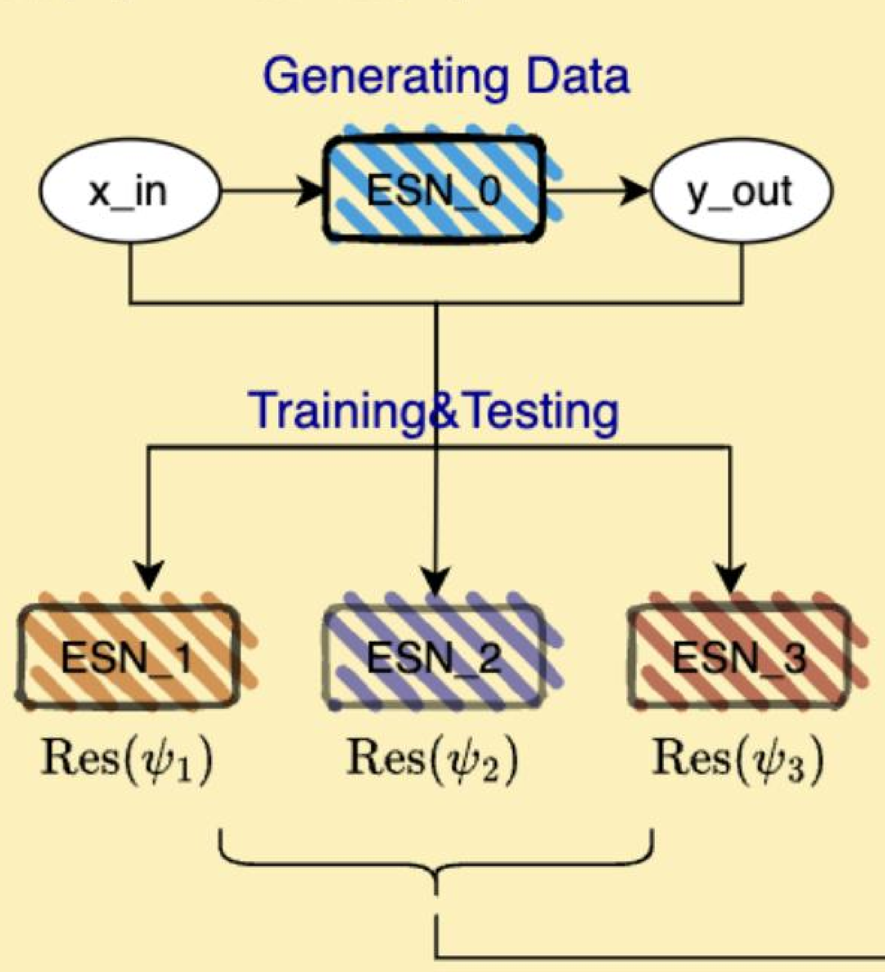
$$X_{\text{res}}[m] = \psi(W_{\text{res}} X_{\text{res}[m-1]} + W_{\text{in}} X_{\text{in}}[m-1] + b_{\text{in}})$$

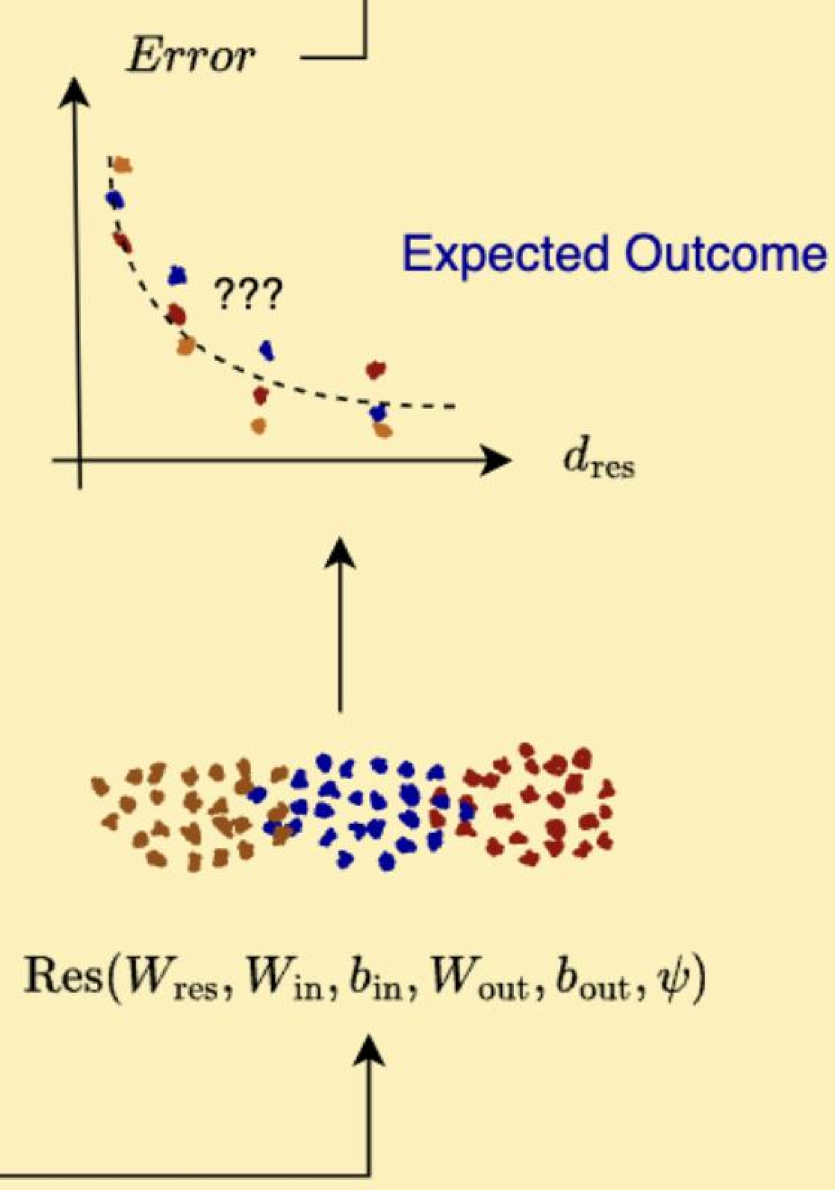
$$\psi_1 : x \mapsto x$$
$$\psi_2 : x \mapsto tanh(x)$$
$$\psi_3 : x \mapsto relu(x)$$
$$relu(x) = max(x, 0)$$

$$X_{\text{out}}[m] = W_{\text{out}} X_{\text{res}}[m]$$

$$\min \frac{1}{N} \sum_{l=1}^{N} \|Y^{(i)} - W_{\text{res}} X^{(i)}\|^2$$

Error

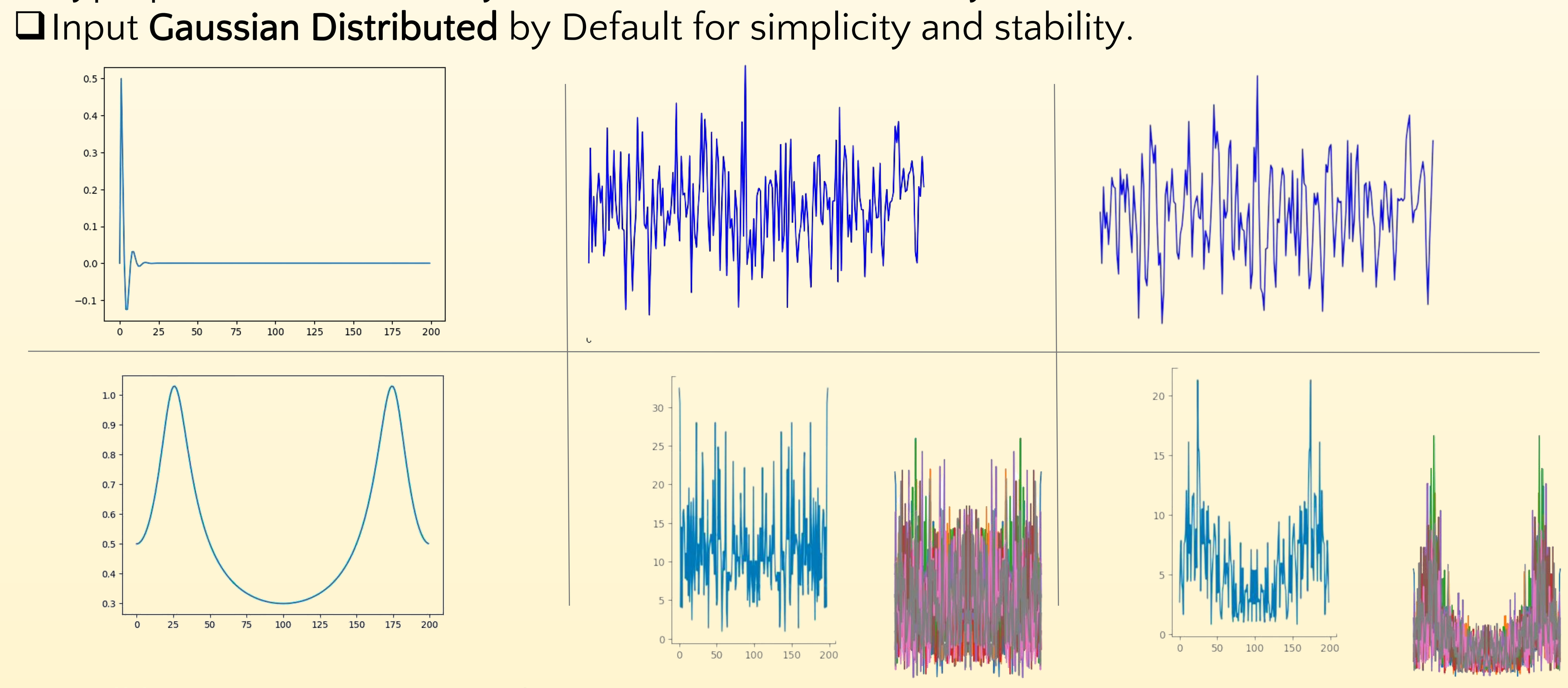




Results

❖ESN_0, Impulse Response & Frequency Response

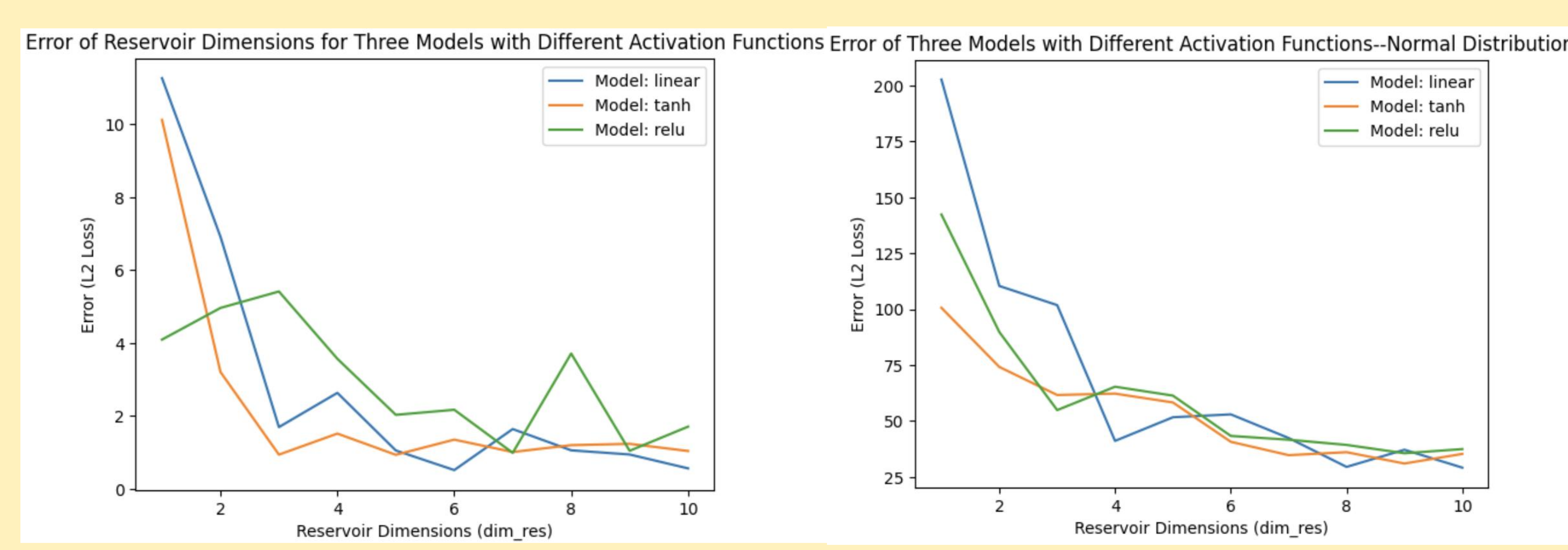
- Here we use default dimension of 1 for input and output.
- Using **Linear Input Transformation (LIT)** as ESN_0 simplified initialization,reduced hyperparameter sensitivity and maintained Consistency Across Tasks:
- Input **Gaussian Distributed** by Default for simplicity and stability.



- In all linear system, an **impulse response** represents how ESN_0 reacts to an instantaneous change or impulse in the input signal
  ⟹ how the system's behavior evolves over time.
- **impulse_response = ESN_0(impulse_input)**
  - The impulse response inherently contains frequency components involved in the system's response.
- **freq_response = FFT(impulse_response)**
  - The **spectrum** of a signal refers to the distribution of its frequency components.
  - Utilize **Fast Fourier Transform (FFT)** to uncover the spectrum of the impulse response of ESN_0.

❖Experiment of Different Parameters (Activation Function, Weight Initialization, Reservoir Dimension)

- 10,000 of total Test Cases of Length 1,000; 10 Epochs
- controling variable: generated training data from ESN_0
- Dimension of reservoir (d_res) from 1-10



- We set weights and freeze the value of W_res through normal distribution and redo the experiment; we devided the result according to the dimension of reservoir and sqeezing its size.

Conclusion

- The error is decreasing as dimension of the reservoir (d_res) increases, which indicates **Capacity Increase** and **Better Representation.**
- The optimal choice of activation function relies on on the complexity of the reservoir.
  - **Linear activation** might be well-suited for **lower dimensions** where simpler transformations suffice.
  - **ReLU** might be more effective for **larger dimensions** to capture more intricate patterns.
  - The **consistence** of **tanh** function could indicate its **balanced performance** across dimensions -> tanh might be a reliable choice when the optimal activation function isn't apparent.
- The significant error expansion due to normal distribution initialization underscores the importance of **weight initialization** in reservoir computing.
- This platform we build can be a paradigm for further investigation about Reservoir Computing, including impacts of data scale, optimization methods and generalization errors.

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